International Journal of Progressive Education, Volume 18 Number 4, 2022

Comparative Analysis of The History of Mathematics Content in The Secondary School Mathematics Textbooks of Turkey, Singapore, Ireland and Canada

Nazan Mersin [i]
İstanbul Medeniyet University

Mehmet Akif Karabörk [ii]
Bolu Abant İzzet Baysal University

Abstract

This study seeks to offer a comparative analysis of the History of Mathematics (HoM) elements identified in the secondary school mathematics textbooks of different countries. Drawing on document analysis method, this study analyzes the secondary school mathematics textbooks of Turkey, Singapore, Ireland and Canada. The HoM elements in the textbooks are examined in terms of famous mathematicians they present, civilization they are related to, content type, associated learning area and whereabouts they are inserted in the text. This study concludes that the textbooks of Ireland present the highest number of HoM elements quantitatively, and Ireland is followed by Turkey, Singapore and Canada, respectively. The most mentioned mathematicians in the HoM elements are Al-Khwarizmi and Pythagoras; further, the most mentioned civilization is Ancient Egypt. Further, Singapore and Canada prioritize discussion-project whereas Ireland and Turkey focus on history of concepts. Moreover, Turkey, Ireland and Canada present the highest number of HoM elements in the learning area of geometry and measurement. Singapore has the highest number of HoM elements in the area of numbers and operations. This study reveals that the countries do not sufficiently incorporate HoM into their textbooks. The countries with relatively higher number of HoM elements like Ireland use HoM for motivational purposes, rather than for teaching purposes.

Keywords: History of Mathematics Textbooks, International Comparison, Comparative Education Mathematics Education

DOI: 10.29329/ijpe.2022.459.5

----

[i] **Nazan Mersin,** Asst. Prof., Mathematics Education, İstanbul Medeniyet University, ORCID: 0000-0002-4208-3807

Correspondence: nazan09gunduz@gmail.com

[ii] **Mehmet Akif Karabörk,** PhD Student, Mathematics Education, Bolu Abant İzzet Baysal University

INTRODUCTION

Offering rich opportunities for learning and teaching, in addition to all materials and resources, textbooks play an invaluable resource, designed for use by both students and teachers, as well as an effective tool and a major support (Amaral et al., 2014; de Almeida &da Silva, 2019; Fan et al., 2013; Glasnović Gracin & Jukić Matić, 2016; Ishii, 2019; Jahnke, 2019; Kajander & Lovric, 2009; Kim, 2014; Lin & Tsai, 2014; Meinerz & Doering, 2019; Yang & Sianturi, 2017). Textbooks that can be used as a guide in designing and presenting courses (Gueudet & Trouche, 2009; Xenofontos & Papadopoulos, 2015) not only provides teachers and students with the interpretation of the curriculum of the course, but also offer an organized array of ideas and information to the learning process with structured instruction to guide readers to understand, think and feel and to allow them to access information (Meinerz & Doering, 2019).

It is reported that textbooks have a great impact on classroom work and form the backbone of mathematics teaching (Fan et al., 2013; Kajander & Lovric, 2009; Yang & Sianturi, 2017). Textbooks guide systematically building the bridges between curricula, teachers, students and teaching environments (Valverde et al., 2002). For guiding purposes, the elements in the textbook play a key role in incorporating the concepts and skills in the curriculum into learning environments and in helping teachers plan the practices that will shape learning environments (Fan et al., 2013). Similarly, the fact that mathematics textbooks are important materials in the construction of mathematical knowledge through enumeration, presentation and explanation of mathematical concepts and problems (O'Halloran et al., 2018) and in the prediction of student performance in mathematics implies that it has a potential of being a factor that greatly affects student achievement (Zhu & Fan, 2006).

Given their importance as teaching and learning tools in mathematics, mathematics textbooks have become a core subject of research in recent years (Lin & Tsai, 2014; Schubring &Fan, 2018; Senk et al., 2014). Indeed, ZDM published two special issues: "Textbook Research in Mathematics Education" and "Recent Advances in Mathematics Textbook Research and Development" in 2013 and 2018, respectively. England, Brazil and Germany organized the International Conference on Mathematics Textbook Research and Development (ICMT), in 2014, 2017 and 2019, respectively. One of the research themes regarding textbooks in these conferences and issues is analysis of textbooks.

Analysis of textbooks, which mostly shape what and how to teach in the learning and teaching process, potentially provides insight into the causes of differences in student achievement (Reys et al., 2004; Zhu and Fan, 2006). Accordingly, student performance in international comparative studies and examinations such as PISA and TIMSS can be explained through analysis of mathematical textbooks in different countries (Ding and Li, 2010; Li, 2000; Zhu and Fan, 2006). Analysis of textbooks can reveal different performance expectations of students in different countries, the extent to which textbooks in a country prioritizes conceptual understanding or procedural fluency and how the treatment of mathematical content differs among countries (Charalambous et al., 2010). Indeed, there is little disagreement over the great influence of textbooks on learning, including problem contexts, types of problems, and the order of presenting concepts (McNeil et al., 2006). This led the researchers to compare the mathematics textbooks of countries with higher success in international exams such as PISA and TIMSS and those with lower success. Also, textbooks are concrete tools, widely used by teachers and students, that reflect cultural and educational traditions, which spurred research to examine textbooks (Gracin, 2014). Analysis of textbooks has been performed by various researchers focusing on different aspects. For example, textbooks were compared in terms of technology integration (Mersin and Karabork, 2021), problems and sample solutions (Toprak and Ozmantar, 2019), and discussion of mathematical concepts such as fraction, probability, function, ratio-proportion (Charalambous et al., 2010; Hong and Choi, 2018). This study seeks to compare the place of history of mathematics (HoM), which affects mathematical learning in mathematics textbooks of different countries.

Why History of Mathematics?

HoM is a kind of epistemological laboratory to explore mathematics in its philosophical, scientific and social context and to understand its nature as a sociocultural product (Byers, 1982; Ju et al., 2016; Radford, 1997). Considering that mathematics progresses as a whole of the information transferred from one society to another, it is known that it is socio-culturally constructed, an endeavor greater than an individual's life and reveals the neutrality of the mathematical system through multi-cultural participation (Radford et al., 2007). Therefore, HoM can provide us with a unique experience in problematizing the moment we live in and uncovering the depth of conceptualizations shared among people living in the same period (Ju et al., 2016). These experiences enable students to acquire different perspectives on learning mathematics and to gain cognitive and affective tendencies.

One of the major benefits of teaching based on HoM is that one may have a deeper and greater understanding by experiencing the fundamentals of mathematical concepts used today, various methods and the development processes of these methods, contrary to the belief that HoM is only about memorization (Marshall and Rich, 2020). For example, as an alternative to the method stated in the curriculum in the solution of quadratic equations, the use of Khwarizmi’s completing the square method as a geometric solution will provide a different perspective for students to achieve deep and permanent learning (Genc & Karatas, 2018; Mersin & Durmus, 2020).

In fact, HoM would ensure that students, who have the opportunity to examine the challenges experienced by famous mathematicians, their successes and failures, clearly see the struggles of these mathematicians to reach the point they have come to, and that students acknowledge that failing to understand something is not an indicator of failing at math and that they need to work harder to achieve their goals.

Also, the integration of HoM elements into lessons can enable students to observe that famous mathematicians also made mistakes in mathematics but did not give up discovering mathematics and focused on success; thus, students become aware that making mistakes is normal and gain confidence that they can succeed too (Philippou & Christou, 1998).

One of the strengths of mathematics is the ability to reveal the cultural diversity of different societies in their contribution to mathematics (Fried, 2007; Swetz, 2009). For example, Fibonacci's openness to the work of Arab mathematicians and his spread of ideas to Europe served as an excellent point of departure for the work of Cardano and other mathematicians (Ju et al.2016). This shows that mathematics does not belong to a single culture, instead it draws its strength from intercultural interaction.

From another perspective, HoM can help students expand their perspectives beyond their own cultural background by showing that mathematics is a cultural hybrid between various civilizations (Grugnetti et al.2002; Jahnke et al. 2002). Thus, they can learn the strengths and weaknesses of each cultural mathematical system. Besides, this shows the dimension of mathematics that emphasizes mathematics is an intellectual human activity, and highlights those mathematicians reach these results, overcoming various difficulties and with strong endeavor and determination. Further, they would realize that mathematical and scientific information should be evaluated within the conditions of the period when they were developed. Achieving all these should not be left to chance or any teacher’s knowledge and effort, and textbooks need to encourage these aspects.

