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The Use of Error Based Activities to Improve the Mathematization Competency

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Abstract

The purpose of the study is to investigate how the error based activities improve the mathematization competency of preservice mathematics teachers. The study was designed as a case study which is one of the qualitative study design. The sample consists of 38 third grade elementary mathematics teacher candidates studying in a university in Turkey. The study group consisted of 20 pre-service teachers; the comparison group consisted of 18 pre-service teachers. Data were collected through PISA questions consisting of 11 questions total. Data were analysed through descriptive analysis. The findings of the study indicates that the study group performed better than the comparison group on getting the full score for most of the questions. Both of the groups performed mostly on getting full and zero scores and rarely getting partial scores. The study group performed mostly at level 3 when the comparison group performed mostly at level 2 and level 1.

Keywords: Error Based Activities, Mathematization, Mathematical Literacy, Preservice Mathematics Teachers

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INTRODUCTION

21st century citizens are expected to be creative and reflective and this increases the need for individuals having developed mathematical literacy skills (Edge, 2009; Jablonka, 2011; Steen, Turner & Burkhardt, 2007). Likewise, the Program for International Student Assessment (PISA) emphasizes the importance of mathematical literacy by aiming to determine the capacity of individuals to apply mathematics to life problems. In this connection, it is stated that today's newly published mathematics curriculum for primary and middle school students aims to teach students how to develop their mathematical literacy skills and use them effectively (Ministry of National Education, MEB, 2018a). This shows that the implemented mathematics curriculum focuses on how to develop the ability to apply mathematics to the real world and how to comprehend the role of mathematics in daily life. Mathematical literacy refers to the capacity of an individual to formulate, apply and interpret mathematics in order to solve various real-life problems. In other words, it is about understanding the role of mathematics in daily life and being able to use mathematics in solving problems encountered in daily life (McCrone & Dossey, 2007; Organisation for Economic Co-operation and Development (OECD, 2019). In this context, mathematical literacy is a multi-faceted and dynamic process that includes formulating, applying, interpreting and evaluating. As a product of this process, mathematical literacy can be perceived as a set of competencies that enable citizens to solve current problems they face by using mathematics in the century we live in. These competencies can be listed as mathematical communication, representation, strategy production, mathematization, reasoning and argument, using symbolic language and operations (PISA, 2016; OECD, 2019; Kabael et al., 2019).

In order for the mathematical competencies to be active in the student, it is necessary to learn how to transform a real-life problem into a mathematical format (Gould & Wasserman, 2014). This view is also supported by PISA data (Stacey & Turner, 2015). In order for this to happen, these competencies should be included in the teaching and learning of mathematics and efforts should be made for students to develop themselves consciously in this regard (Turner et al., 2015). These competencies should be directly targeted and promoted in mathematics lessons (Turner, 2010). De Lange (2003) stated that it was observed that in order to develop mathematical literacy, the educational objectives should also include these competencies.

In the learning environments in our country, importance is given to the teaching of the components of the competence of using the symbolic language and operations and the competence of representation, albeit partially (Ülger, 2021). Practice of the mathematization competency does not take place in most learning environments. In fact, in order for the individual to solve real life problems, he/she needs to be able to formulate the problem, solve it by applying it and mathematize it by interpreting it (OECD, 2019). Strong and relevant connections between realities, life contexts and students' mathematical learning are a key feature of Realistic Mathematics Education. Mathematization in Realistic Mathematics Education was expressed by Hans Freudenthdal in two ways, horizontal and vertical. Horizontal mathematization is expressed as turning a real life problem into a mathematical formula and vertical mathematization is expressed as reaching a solution by establishing communication and relationships between mathematical expressions (Gravemeijer & Terwel, 2000). With horizontal mathematization, the targeted concept is reached while with vertical mathematization, this concept is used to progress onto more general concepts, the obtained concept is generalized and formulas are reached by working with symbols and establishing relationships between concepts (Van den Heuvel-Panhuizen & Drijvers, 2014). The knowledge generated by horizontal mathematization forms the basis for vertical mathematization. Therefore, in order for vertical mathematization to take place, horizontal mathematization must first be performed (Freudenthal, 1973). In this context, when the fact that mathematical literacy is important in all the stages of schooling from pre-school to adulthood is considered, it becomes clear that the necessary importance should be given to mathematization in each stage of education (Drabekova et al., 2014; OECD, 2019).

In order to train students who have the competency of mathematization, which is one of the competencies that mathematical literacy requires, first teachers should be trained in such a way as to have these competencies. To do so, the main responsibility should be taken by teacher educators. A qualified teacher should seek for the ways of creating an environment suitable for mathematical meaning-making (Schoenfeld, 2002). At this point, it is expected from pre-service teachers to learn the ways of creating these environments before starting their professional career, to realize the importance of mathematical literacy and correspondingly the importance of mathematization and to improve themselves in this regard (Güneş & Gökçek, 2013; Hobden, 2007; Tekin & Tekin, 2004).

Being trained as mathematically literate and having the competency of mathematization depend on the reshaping of in-class relationships in such a way as to openly focus on researching, explanation and justification (Solomon, 2009). It is important to verify the claims of the individual or to make him/her realize the falsity of his/her own opinion during inquiries and discussions to be conducted in mathematical environments (Höfer & Beckmann, 2009; Johnson et al., 2011). In order for such mathematical environments to be formed, mathematical activities should be organized in the classrooms (Borasi, 1996; Gellert, 2004). Through mistake-handling activities, the individual can improve his/her mathematization skill by investigating different mathematical relationships by going over the problem when he/she has committed a mistake while solving the problem (Bilgili et al., 2021). Therefore, mathematization provides a two-way interaction, making mathematical processes more active. These activities, which lead the individual to questioning, also positively affect the individual's having different perspectives by trying different solutions (Borasi, 1987).

Mistake-handling activities allow the creation of correct information by presenting negative information to the individual alongside positive information (Heinze, 2005). Negative information is defined as the experiential knowledge of what is wrong and to be avoided (Gartmeier et al., 2008; Parviainen & Eriksson, 2006), while positive information is defined as information of what works (Martignoni, 2015). Learning involves positive information, but negative information is also needed. Mistake-handling activities make a dynamic contribution to the process of doing mathematics by enabling teachers, students and pre-service teachers to be engaged in higher-order thinking (Gedik & Konyalıoğlu, 2019; Özkaya & Konyalıoğlu, 2019; Santagata, 2005). Mistake-handling activities, which are thought to have a positive effect on the development of thinking skills, are believed to improve mathematical literacy and the competence of mathematization within the context of mathematical literacy.

Conceptual Framework

In the current study, it is thought that through mistake-handling activities, the pre-service teachers will try different ways, see different solutions, use different representations and think from different perspectives (Palincsar & Brown, 1984 as cited in Santagata, 2005). Considering that mathematization is the basis of solving a mathematical problem, it is thought that this application conducted on the basis of mistake-handling activities may have a positive effect on the development of the competence of mathematization. There are two types of mathematization in the literature: horizontal and vertical. In the current study, it was thought that the pre-service teachers had already mastered horizontal mathematization and it was aimed to examine how the vertical mathematization developed by applying mistake-handling activities that support the development of higher-order thinking skills. To this end, the theoretical framework created by Turner et al. (2015) by defining activities at four levels (0-3) in order to determine their vertical mathematization competency levels, was taken into consideration. This framework used in the study is given in detail below.

Level 0: Either the situation is purely intra-mathematical, or the relationship between the extra-mathematical situation and the model is not relevant to solving the problem.

Level 1: The individual constructs a model where the required assumptions, variables, relationships and constraints are given; or draws conclusions about the situation directly from a given model or from the mathematical results.

Level 2: The individual constructs a model where the required assumptions, variables, relationships and constraints can be readily identified; or modifies a given model to satisfy changed conditions; or interprets a model or mathematical results where consideration of the problem situation is essential.

Level 3: The individual constructs a model in a situation where the assumptions, variables, relationships and constraints need to be defined; or validate or evaluate models in relation to the problem situation; or link or compare different models.

METHOD

Research Model

The study is a case study, one of the qualitative research methods. As stated by Subaşı and Okumuş (2017), a case study aims to analyze an event in a short time of a few weeks or in a long period of one or two years and to describe and interpret it in depth on the basis of written documents. In the current study, from among the two groups taking a course focused on mathematics teaching, one was selected as the study group and the other as the comparison group. The academic grade point averages of the study group were found to be lower than those of the comparison group. Mistake-handling activities were administered to the study group for three weeks. The improvement caused by mistake-handling activities in mathematical literacy and mathematization competency is described.