Rationale for the Study

Considering the importance of mathematics textbooks for both students and teachers in teaching mathematics, textbooks are utilized to benefit from HoM, which caused the need to examine the qualities of HoM content in textbooks. Indeed, it is claimed that HoM content to be integrated into mathematics textbooks must contain certain features (Wang and Yang, 2015). For example, HoM information in textbooks must be correct, objective and closely related to mathematical content; HoM must be represented with different periods, countries, nations and cultures, and must be appropriate to the cognitive abilities of students, interesting and unique. HoM can be presented in textbooks in different ways and for different purposes considering these features. There are some studies in the literature that examine HoM elements in mathematics textbooks based on various categorizations.

Various studies have been carried out on the national and international platform, where the content of the history of mathematics in the textbooks is examined (Baki & Butuner, 2013; Ekawati et al., 2018; Erdoğan et al., 2015; Ju et al., 2015; Incikabi et al., 2019; Mersin & Durmus, 2018; Sahin et al., 2016; Schorcht, 2018; Smestad, 2003; Tan Sisman & Kirez, 2018 Xenofontos & Papadopoulos, 2015). These studies analyzed HoM elements under the categories such as where it is presented, learning area, content, purpose, the mathematical skill targeted, cognitive skills, teaching method, ways of use, grade level and frequency of use in textbooks. In general, they indicated that HoM elements in the textbooks were not sufficient and were mostly in the form of historical snippets. On the other hand, all these studies focused only on the textbooks used in teaching secondary school mathematics in Turkey. There has been no study that compares and reviews the HoM elements in the secondary school mathematics textbooks of Turkey and the textbooks of other countries. Given that mathematics has cumulatively developed through interaction between civilizations, in other words, mathematics is a cultural heritage, it is expected that this heritage is reflected in the textbooks of different nations. Besides, HoM serves as an alternative and complementary tool in mathematics teaching, encourages positive attitudes and motivation towards mathematics and eliminates racial discrimination; given all these advantages of HoM, different countries are expected to include HoM elements in their mathematics textbooks. Indeed, the textbooks of some countries such as Germany, Greece, China, Korea, Indonesia and Norway, have been examined for HoM elements, as mentioned above. In this regard, in the present study, it is aimed to compare the HoM content in middle school mathematics textbooks of Turkey and Singapore, Ireland and Canada, which are located in different continents, have different cultures and different success levels.

Research Questions

This study is designed based on two key research questions. These questions focus on the similarities and differences in how HoM elements in the secondary school mathematics textbooks of Turkey and of Singapore, Ireland and Canada are presented (content type, when it is presented in the book, and learning area). These research questions are as follows:

1.How does the way the HoM elements in the secondary school textbooks of Turkey, Singapore, Ireland and Canada are presented vary?

a. How do HoM elements vary by their references to famous mathematicians?

b. How do HoM elements vary by their association with civilizations?

c. How do HoM elements vary by content type (their presentation)?

d. How do HoM elements vary by the place where it is presented in the book (whereabouts they were inserted in the text)?

e. How do HoM elements vary by their associated learning area?

METHOD

This study, which analyzes and compares the HoM elements in the secondary school textbooks of Turkey, Singapore, Ireland and Canada, draws on document review, one of the qualitative research methods. Document analysis, closely related to thematic analysis and content analysis, involves an iterative process of carefully examining, reviewing and interpreting content, recognizing patterns within the data, shaping emerging themes and creating categories (Bowen, 2009). This study uses textbooks as a resource for data analysis.

Selection of Turkey, Singapore, Ireland and Canada Textbooks

Singapore, Ireland and Canada countries are on different continents (Asia, Europe, America), they have different cultural structures, they show higher results in international exams such as PISA and TIMSS, and English-language mathematics textbooks are easily accessible, which are the reasons for their preference in this study.

Table 1. Textbooks Included in This Study

Country	Textbooks
Turkey	Middle School Mathematics Textbook 6, MoNE Publishing, 2019
	Middle School Mathematics Textbook 7th grade, MoNE Publishing, 2019
	Middle School Mathematics Textbook 8th grade, MoNE Publishing, 2019
Singapore	New Syllabus Primary Mathematics 6A-6B, Shing Lee, 2018
	New Syllabus Mathematics 1(7th grade), Shinglee, 2018.
	New Syllabus Mathematics 2(8th grade), Shinglee, 2018
Ireland	Exploring Project Maths Book 1(7th grade), CJ Fallon Ltd, 2013
	Exploring Project Maths Book 2(8th and 9th grade), CJ Fallon Ltd, 2013
Canada	Math Make Sense 7, Pearson, 2007
	Math Make Sense 8, Pearson, 2008
	Math Make Sense 9, Pearson, 2009

Data Analysis Framework and Process

Prior to the comprehensive analysis of the HoM elements in the secondary school mathematics textbooks of Turkey, Singapore, Ireland and Canada, the textbooks were analyzed in general to find out how much they cover HoM elements. It was found that the mathematics textbooks of Turkey, Singapore, Ireland and Canada contained HoM elements in 26 (3,4%), 30 (3,6%), 48 (6,7%) and 17 (1,2%) of their pages, respectively.

To analyze the identified HoM elements, a framework was formed by the researchers based on the studies in the literature (Ekawati et al., 2018; Incikabi et al., 2019; Mersin and Durmus, 2018; Riley, 2018; Sahin et al., 2016; Tan Sisman and Kirez, 2018; Tzanakis et al., 2002).

Accordingly, the HoM elements in the textbooks were analyzed to reveal how they were presented. To determine how they were presented, as stated in the first research question, the references to famous mathematicians in the HoM elements were examined. Considering the effect of the life stories of famous mathematicians, their determination in the process of scientific discovery and the impact of their contribution to science on student motivation, it is important to reveal how much these are presented in the textbooks.

Secondly, the HoM elements were analyzed to determine whether they emphasize any civilization or not. Throughout the history, many civilizations influenced the development of mathematics. Due to the fact that some countries do not have a single ethnicity, their textbooks present the contributions of various nations, which perhaps helps students embrace the lesson more and prevents cultural nationalism that is hypothetically caused by HoM. Thus, the HoM elements were reviewed for emphasis on any civilization.

Thirdly, regarding the first research questions, content type was examined to reveal how the HoM elements were presented. It is reported by various researchers that HoM can be included to various different contents in the textbooks (Gulikers & Blom, 2001; Jankvist, 2009; Man-Keung, 2000; Sahin et al., 2016; Tan Sisman & Kirez, 2018; Tzanakis et al., 2002; Wang & Yang, 2015).

A framework was designed to classify the HoM elements in terms of content type, considering the studies by Tan-Sisman and Kirez (2018), Şahin et al. (2016), Xenofontos and Papadopoulos (2015), Tzanakis et al. (2002). This framework for content classification consists of the following categories: basic historical biographical information, formula-rule-solution, history of concepts, historical work, tools-equipment, discussion-project and game. While classifying the HoM elements, it was seen that some elements could fall into multiple categories. The HoM elements in the textbooks were lastly analyzed to determine whereabouts they were inserted in the text.

Under the theme of how HoM elements are presented, the first category that deals with the question of when is where abouts these elements were inserted in the text. Accordingly, HoM elements were presented either in the introduction section of the subject or as a part of the subject itself, or at the end of the subject/assessment sections, or individually, outside the subject. The framework used for this was designed based on the framework utilized by Incikabı et al.(2019), Tan-Sisman and Kirez (2018), Mersin and Durmuş (2018), Şahin et al. (2016) for the classification of HoM elements according to whereabouts they were inserted.

Another category is the learning areas where HoM elements were frequently used. It was observed that HoM elements were used in similar learning areas with common names in the countries included in this study. For that reason, HoM elements were classified according to the learning areas of numbers and operations, algebra, geometry and measurement, data processing and probability, with reference to the learning areas in Turkey.

Below is a summary table on the classification used in this study.