Sample

The participants of the study are 38 third-year pre-service teachers attending the Department of Elementary Math Teaching in a university located in Central Anatolia. In the study, pre-service teachers' having taken essential mathematics field courses was taken as the criterion of inclusion, since it was thought that they would have a positive effect on the mathematization process and thus individuals who met this criterion were preferred. The pre-service teachers were taking the course of Special Teaching Methods in two different classes. Of these groups, one group was randomly selected as the study group and the other group was selected as the comparison group. While the study group consisted of 20 pre-service teachers, the comparison group consisted of 18 pre-service teachers.

Data collection tools

PISA questions are prepared by targeting mathematical literacy, which forms the basis of mathematics learning. Accordingly, a test consisting of four questions was prepared using the PISA exams in order to see the development of their mathematical literacy and mathematization competency after the administration of three-week mistake-handling activities to the group. Three of these four questions have three sub-questions and one has two sub-questions. In this context, eleven questions were presented to the pre-service teachers in total. The questions administered to the pre-service teachers during the data collection process are given in the findings section. The pre-service teachers were asked to solve these questions without any time restriction during the application.

Application process

The academic grade point averages of the study and comparison groups, which were randomly selected from among the students taking the course in which the application would be conducted, were checked before the application. It was determined that the academic grade point averages of the participants in the study group were lower than those of the participants in the comparison group.

In the three-week application, the mistake-handling activities were carried out within the scope of the course. No extra application was conducted on the comparison group other than what was required in the curriculum. After three weeks of application, necessary explanations were made by the researcher and the data collection tool consisting of PISA questions was filled under the supervision of the researcher. During the application process, necessary environmental conditions were provided for the students to individually respond to the prepared test. The pre-service teachers were told to solve the questions without any time restriction. It was observed that the study and comparison groups answered the test questions within one hour and handed them in to the researcher.

One of the points taken into consideration in the application was that the pre-service teachers could ask any question to the researcher whenever they needed to. The pre-service teachers were able to easily communicate with the researcher. Thus, it is thought that they seriously responded to the data collection tool. In this way, the validity of the study was increased and data loss was prevented.

Data Analysis

The data obtained from the pre-service teachers' solutions to the PISA questions were scored by assigning a full point, partial point and zero point, taking into account the scoring of PISA. Then, in order to determine the vertical mathematization levels of the pre-service teachers (Turner, Blum, & Niss, 2015), the conceptual framework of mathematization levels was used. In this framework, four levels were determined as Level 0, Level 1, Level 2 and Level 3. According to the determined levels, the level of the pre-service teachers in the vertical mathematization process depending on the scores they received from the answers they gave to the data collection tool is given in the tables in the findings section and explained in writing.

FINDINGS

While the participants in the study group were engaged in mistake-handling activities for three weeks, the participants in the comparison group were instructed as required in the curriculum of the course for the same period. After the application, the pre-service teachers were asked to answer the questions selected from the PISA exam. The answers given by the study and comparison groups were analyzed on the basis of the scoring of PISA. The scoring table of the groups is given below.

Table 1. Scoring Table of the Study and Comparison Groups (CG: Control group,SG: Study group)

Score / Questions		*Full Score (f)*			*Partial Score (f)*			*Zero Score (f)*		
		Q1	Q2	Q3	Q1	Q2	Q3	Q1	Q2	Q3
Ice Cream Shop	CG	12	0	0	1	1	0	5	17	18
	SG	15	7	0	2	3	0	3	10	20
Hale the Bike Rider	CG	15	10	2	0	0	0	3	8	16
	SG	17	13	5	0	0	0	3	7	15
Climbing Mount Fuji	CG	16	5	1	0	0	0	2	13	17
	SG	14	13	8	0	0	1	6	7	11
Drop Rate	CG	4	9		3	0		1	9	
	SG	16	10		3	0		1	10	

As can be seen in Table 1, when the solutions produced for the first one of the subcategory questions of the Ice Cream Shop question are examined, it is seen that 62% (12 people) of the comparison group have a full point, 6% partial point and 28% zero point while 75% of the study group have a full point, 11% partial point and 15% zero point. For the solutions produced for the second one of the subcategory questions of the Ice Cream Shop question, 6% of the comparison group received a partial point and 17% received a zero point, while 35% of the study group received a full point, 15% partial point and 50% zero point. For the third of the subcategory questions, all the participants in the study and comparison group received a zero point as they could not produce any solution and thus, related parts in the table are seen to be empty.

When Table 1 is examined for the scoring of the solutions to the Hale the Bike Rider question, it is seen that 83% of the comparison group received a full point and 17% zero point in the first sub-question, while 85% of the study group received a full point and 15% zero point. In the second sub-question, while 56% of the comparison group received a full point and 44% received a zero point, 65% of the study group received a full point and 35% received a zero point. In the third sub-question, while 11% of the comparison group received a full point and 89% received a zero point, 25% of the study group received a full point and 75% received a zero point.

When the scoring of the Climbing Mount Fuji question is examined, it is seen that in the first sub-question, 89% of the comparison group received a full point, 11% received a zero point, while 70% of the study group received a full point and 30% received a zero point. In the second sub-question, 28% of the comparison group received a full point and 72% received a zero point, while 65% of the study group received a full point and 35% received a zero point. When we examine the third sub-question, we see that 6% of the comparison group received a full point and 94% received a zero point, while 40% of the study group received a full point, 5% partial point and 55% zero point.

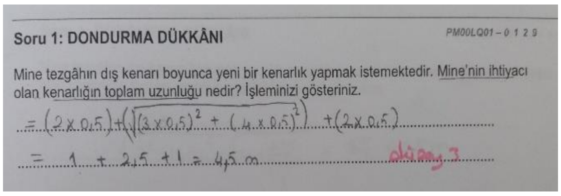
When the scores obtained for the Drop Rate question are examined, it is seen that in the first sub-question, 22% of the comparison group received a full point, 17% partial point and 6% zero point, while 55% did not answer the question. On the other hand, 80% of the study group received a full point, 15% partial point and 5% zero point. In the second sub-question, 50% of the comparison group received a full point and 50% received a zero point, while 50% of the study group received a full point and 50% zero point.

The answers given by the study and comparison groups to the PISA questions are given in a table based on the scoring key of PISA. The answers given by the groups to the questions were then analyzed according to the levels created by Turner, Blum, and Niss (2015) regarding the competency of mathematization. Table 2 shows how many people are in the groups and at what level they are. Afterwards, sample solutions showing the levels of some people in the study and comparison groups are included for each question.

Table 2. Level Graph of the Study and Comparison Groups

Groups / Questions		*Comparison Group (f)*				*Study Group (f)*			
		Level0	Level 1	Level 2	Level 3	Level 0	Level 1	Level 2	Level 3
Ice Cream Shop	Q1	2	2	0	13	1	1	0	17
	Q2	1	4	9	1	1	5	4	9
	Q3	0	0	0	0	0	0	0	0
Hale the Bike Rider	Q3	15	10	2	0	3	3	9	5
Climbing Mount Fuji	Q1	0	12	2	4	1	3	0	14
	Q2	1	3	1	5	1	4	0	13
	Q3	5	7	1	0	2	6	3	8
Drop Rate	Q1	0	0	9	8	1	0	2	17
	Q2	1	1	4	9	0	2	5	12

When the responses given to the first sub-question of the Ice Cream Shop question are examined in Table 2, it is seen that 11% of the comparison group could perform mathematization at Level 0, 11% at Level 1 and 72% at Level 3, while 6% left the question unanswered and thus were not placed at any level. On the other hand, it is clearly seen that while 5% of the study group could perform mathematization at Level 0, 5% at Level 1 and 85% at Level 3, while 5% of them left the question unanswered and thus were not placed at any level. In both groups, the pre-service teachers mostly performed at Level 3.