Table 2. Analysis of the Framework for Study

Theme	Question answered	Category	Sub-category
How HoM elements are presented	How	Famous mathematicians	
		Civilizations	
		Content Type	Biographical information Formula/Rule/Solution History of concepts Historical work Tools-equipment Discussion/Project Game
	When	Whereabouts they were inserted	Introduction As a part of the subject At the End/Assessment Off-subject
		Learning area	Numbers and Operations Algebra Geometry and Measurement Data Processing Probability

Reliability and Validity

A framework for analysis was first developed for a valid and reliable identification of the HoM elements in mathematics textbooks. Then, the relevant literature was examined and categories were determined by the researchers, who reached a consensus. Following that, 12 textbooks used in Turkey, Singapore, Ireland and Canada were coded according to these categories by two different researchers separately; the researchers compared their coding and reached a consensus on the codes they first disagreed. The agreement percentage between these two coders was found to be 81.68% according to (Miles and Huberman, 1994). This implies that the coding was reliable.

FINDINGS

The findings of this study, which seeks to compare the HoM elements in the secondary school mathematics textbooks, are presented as sub-problems. To facilitate the comparison of the findings, each finding for each sub-problem was proportioned with the number of tasks and sections that contained HoM elements in each country's textbook and expressed by percentage. Table 3 shows the total number of the HoM elements for each country. The percentages given in the sub-problems were calculated based on this number. When the sub-problem included another comparison, it was proportioned within itself.

Table 3. Total Number of HOM Elements in Secondary School Math Textbooks

	TR	IR	SGP	CAN
Total elements	38	50	27	16

As seen in Table 3, Ireland (IR), with 50 elements, had the highest number of HoM elements in their secondary school mathematics textbooks whilst Canada (CAN), with 16 elements, had the lowest number of HoM elements. On the other hand, Turkey (TR) presented 38 HoM elements and Singapore (SGP) presented 27 HoM elements in their secondary school mathematics textbooks.

How does the way the HoM elements in the secondary school textbooks of Turkey, Singapore, Ireland and Canada are presented vary?

How do HoM elements vary by their references to famous mathematicians?

The findings, which include a large number of names, are summarized with multiple tables. Table 4 demonstrates the number of mathematicians and the number of the HoM elements in each country's mathematics textbooks by frequency and percentage. The percentages given in Table 5 and Table 6 were calculated based on the total number of mathematicians in Table 4.

Table 3. Total Frequency of Famous Mathematicians

	TR	IR	SGP	CAN	Mean
Total famous mathematicians	27	42	25	12	26.50
Number of elements include famous mathematicians	27	27	22	12	21.75
Mathematicians per elements	1	1.55	1.14	1	1.17
Percentage to total elements	%71.05	%54.00	%81.48	%75.00	%70.38

Table 4 indicates the percentage of the total number of elements of each country presented by famous mathematicians. With 81.48%, SGP had the highest percentage; with 54.00%, IR had the lowest percentage. The countries were compared by frequency and average percentage, which revealed that IR, whose the number of HoM elements containing famous mathematicians was above average, was proportionally below the average; CAN, whose the number of HoM elements was below average, was proportionally above the average.

The famous mathematicians were specially examined and it was found out that some of them were commonly presented in elements in multiple textbooks, some were included only in the textbooks of a single country. The famous mathematicians in Table 5 were mathematicians commonly presented in the math textbooks of multiple countries.

Table 4. Mathematicians Commonly Presented in Multiple Countries

	TR		IR		SGP		CAN	
	N	%	N	%	N	%	N	%
Descartes	1	3.70	2	4.76	1	4.00	1	8.33
El-Khwarizmi	4	14.81	3	7.14	1	4.00	1	8.33
Pythagoras	2	7.41	2	4.76	2	8.00	3	25.00
Archimedes	1	3.70	1	2.28	2	8.00	-	-
Euclid	1	3.70	4	9.52	1	4.00	-	-
Fibonacci (Leonardo of Pisa)	3	11.11	2	4.76	1	4.00	-	-
Eratosthenes	3	11.11	2	4.76	-	-	-	-
Robert Record	1	3.70	1	2.38	-	-	-	-
Thales	1	3.70	1	2.38	-	-	-	-
Diophantus	1	3.70	-	-	1	4.00	-	-
Cristian Goldbach	-	-	1	2.38	1	4.00	-	-
Total	18	66.67	19	45.24	11	44.00	5	41.67

The data in Table 5 on the comparison of mathematicians commonly presented in the textbooks show that TR with 66.67% proportionally presented these common mathematicians most whereas CAN with 41.67% proportionally presented them the least frequently. The most commonly presented mathematician among these mathematicians was Al-Khwarizmi (14.81%) in TR, Euclid (9.52%) in IR, Pythagoras and Archimedes in SGP (8%) and Pythagoras in CAN (25%). 1 one of 11 commonly presented mathematicians was not included in the textbooks of both TR and IR; on the other hand, SGP and CAN did not include 3 and 8 of these mathematicians, respectively.

Table 6 shows the findings on the mathematicians other than these common mathematicians.

Table 5. Famous Mathematicians only Presented in the Textbooks of One Country

	TR		IR		SGP		CAN	
N	Mathematician	%	Mathematician	%	Mathematician	%	Mathematician	%
mentioned 3 or more times	Atatürk	11.11	Thomas Harriot	7.14	-	-	M.C. Escher (4)	33.33
mentioned 2 times	Cahit Arf, Florence Nightingale	14.81	Johann Heinrich Lambert, Edward Kasner, Brahmagupta	14.29	Blaise Pascal	8.00		-
mentioned 1 times	Albert Einstein, Ömer Hayyam	7.41	Akira Haraguchi, Bartholomea Pitiscus, George Cantor, Hipparcus, James Newman, Johannes Widmann, John Graunt, John Venn, Ludolp van Ceulen, Marin Mersenne, Ptolemy, William Jones, William Oughtred, Zu Chong	33.33	Andrew Wiles, Chevalier de Mere, Dr. Curtis cooper, Giorgio Chiarino, Girolama Cardano, Henry Dudeney, Olivera de Silva, Pierre Fermat, Leonardo da Davinci, Lonenzo Mascheoni, Newton, Sir William Petty,	44.00	Dr. Edward Doolittle, Theodorus, Sierpinski	25.00
Toplam	9	33.33	23	54.76	13	52.00	7	58.33

The frequency and percentage of the mathematicians only presented in the textbooks of one country were examined, and it was ascertained that CAN (58.33%) included the mathematicians other than the above-mentioned common mathematicians in their textbooks most whilst TR (33.33%) included such mathematicians the least frequently. The following mathematicians were remarkable: Atatürk in TR, Thomas Harriot in IR, Blaise Pascal in SGP and M. S. C. Esher in CAN. Figure 1 gives examples of the famous mathematicians in the textbooks.

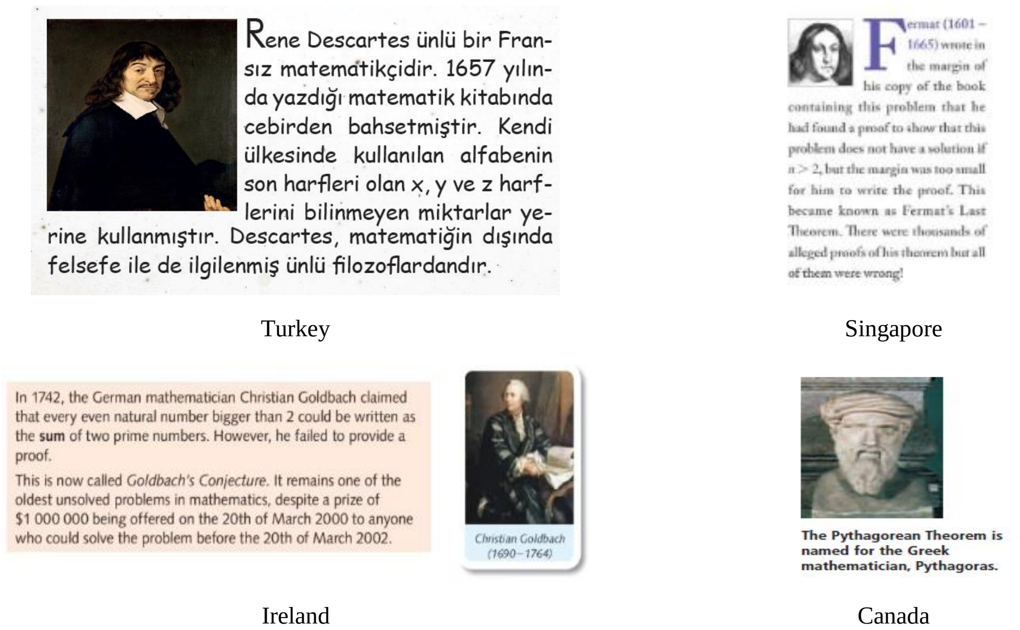


Figure 1. Examples of the famous mathematicians in the textbooks

How do HoM elements vary by their association with civilizations?