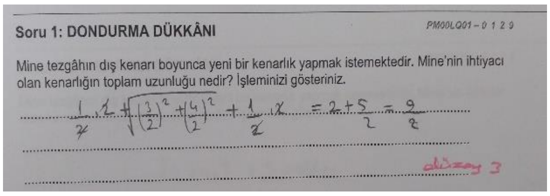
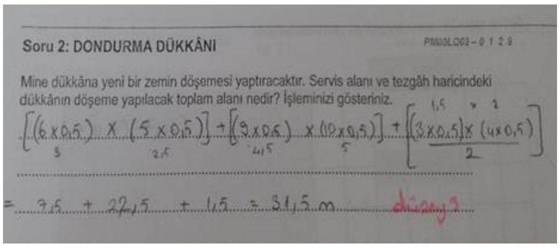


Figure 1: A solution from the study group **Figure 2: A solution from the comparison group**

In the second sub-question of the Ice Cream Shop question, 6% of the comparison group were able to perform mathematization at Level 0, 22% at Level 1, 50% at Level 2 and 6% at Level 3, while 16% did not answer the question. On the other hand, in the same question, 5% of the study group were able to perform mathematization at Level 0, 25% at Level 1, 20% at Level 2 and 45% at Level 3 and 5% left the question unanswered. The biggest portion of the pre-service teachers in the comparison group performed at Level 2, while the biggest portion of the pre-service teachers in the study group performed at Level 3.



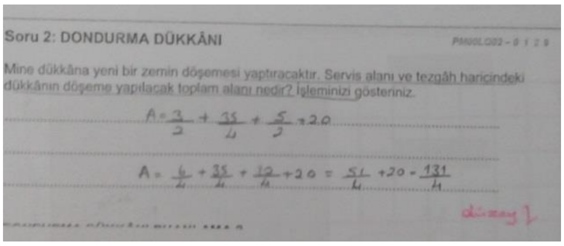
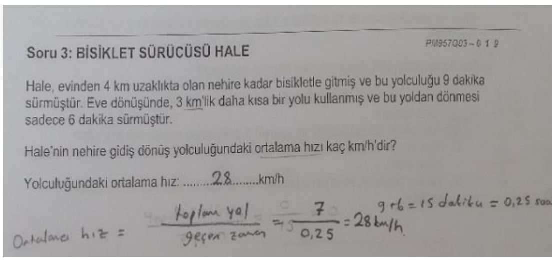


Figure 3: A solution from the study group **Figure 4: A solution from the comparison group**

Since both groups did not answer the third question, the number of people at the levels related to this question is indicated as zero in Table 2.

In the Hale the Bike Rider question, first and the second sub-questions are multiple choice questions. In these questions, the pre-service teachers in the study and comparison groups marked an option but did not perform any operation or make any explanation. Therefore, the level of the answers given in these two questions could not be determined. When Table 2 is examined, it is seen that in the third sub-question of the Hale the Bike Rider question, 28% of the comparison group were at Level 0, 50% were at Level 1, while there were no pre-service teachers performing at Level 2 and Level 3 and 22% of the pre-service teachers did not answer the question. On the other hand, 15% of the study group performed at Level 0, 15% at Level 1, 45% at Level 2 and 25% at Level 3. For this question, it was observed that more pre-service teachers from the comparison group performed at Level 0 and Level 1, while more pre-service teachers from the study group performed at Level 2 and Level 3.



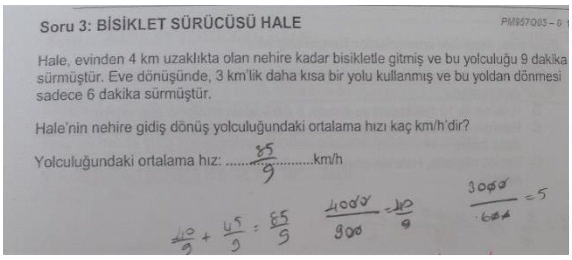
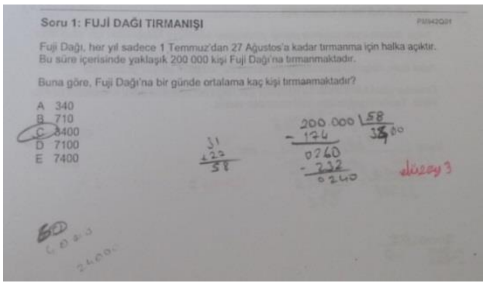


Figure 5: A solution from the study group **Figure 6: A solution from the comparison group**

When the data given for the Climbing Mount Fuji question in Table 2 are examined, it is seen that while there was no pre-service teacher performing at Level 0 in the comparison group for the first sub-question, 67% of the pre-service teachers performed at Level 1, 11% at Level 2 and 22% at Level 3. On the other hand, while 5% of the study group performed at Level 0, 15% at Level 1 and 70% at Level 3, there were no pre-service teacher performing at Level 2 and 10% of the pre-service teachers left the question unanswered. According to the findings in Table 2, the majority of the comparison group performed at Level 1 and the majority of the study group performed at Level 3 for this question.