The HoM elements were analyzed by notable civilizations and the relevant findings are given in Table 7. The percentages in Table 7 were proportioned with the total number of HoM elements in Table 1. The mean values are the averages of the data presented in the table.

Table 6. Total Frequency by Civilization Emphasized in the HoM Elements

	TR	IR	SGP	CAN	Mean
Total frequency of civilization	8	37	6	4	13.75
Number of elements include civilization	7	25	4	4	10
Civilization per elements	1.14	1.48	1.5	1	1.28
Percentage to total elements	%18.42	%50.00	%14.81	%25.00	%27.20

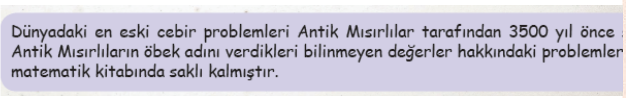
The total number of the statements that emphasized any civilization in the HoM elements was calculated; IR with 37 statements related to civilization outperformed other countries by far. It is striking that IR increased the average value; still, even if this was not the case, the country with the least emphasis on civilization was SGP (14.81%). However, it was SGP that had the highest density of elements. CAN and TR presented statements related to civilization below average percentage and frequency.

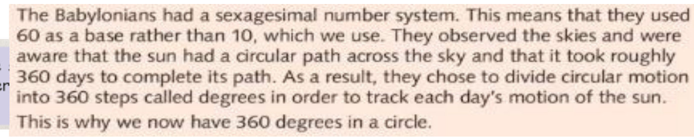
Table 8 shows which civilization was prominent in which country. The rates in this table were proportioned with the total number of the statements related to civilization.

Table 7. Distribution of the Statements Related to Civilization in the HoM Elements

Civilizations	TR		IR		SGP		CAN	
	N	%	N	%	N	%	N	%
Ancient Egypt	3	42.86	10	27.03	2	33.33	1	25.00
China	1	14.29	6	16.22	1	16.67	-	-
Babylonian	-	-	7	18.92	1	16,67	1	25.00
Ancient Greece	2	28.57	5	13.51	-	-	-	-
Indian	1	14.29	3	8.11	-	-	-	-
Latin	-	-	-	-	1	16.67	1	25.00
Roma	-	-	3	8.11	-	-	-	-
European	-	-	2	5.40	1	16.67	-	-
U.S.	-	-	1	2.70	-	-	-	-
Canada’s Inuit people	-	-	-	-	-	-	1	25.00
Total	8	100	37	100	6	100	4	100

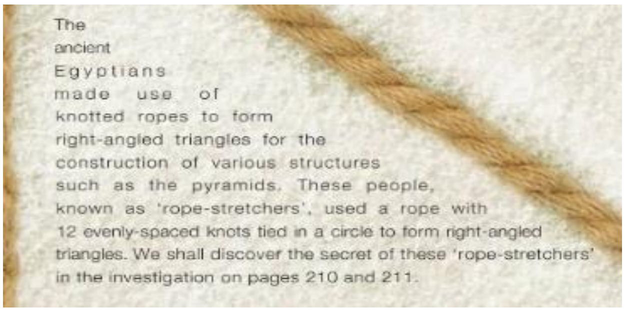
As seen in Table 8, the civilization of “Ancient Egypt” was included in the statements related to civilization of all countries whilst CAN did not present any statement related to “China.” As for Turkey, the civilization of “Babylonian” was not included. The civilization of “Ancient Greece” was the civilization included by both IR and TR; also, “Latin” was commonly available in the textbooks of SGP and CAN. The civilization of “Ancient Egypt” was the civilization most commonly included in the textbooks; the least commonly used civilizations were the civilizations only included in the textbooks of one country. CAN, exceptionally, equally expressed all the civilizations included in the HoM elements. Figure 2 demonstrates the examples of the civilizations highlighted in the textbooks.

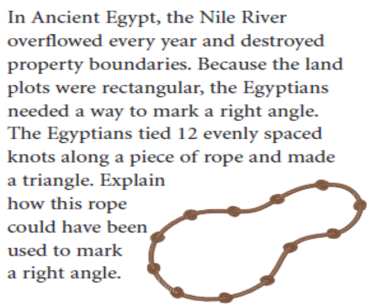

Dünyadaki en eski cebir problemleri Antik Mısırlılar tarafından 3500 yıl önce
Antik Mısırlıların öbek adını verdikleri bilinmeyen değerler hakkındaki problemler
matematik kitabında saklı kalmıştır.


The Babylonians had a sexagesimal number system. This means that they used 60 as a base rather than 10, which we use. They observed the skies and were aware that the sun had a circular path across the sky and that it took roughly 360 days to complete its path. As a result, they chose to divide circular motion into 360 steps called degrees in order to track each day's motion of the sun. This is why we now have 360 degrees in a circle.

Turkey (MoNE 6, s.155)

Ireland (EP 1, s.229)


The ancient Egyptians made use of knotted ropes to form right-angled triangles for the construction of various structures such as the pyramids. These people, known as 'rope-stretchers', used a rope with 12 evenly-spaced knots tied in a circle to form right-angled triangles. We shall discover the secret of these 'rope-stretchers' in the investigation on pages 210 and 211.


In Ancient Egypt, the Nile River overflowed every year and destroyed property boundaries. Because the land plots were rectangular, the Egyptians needed a way to mark a right angle. The Egyptians tied 12 evenly spaced knots along a piece of rope and made a triangle. Explain how this rope could have been used to mark a right angle.

Singapore (NS 2, s. 208)

Canada (MS 8, s.45)

Figure 2. Examples of the civilizations included in the textbooks

How do HoM elements vary by content type (their presentation)?

The information that the HoM elements contained were analyzed and Table 9 presents the headings and statements used in this analysis. Each content was proportioned with the total HoM elements. As one of the elements featured more than one content characteristics, the sum of the percentages was above 100%. This is a finding that may indicate the richness of the HoM elements in terms of content. In other words, one of the criteria in the analysis was whether an element features more than one relevant characteristic or is unidirectional or multidirectional.

The richness of the HoM elements in terms of content was analyzed; SGP had the highest percentage (203.70%) and CAN had the lowest percentage (143.75%). The total content percentage on country basis was compared with the total average (171.21%); IR and SGP were above average whilst TR and CAN were below average.

Table 9. The Comparison of the HoM Elements by Content

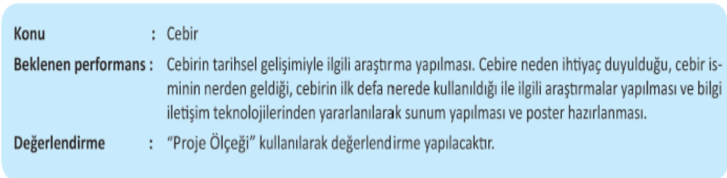
	TR		IR		SGP		CAN		Mean	
	N	%	N	%	N	%	N	%	N	%
Biographical information	17	44.74	10	20.00	7	25.93	5	31.25	9.75	30.48
Formula-rule-solution	9	23.68	24	48.00	13	48.15	5	31.25	12.75	37.77
History of concepts	19	50.00	32	64.00	13	48.15	5	31.25	17.25	48.35
Historical work	2	5.26	17	34.00	3	11.11	2	12.50	6.00	15.72
Tools-equipment	0	0.00	1	2.00	1	3.70	0	0.00	0.5	1.43
Discussion-project	9	23.68	10	20.00	18	66.67	6	37.50	10.75	36.96
Game	0	0.00	1	2.00	0	0.00	0	0.00	0.25	0.50
Total	56	147.66	95	190.00	55	203.70	23	143.75	57.25	171.21

The countries were compared based on the sub-headings. The richest content was related to the sub-heading(s) of biographical information (44.74%) in TR, formula-rule-solution (48.15%), tools-equipment (3.70%) and discussion-project (66.67%) in SGP, history of concepts (64%), historical work (34%) and game (2%) in IR, compared to other countries.

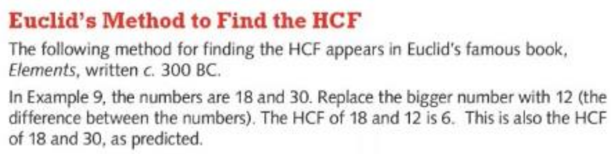
The sub-headings were compared within each country itself; the sub-heading of history of concepts was foregrounded in TR and IR whereas the sub-heading of discussion-project was foregrounded in SGP and CAN. This sub-heading was further prioritized by CAN. The distribution of the sub-headings was examined; IR offered content in all sub-headings whereas CAN had a Homogenous distribution.

The sub-headings were proportionally compared to the average of the sub-headings; the sub-heading of biographical information was above average percentage in TR and CAN and below average in other countries. The sub-headings of formula-rule-solution and tools-equipment were above average in IR and SGP and below average in TR and CAN. Besides, the sub-heading of history of concepts was below average only in CAN; the sub-headings of historical work and game were above average only in IR whereas the sub-heading of discussion-project was above average only in SGP.

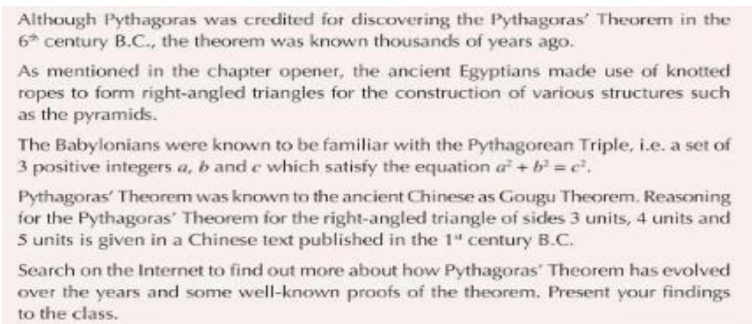
Figure 3 shows examples of HoM elements by content type and learning area.


Konu : Cebir
Beklenen performans : Cebirin tarihsel gelişimiyle ilgili araştırma yapılması. Cebire neden ihtiyaç duyulduğu, cebir isminin nerden geldiği, cebirin ilk defa nerede kullanıldığı ile ilgili araştırmalar yapılması ve bilgi iletişim teknolojilerinden yararlanılarak sunum yapılması ve poster hazırlanması.
Değerlendirme : "Proje Ölçeği" kullanılarak değerlendirme yapılacaktır.

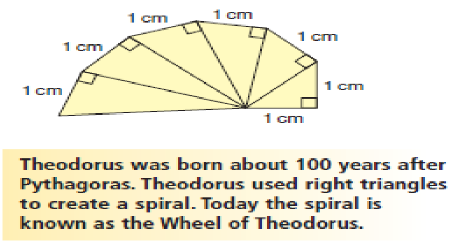
Turkey (MEB 8, s.234), discussion-project, Algebra learning area


Euclid's Method to Find the HCF
The following method for finding the HCF appears in Euclid's famous book, *Elements*, written *c.* 300 BC.
In Example 9, the numbers are 18 and 30. Replace the bigger number with 12 (the difference between the numbers). The HCF of 18 and 12 is 6. This is also the HCF of 18 and 30, as predicted.

Ireland (EP 1, s.40), formula-rule-solution, Numbers and operations learning area


Although Pythagoras was credited for discovering the Pythagoras' Theorem in the 6th century B.C., the theorem was known thousands of years ago.
As mentioned in the chapter opener, the ancient Egyptians made use of knotted ropes to form right-angled triangles for the construction of various structures such as the pyramids.
The Babylonians were known to be familiar with the Pythagorean Triple, i.e. a set of 3 positive integers a, b and c which satisfy the equation $a^2 + b^2 = c^2$.
Pythagoras' Theorem was known to the ancient Chinese as Gougu Theorem. Reasoning for the Pythagoras' Theorem for the right-angled triangle of sides 3 units, 4 units and 5 units is given in a Chinese text published in the 1st century B.C.
Search on the Internet to find out more about how Pythagoras' Theorem has evolved over the years and some well-known proofs of the theorem. Present your findings to the class.

Singapore (NS 2, s. 212), History of concept,



Canada (MS 8, s.36), Biographic information and formula-rule-solution, Geometry and measurement learning area

Figure 3. Examples of HoM elements by content type and learning area

How do HoM elements vary by the place where it is presented in the book (whereabouts they were inserted in the text)?

Whereabouts an element presented in a textbook is inverted in that textbook affects the message given to students and the content of the element. Presenting the same element in different parts may create different messages and advantages; whereabouts a task is inserted changes the content and nature of that task. Table 10 indicates the distribution of the elements in the textbooks analyzed; based on the averages of the countries, the HoM elements were most frequently presented as a part of the subject (42.11%) and least frequently as off-subject (1.56%).

Table 10. Distribution of the Presentation of HoM Elements in the Subject

	TR		IR		SGP		CAN		Mean	
	N	%	N	%	N	%	N	%	N	%
Introduction	13	34.21	9	18.00	9	33.33	-	-	7.75	21.39
As a part of the subject	8	21.05	34	68.00	13	48.15	5	31.25	15	42.11
At the End/Assessment	17	44.74	7	14.00	5	18.52	10	62.50	9.75	34.94
Off-subject	-	-	-	-	-	-	1	6.25	0.25	1.56
Total	38	100	50	100	27	100	16	100	32.75	100

Whereabouts the HoM elements were presented in the textbooks were compared based on the averages of the countries; the HoM elements presented in the introduction were above average (21.79%) in TR (34.21%) and SGP (33.33%) whilst the HoM elements presented as a part of the subject were above average (42.11%) in IR (68%) and SGP (48.15%). Lastly, the HoM elements presented at the end/in the assessment part were above average (34.94%) in CAN (62.50%) and TR (44.74%) and below average in other countries. Similarly, the distribution of the HoM elements within each country was analyzed, and it was found that TR and CAN presented them most at the end/in the assessment part whilst IR and SGP presented them most as a part of the subject.

How do HoM elements vary by their associated learning area?

The findings on the learning areas in Table 11 indicate that the learning area of data processing and probability presented the lowest number of the HoM elements in the textbooks of all countries whereas the learning area of geometry and measurement included the highest number of the HoM elements. IR (58%) prioritized the learning area of number and operations, but CAN did not include any HoM element in this learning area. TR (26.32%) and SGP (25.93%) were proportionally close to each other, yet both were below average in the learning area of numbers and operations since IR increased the average. The only country above average in the learning area of algebra was TR.

Table 11. Distribution of HoM Elements by Learning Area

	TR		IR		SGP		CAN		Mean	
	N	%	N	%	N	%	N	%	N	%
Numbers and operations	10	26.32	29	58.00	7	25.93	-	-	11.5	27.56
Algebra	12	31.58	4	8.00	4	14.81	2.00	12.5	5.5	16.72
Geometry and measurement	14	36.84	15	30.00	13	48.15	14.00	87.5	14	50.62
Data processing and probability	2	5.26	2	4.00	3	11.11	-	-	1.75	5.09

Disregarding the learning area of data processing and probability, TR had a balanced distribution in terms of language areas; yet, SGP, IR and CAN showed a distribution from homogeneous towards heterogeneous, respectively.

DISCUSSION AND CONCLUSION

This study, which analyzes and compares the HoM elements in the secondary school mathematics textbooks of Turkey, Singapore, Ireland and Canada. The mathematics curricula of the countries were examined; Ireland explained the purpose for which HoM elements are used in math lessons as follows: "A student should be aware of history of mathematics and hence of its past, present and future role as a part of our culture" (Department of Education and Science (DES/National Council for Curriculum and Assessment (NCCA), 2014). Moreover, their teacher's book offers various content related to HoM, references to HoM resources and helpful web-sites for teachers (DES/NCCAA, 2000). Although Turkey did not articulate the use of HoM in its current math curriculum, one of the major purposes of its curriculum is that "A student should be aware of the fact that that mathematics is a common value of humanity, thus value mathematics" which clearly emphasizes HoM (Ministy of National Education (MoNE), 2018). No specific statement on HoM is available in the math curricula of Singapore (Ministry of Education (MoE,) 2012; MoE, 2020). Canada highlights HoM both in primary and secondary school mathematics curricula (MoE, 2020)). The finding that Ireland and Turkey used more HoM elements than other countries is significant as these countries mentioned HoM in their math curricula.