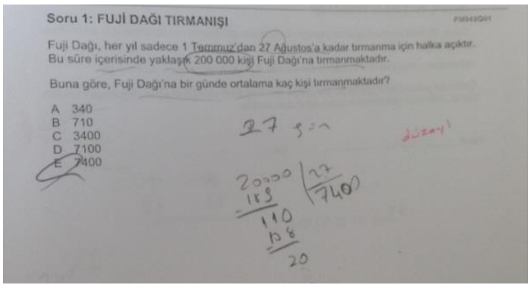
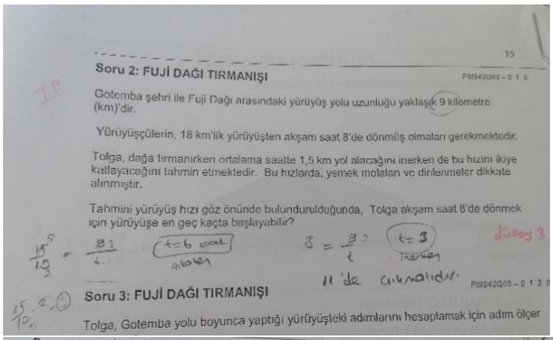


Figure 7: A solution from the study group **Figure 8: A solution from the comparison group**

In the second sub-question, 6% of the comparison group performed at Level 0, 17% at Level 1, 6% at Level 2 and 28% at Level 3 and 43% of them left the question unanswered. While 5% of the study group performed at Level 0, 20% at Level 1 and 65% at Level 3, there was no pre-service teacher performing at Level 2 and 10% of the pre-service teachers did not respond to this question. While the majority of the comparison group left the question unanswered, most of the participants who answered the question performed at Level 3. Although the number of the pre-service teachers who left the question unanswered in the study group was low, it was observed that the majority of the pre-service teachers giving an answer performed at Level 3.



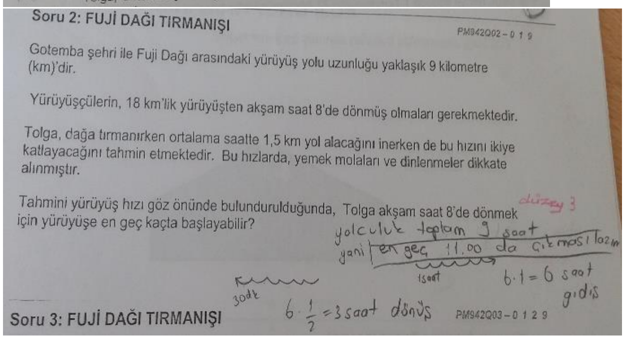
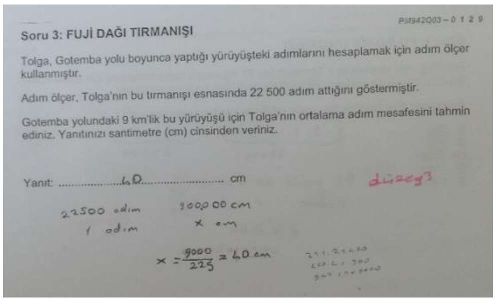


Figure 9: A solution from the study group **Figure 10: A solution from the comparison group**

In the third question, while 28% of the comparison group performed at Level 0, 39% at Level 1 and 6% at Level 2, there was no pre-service teacher who performed at Level 3 and 27% of the comparison group left the third question unanswered. On the other hand, 10% of the pre-service teachers in the study group performed at Level 0, 30% at Level 1, 15% at Level 2 and 40% at Level 3, 5% of the pre-service teachers did not answer the question. As can be seen in Table 2, while the majority of the participants in the comparison group performed at Level 1, the majority of the participants in the study group performed at Level 3.