Furthermore, comparing with the previous studies, one can say that there has been an increase in the number of HoM elements in mathematics textbooks (Erdoğan et al., 2015; Incikabi et al., 2019; Mersin & Durmus, 2018; Tan Sisman & Kirez, 2018). Singapore, which gained its national independence in 1965, enforced many educational reforms as well, but there was no mention of HoM at primary school level, and there were historical anecdotes at the secondary school level, such as the number systems used by Ancient Egypt and Ancient China after the 1980s (Ho, 2008). For example, the New Syllabus series presented mathematical stories and snippets. Emphasizing that it is not clear enough how to use of HoM in math lessons, Ho (2008) made suggestions on the ways it can be used in lessons. Thus, the emphasis that HoM was not sufficiently used about ten years ago caused the increased use of HoM in textbooks, but this still seems not sufficient for Singapore. Though Canada recommends the use of HoM in math textbooks, this is not practically applied in textbooks. Indeed, Canada is the country that least frequently used HoM elements relative to other countries.

The way the HoM elements in the math textbooks were presented was analyzed under the headings of famous mathematician, civilization, content type, whereabouts element is inserted and learning area. The findings of this study indicated that the HoM elements were concentrated around the learning areas of geometry, numbers and operations; the distribution of famous mathematicians and civilizations was closely related to the learning areas. The learning area of numbers and operations was only prioritized in the subject related to famous mathematicians in the textbooks of only one country. The common mathematicians were presented most in the learning areas of geometry and algebra. A total of 106 famous mathematicians were mentioned in the math textbooks. The famous mathematicians most mentioned among them and included in the textbooks of all countries were Al-Khwarizmi and Pythagoras. Given the importance of Al-Khwarizmi for algebra and Pythagoras for geometry as well as grade levels, this content perhaps meets the need in the curricula (MoNE, 2018). Also, famous mathematicians including Descartes, Archimedes, Euclid, Fibonacci, Eratosthenes, Robert Record, Thales, Diophantus and Golbach were identified in the mathematics textbooks of multiple countries. Besides, mathematicians Cahit Arf, Thomas Harriot, Andrew Wiles, Theodorus were mentioned in the textbooks of only one country. These findings imply that the mathematics mentioned only in one textbook were proportionally presented most frequently in the textbooks of Canada and less frequently in the textbooks of Turkey. The mathematicians commonly mentioned in the textbooks of all four countries were proportionally compared with the HoM elements in these textbooks; in this comparison, Turkey ranked first whereas Canada ranked last.

According to Smestad (2003), HoM can allow for the understanding of the importance and role of mathematics in the society, and also ensure the students perceive mathematics is a human product, that is, it has a human side. Thus, HoM is perhaps a suitable tool to give students the message that they can do what their ancestors did or succeed like them; in this way, students can be motivated through examples of their ancestors (Fauvel, 1991; Jankvist, 2009; Ju et al., 2016; Mersin, 2019; Philippou & Christou, 1998). The tendency of the countries to present famous mathematicians was also examined; other countries, except for Turkey, presented mathematicians close to their culture and less frequently mentioned the mathematicians around Anatolia. The mathematicians commonly mentioned by the countries and only mentioned by one country were analyzed; the common mathematicians were close to the Anatolian or Middle Eastern geography whereas the mathematicians only mentioned by one country were somehow associated with the past of that country or notable in the near century. Moreover, factors such as geography, religion, racial and/or political reasons may be influential in the selection of famous mathematicians as well.

Inclusion of HoM elements and the mathematicians that students can relate to in these HoM elements would positively contribute to mathematics education, thus is supported by various researchers (Li & Fan, 2019; Philippou & Christou, 1998; Smestad, 2003). However, an unbalanced inclusion of these mathematicians may lead to the evaluation of mathematics as a corpus of efforts and achievements of a few genius mathematicians, as Ju et al. (2016) stated. HoM not only includes mathematicians, but also the period and civilization these mathematicians lived and offers a broader perspective towards how and why they worked under which circumstances, why they succeeded, how they practiced mathematics and what mathematics was all about (Toeplitz, 1963).

The civilization of Ancient Egypt, which was included in the textbooks of all countries, was the most presented civilization. Given the contributions of the civilization of Ancient Egypt to the learning areas of numbers and operations as well as geometry and measurement at secondary school level, it is plausible that the textbooks of all countries mentioned the civilization of Ancient Egypt. Moreover, the fact that Ireland and Turkey mentioned the civilization of Ancient Greece, that Ireland mentioned Rome, that Ireland and Singapore mentioned Europe, and that Canada mentioned Canada’s Inuit supports the idea that the countries were affected from factors such as geography, ethnicity, religion and so forth. The educational importance of HoM lies in the fact that it helps us rediscover ourselves as cultural beings and understand the epistemological norms of others' performing mathematics as a historical-cultural construct Ju et al. (2016). For that reason, mathematics, as an information system that can be improved based on differences through communication, is a valuable tool for teaching students the power of diversity and the egalitarian nature of mathematics. In this regard, especially multicultural countries such as Canada should include more examples of HoM in their mathematics textbooks.

The findings of the country averages in relation to the order through which the HoM elements were presented in the subject show that HoM elements were presented either in the introduction section of the subject or as a part of the subject itself, or at the end of the subject/assessment sections, or individually, outside the subject. This finding is congruent with the findings of Incikabi et al. (2019). Only Canada presented HoM elements outside the subject in only one element. The reason that HoM elements were second mostly presented at the end/in the assessment part is that Turkey and Canada dramatically differed from other countries in this regard. However, as mentioned earlier, Turkey and Canada had different purposes. While Canada prioritized in-class activities such as measurement and intensifying, Turkey focused on out-of-class activities. The averages of the country percentages, purpose of use and presentation of the HoM elements were analyzed; the sum of the use for intensifying and project-research was close to the presentation of the HoM elements at the end/in the assessment part. The idea that it is reasonable to present HoM elements in the introduction or as a part of the subject for motivation is supported by the finding that the sum of this presentation order is related to the purpose of motivation.

The HoM elements in the textbooks were generally presented in the introduction or as a part of the subject, for the purpose of motivation or informing, as history of concepts or formula-rule-solution. Analysis of the presentation of the HoM elements in terms of content type revealed contrary to the previous studies that (Baki & Butuner, 2013; Incikabi et al., 2019; Tan Sisman & Kirez, 2018) the countries focused on other categories, rather than providing only biographical information. However, this was not the case for Turkey; Turkey prioritized biographical information most among the other countries. These findings are congruent with the previous research on the HoM elements in the math textbooks of Turkey (Baki & Butuner, 2013; Erdoğan et al., 2015; Incikabi et al., 2019; Mersin & Durmus, 2018; Sahin et al., 2016; Tan et al., 2018).

Considering the average percentages, history of concepts was the most commonly used content type; Turkey and Ireland mostly used this content type, but Singapore and Canada preferred discussion-project most. Regarding the history of concepts, Ireland presented more than one famous mathematician and civilization within an element and thus focused on the developmental process of the concepts; on the other hand, Turkey only included brief snippets on the origins of the concepts, instead of the development of the concepts. Although Canada was lacking in terms of content, it, like Singapore, focused on discussion-project in presenting the content and had a balanced distribution in other areas. The current literature argues that integrating HoM into lessons as snippets is ineffective and insufficient to promote higher-order thinking skills (Ju et al., 2016; Xenofontos & Papadopoulos, 2015). To encourage higher-order learning, HoM elements should be presented in forms, other than snippets, to provide content that ensure that the relations between the past and the present are established in a solid way (Jahnke et al., 2002; Lawrence, 2008). Assessing the success of countries in TIMMS and similar exams through HoM elements may appear as a controversial method; but there are some studies that reveal that mathematics lessons with HoM increase student achievement (e.g. Albayrak, 2011; Awosanya, 2001; Bahadir Varol, 2019; Bayam, 2012; Ersoy, 2015; İdikut, 2007; Lim & Chapman, 2015; Ozcan, 2014).

This study also analyzed the distribution of the HoM elements in terms of learning area. Similar to the previous studies, it was found that the learning areas of geometry, numbers and operations were far more used than other learning areas. The lack of enough HoM elements or a balanced distribution of HoM elements in the learning area of data processing and probability in all countries may be related to the reasons such as the time allocated to the learning area, the inability to find enough content for this area, etc. The finding that Turkey, unlike other countries, had a more balanced content in the learning area of algebra may be associated with the advantage of presenting common mathematicians.

In a nutshell, the findings indicated that the countries prioritized affective aspects rather than cognitive and socio-cultural characteristics in the selection of HoM elements. They presented the mathematicians related to the subject based on affective characteristics and incorporated skills and cognitive characteristics into the elements as far as their choice allowed. From the textbooks examined in the past years, one may observe that HoM elements in the textbooks of the countries have improved. Further, this study suggested that Turkey and Canada review their HoM elements in terms of content. This study has examined the mathematics textbooks of Turkey and Singapore, Canada and Ireland, which were more successful than Turkey in international exams. Therefore, further studies may choose their sample based on different criteria and compare the HoM elements in the textbooks of different countries.

REFERENCES

Albayrak, O. (2011). *Effects of history of mathematics integrated instruction on mathematics self-efficacy and achievement* [Unpublished master thesis]. Boğazici University, İstanbul.

Amaral, R.B., Ribeiro, C.M. &Godoy, J.S. (2014). Choosıng textbooks wı-ithout looking at the textbooks: the role of the other's interpretatıons, in: Jones, K., Bokhove, C., Howson, G., Fan, L. (Eds.), *Proceedings of the International Conference on Mathematics Textbook Research and Development (ICMT-2014*). University of Southampton, Southampton, pp. 153–158.

Awosanya, A. (2001). *Using history in the teaching of mathematics*. Florida State University, Florida, USA.

Bahadir Varol, G. (2019). *The effects of integrating the history of math into the learning domain of arithmetic* [Unpublished master thesis]. Aydin Adnan Menderes University, Aydin, Turkey.

Baki, A. &Butuner, S.O. (2013). The ways of using the history of mathematics in 6th, 7th and 8th grade mathematics textbooks. *Elementary Education Online 12*, 849-872.

Bayam, S.B. (2012). *The impact of a knowledge of the history of mathematics on primary school student mathematics achievement and attitudes* [Unpublished master thesis]. Kastamonu University, Kastamonu, Turkey.

Bowen, G.A. (2009) Document analysis as a qualitative research method. *Qualitative Research Journal, 9*, 27–40. https://doi.org/10.3316/QRJ0902027

Byers, V. (1982). Why study the history of mathematics? *International Journal of Mathematical Education in Science and Technology. 13*(1), 59-66. https://doi.org/10.1080/0020739820130109

Charalambous, C.Y., Delaney, S., Hsu, H.Y. &Mesa, V. (2010). A comparative analysis of the addition and subtraction of fractions in textbooks from three countries. *Mathematical Thinking and Learning. 12* (2), 117-151. https://doi.org/10.1080/10986060903460070

de Almeida, L.M.W.& da Silva, K.A.P. (2019). The learnıng of double ıntegral concept usıng a textbook. *Platforms as Infrastructures for Mathematics Teachers' Work with Resources, 16*, 149.

DES/NCCAA (Department of Education and Science/National Council for, Assessment) (2000). *Mathematics syllabus: Higher, Ordinary, and Foundation level*. Dublin: Stationery Office.

Ding, M. &Li, X. (2010). A comparative analysis of the distributive property in US and Chinese elementary mathematics textbooks. *Cognition and Instruction, 28*(2) 146–180.

Ekawati, R., Fiangga, S., Siswono, T.Y.E. (2018). Historical aspect of mathematics on Indonesian mathematics textbook, in: *IOP Conference Series: Materials Science and Engineering, 434*(1) https://doi.org/10.1088/1757-899X/434/1/012001

Erdoğan, A., Eşmen, E. &Findik, S. (2015). History of mathematics in secondary school textbooks: an ecological analysis. *Marmara Universsity Atatürk Education Faculty Education Science Journal, 42*(42), 239–259.

Ersoy, E. (2015). *Effect of using history of mathematics on the elementary school 4th grade students' academic success, retention and motivation level* [Unpublished master thesis]. Adnan Menderes University, Aydin, Turkey.

Fan, L., Zhu, Y. &Miao, Z. (2013). Textbook research in mathematics education: *Development status and directions. ZDM Math. Education. 45*(5) . https://doi.org/10.1007/s11858-013-0539-x

Fauvel, J. (1991). Using history in mathematics education. *For the Learning Mathematics, 11*(2), 3–6.

Fried, M.N., 2007. Didactics and history of mathematics: Knowledge and self-knowledge. *Educational Studies in Mathematics, 66*(2), 203-223. https://doi.org/10.1007/s10649-006-9025-5

Genc, M. &Karatas, İ. (2018). Integrating history of mathematics into mathematics teaching: Al-Khwarizmi's completing the square method. *Kastamonu Education Journal. 26*(1), 219–230.

Gracin, D. G., Matić, L. J. (2016.) The role of mathematics textbooks in lower secondary education in Croatia: An empirical study. *The Mathematics Educator 16*(2), 31–58.

Gracin, D.G. (2014). What can textbook research tell us about national mathematics education? Experiences from croatia, in: Jones, K., Bokhove, C., Howson, G., Fan, L. (Eds.), *Proceedings of the International Conference on Mathematics Textbook Research and Development (ICMT-2014*). University of Southampton, Southampton, pp. 251–256.

Grugnetti, L., Rogers, L., Carvalho e Silva, J., Daniel, C., Coray, D., de Guzmán, M., …Vasco, C. (2002). Philosophical, multicultural and interdisciplinary issues, *In History in mathematics education* (pp. 39-62) Springer, Dordrecht.

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for mathematics teachers? *Educational Studies in Mathematics, 71*(3), 199–218. https://doi.org/https://doi.org/10.1007/s10649-008-9159-8

Gulikers, I., Blom, K. (2001). "A historical angle", a survey of recent literature on the use and value of history in geometrical education. Educational Studies in Mathematics. 47 (2), 223-258. https://doi.org/10.1023/A:1014539212782

Ho, W. (2008). Using history of mathematics in the teaching and learning of mathematics in Singapore., *1st RICE, Singapore*:Raffles Junior College.

Hong, D.S. &Choi, K.M. (2018). A comparative analysis of linear functions in Korean and American standards-based secondary textbooks. *International Journal of Mathematical Education in Science and Technology, 49*(7), 1025-1051. https://doi.org/10.1080/0020739X.2018.1440327

İdikut, N. (2007). *The effect of benefiting from history in education of mathematics on the student's attitudes towards mathematics and their success on it* [Unpublished master thesis]. Yuzuncu Yil University, Van, Turkey.

Incikabi, L., Kepceoglu, I.&Kucukoglu, U. (2019). Examination of Mathematical History Contents in Middle School Mathematics Textbooks. Pamukkale University Journal of Education, 45, 144–158.

Ishii, T. (2019). Limit of textbook in conceptual understanding. *Platforms as Infrastructures for Mathematics Teachers' Work with Resources, 16,* 373.

Jahnke, H.N., Arcavi, A., Barbin, E., Bekken, O., Furinghetti, F., El Idrissi, A., da Silva, C.M.S., Weeks, C. (2002). The use of original sources in the mathematics classroom. *In History in mathematics education* (pp.291-328). Springer, Dordrecht.

Jahnke, T. (2019). Report on the design and realisations of the workshop 'writing maths textbooks.' *Platforms as Infrastructures Mathematics Teachers' Work with Resources*. 16, 203–208.

Jankvist, U.T. (2009). A categorization of the "whys" and "hows" of using history in mathematics education. *Educational Studies in Mathematics, 71*(3), 235-261. https://doi.org/10.1007/s10649-008-9174-9

Ju, M.K., Moon, J.E., &Song, R.J. (2016). History of mathematics in korean mathematics textbooks: Implication for using ethnomathematics in culturally diverse school. *International Journal of Science and Mathematics Education, 14* (7), 1321-1338. https://doi.org/10.1007/s10763-015-9647-0

Kajander, A., & Lovric, M. (2009). Mathematics textbooks and their potential role in supporting misconceptions. *International Journal of Mathematical Education in Science and Technology, 40*(2), 173-181. https://doi.org/10.1080/00207390701691558

Kim, S.C. (2014). Utilization plan of primary 3-4 grade mathematics textbooks for 'Flipped Classroom'. *Education of Primary School Mathematics, 17*(3), 265–276.

Lawrence, S. (2008). *History of mathematics making its way through the teacher networks: professional learning environment and the history of mathematics in mathematics curriculum,* in: 10th International Congress on Mathematical Education, International Congress on Mathematics Education, Mexico.

Li, N., &Fan, L. (2019). How are mathematicians represented in chinese mathematics textbooks? *Platforms as Infrastructures for Mathematics Teachers' Work with Resources*, *16*, 227–232.

Li, Y., 2000. A comparison of problems that follow selected content presentations in American and Chinese mathematics textbooks. *Journal for Research in Mathematics Education, 31*(2), 234-241. https://doi.org/10.2307/749754.

Lim, S.Y.& Chapman, E. (2015). Effects of using history as a tool to teach mathematics on students' attitudes, anxiety, motivation and achievement in grade 11 classrooms. Educational Studies in Mathematics, 90(2), 189-212. https://doi.org/10.1007/s10649-015-9620-4

Lin, P.-J., & Tsai, W.H. (2014). Improvement in teachers' interpretation of mathematics textbooks, in: Jones, K., Bokhove, C., Howson, G., Fan, L. (Eds.), *Proceedings of the International Conference on Mathematics Textbook Research and Development (ICMT-2014*). University of Southampton, UK, Southampton, UK, pp. 303–310.

Man-Keung, S. (2000). The ABCD of using history of mathematics in the (undergraduate) classroom. *Paleontological Society Papers, 6*, 3–10.

Marshall, G.L., & Rich, B.S. (2020). The Role of History in a Mathematics Class. The Mathematics Teacher. 93(8), 704-706. https://doi.org/10.5951/mt.93.8.0704

McNeil, N.M., Grandau, L., Knuth, E.J., Alibali, M.W., Stephens, A.C., Hattikudur, S., & Krill, D.E. (2006). Middle-school students' understanding of the equal sign: The books they read can't help. Cognition and. Instruction, 24(3), 367-385. https://doi.org/10.1207/s1532690xci2403_3

Meinerz, F.M., Doering, L.R. (2019). Analysıs of textbooks in three latin countries: resolution of equations and inequalities. *Platforms as Infrastructures Mathematics Teachers'Work with Resources 16*, 239–244.

Mersin, N. (2019). *Designing activities related to the history of mathematics for secondary school students and analysis of student reflections on these activities*. Bolu Abant İzzet Baysal University.

Mersin, N., & Durmus, S. (2020). Prospective Mathematics Teachers' Perceptions of the History of Mathematics on the Role of Mathematics Education. *Amasya Education Faculty Journal, 9*(2), 258–317.

Mersin, N. & Durmus, S. (2018). Place Of Mathematics History in Middle School Mathematics Textbooks. *Bolu Abant Izzet Baysal University Journal of Faculty Education, 18*(2), 997–1019.

Mersin, N.& Karabork, M.A. (2021). The comparison of math textbooks in Turkey and Singapore in terms of technology integration: The comparison of math textbooks. *International Journal of Curriculum and Instruction, 13*(1), 552–573.

Miles, M.B., & Huberman, A.M. (1994). *Qualitative data analysis*: An expanded sourcebook. sage.

Ministry of Education, O. (2020). *The Ontario Curriculum, Grades 1 and 8. Mathematics*. Queen's Printer for Ontario, Ontario.

Ministry of Education, S. (2020). *Mathematıcs Syllabuses Secondary One to Four Express Course Normal (Academic) Course.* Ministry of Education, Singapore, Singapore.

Ministry of Education, S. (2012). *Primary mathematics teaching and learning syllabus. Ministry of Education*, Singapore, Singapore.

Ministy of National Education, T. (2018*). Mathematics Course Teaching Programme (For elementary and middle school, 1-8. grade)*. Ministry of Education, Ankara.

National Council for Curriculum and Assessment (NCCA) (2014). *Maths in Practice – Report and Recommendations*. National Council for Curriculum and Assessment, Dublin.

National Council of Teachers of Mathematics (2000). Principles and Standards for School Mathematics, *School Science Mathematics. 47*.

O'Halloran, K.L., Beezer, R.A., & Farmer, D.W. (2018). A new generation of mathematics textbook research and development. *ZDM, 50*(5), 863-879. https://doi.org/10.1007/s11858-018-0959-8.

Ozcan, D. (2014). *With the history of mathematics applied to the anatolian high school students and the effects of enriched curriculum in mathematics'success.* İstanbul Sabahattin Zaim Üniversitesi,

Philippou, G.N.& Christou, C. (1998). Beliefs, teacher education and the history of mathematics, in: *PME 22 : Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education*, 12-17 July 1998. Stellenbosch, South Africa, pp. 1–4.

Radford, L. (1997). On psychology, historical epistemology, and the teaching of mathematics: Towards a socio-cultural history of mathematics. *For the learning of mathematics, 17*(1), 26-33.

Radford, L., Furinghetti, F. &Katz, V. (2007). Introduction the topos of meaning or the encounter between past and present. *Educational Studies in Mathematics*, *66*(2), 107-110. https://doi.org/10.1007/s10649-006-9076-7

Reys, B.J., Reys, R.E. &Chávez, O. (2004). Why Mathematics Textbooks Matter. Educational Leadership, 61(5), 61-66.

Sahin, Ö., Basibuyuk, H., Soylu, Y. (2016). The place of the history of mathematics in the sixth grade textbooks, in: *International Conference on Education in Mathematics, Science & Technology (ICEMST-2016),* Ersan Resort & Spa, Muğla. Muğla, Turkey, pp. 929–935.

Schorcht, S. (2018). History of Mathematics in German Mathematics Textbooks.*In Mathematics, Education and History* (pp, 143-162). Springer, Cham. https://doi.org/10.1007/978-3-319-73924-3_8

Schubring, G., & Fan, L. (2018). Recent advances in mathematics textbook research and development: An overview. ZDM, 50(5), 765-771. https://doi.org/10.1007/s11858-018-0979-4

Senk, S.L., Thompson, D.R., &Wernet, J.L. (2014). Curriculum and Achievement in Algebra 2: Influences of Textbooks and Teachers on Students' Learning about Functions.In Mathematics curriculum in school education (pp 515-540). Springer, Dordrecht. https://doi.org/10.1007/978-94-007-7560-2_24

Smestad, B. (2003). Historical topics in Norwegian textbooks, in: Study the Masters: *The Abel-Fauvel Conference*. pp. 153–168.

Swetz, F.J. (2009). Word problems: Footprints from the history of mathematics, in: *Words and Worlds. Brill Sense*, pp. 71–91. https://doi.org/https://doi.org/10.1163/9789087909383_006

Tan Sisman, G., & Kirez, B. (2018). History of mathematics in the Turkish middle school mathematics curriculum and textbooks. *Cukurova University Faculty of Education Journal, 47*(1), 188–215. https://doi.org/10.14812/cuefd.361176

Toeplitz, J. (1963). Film Scholarship: Present and Prospective. *Film Quarterly, 16*(3), 27-37. https://doi.org/10.2307/1210727

Toprak, Z. & Ozmantar, M.F. (2019). A Comparative Analysis of Turkey and Singapore 5th Grade Mathematics Textbooks in Terms of Worked Examples and Questions. *Turkish Journal of Computer and Mathematics Education*, *10*(2), 539–566.

Tzanakis, C., Arcavi, A., de Sa, C.C., Isoda, M., Lit, C.-K., Niss, M., de Carvalho, J.P., Rodriguez, M., Siu, M.-K (2002). Integrating history of mathematics in the classroom: an analytic survey. *In History in mathematics education* (pp. 201-240).

Valverde, G.A., Bianchi, L.J., Wolfe, R.G., Schmidt, W.H., Houang, R.T. (2002). *Physical Features of Textbooks, in: According to the Book*. Springer, pp. 21–51.

Wang, J., Yang, Z. (2015). Study on Application of Mathematical History in Mathematics Textbooks in Mainland China, in*: Proceedings of the 2015 International Conference on Management, Education, Information and Control*. https://doi.org/10.2991/meici-15.2015.274

Xenofontos, C., Papadopoulos, C. (2015). Opportunities of learning through the history of mathematics: the example of national textbooks in Cyprus and Greece. *International Journal for Mathematics Teaching and Learning, 16*.

Yang, D.C., & Sianturi, I.A. (2017). An Analysis of Singaporean versus Indonesian textbooks based on trigonometry content. *Eurasia Journal of Mathematics Science and Technology Education*, *13*(7), 3829-3848. https://doi.org/10.12973/eurasia.2017.00760a

Zhu, Y., & Fan, L. (2006). Focus on the representation of problem types in intended curriculum: A comparison of selected mathematics textbooks from Mainland China and the United States. *International Journal of Science and Mathematics Education, 4*(4), 609-626. https://doi.org/10.1007/s10763-006-9036-9