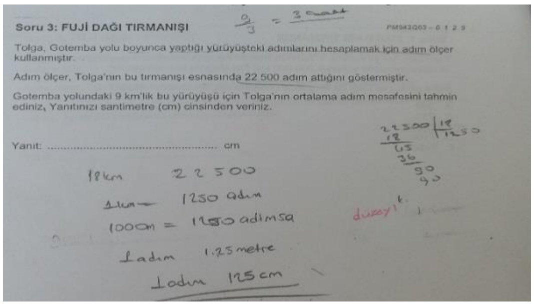
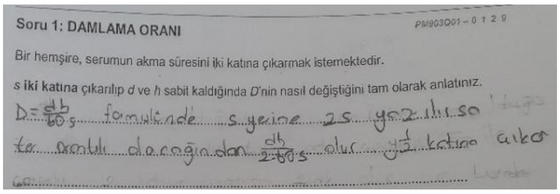


Figure 11: A solution from the study group **Figure 12: A solution from the comparison group**

In the first of the two sub-questions in the Drop Rate question, 50% of the participants in the comparison group performed at Level 2 and 44% at Level 3, while there was no pre-service teacher performing at Level 0 and Level 1 and 6% left the question unanswered. In the study group, while 5% of the pre-service teachers were at Level 0, 10% at Level 2 and 85% at Level 3, no pre-service teacher was found at Level 1. While the pre-service teachers in the comparison group mostly performed at Level 2, the majority of the pre-service teachers in the study group performed at Level 3.



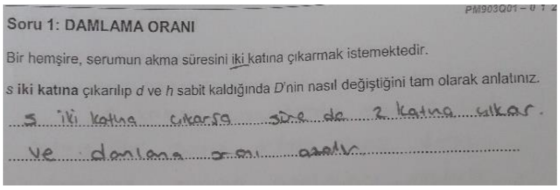
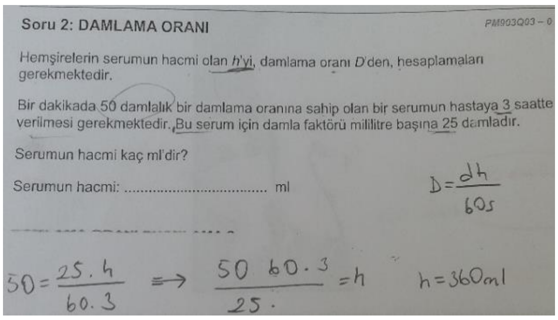


Figure 13: A solution from the study group **Figure 14: A solution from the comparison group**

In the second question, 6% of the comparison group performed at Level 0, 6% at Level 1, 22% at Level 2 and 50% at Level 3, while 16% of the pre-service teachers did not answer the question. On the other hand, 10% of the study group performed at Level 1, 25% at Level 2 and 60% at Level 3 and there was no pre-service teacher who performed at Level 0 and 5% of the pre-service teachers did not answer the question. According to these data, the majority of the pre-service teachers in both groups performed at Level 3.



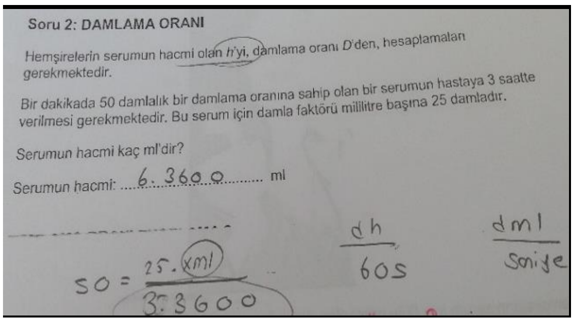


Figure 15: A solution from the study group **Figure 16: A solution from the comparison group**

CONCLUSIONS

When the results of the current study are evaluated considering the importance of mathematical literacy, according to the scoring of the answers they gave to the PISA questions used as a data collection tool, the pre-service teachers in the study group got more full points than the pre-service teachers in the comparison group and thus demonstrated more performances at Level 3. On the other hand, the majority of the pre-service teachers in the comparison group were intensely distributed across Levels 1 and 2.

When the answers given by the groups to the questions are examined in detail, it is seen that both groups performed mostly at Level 3 in the first sub-question of the Ice Cream Shop question. Therefore, they created a solution model by establishing relationships between the data they read and they expressed and validated this model operationally. When the answers they gave to the second sub-question were examined, it was seen that the pre-service teachers in the comparison group were mostly located at Level 2 and the participants in the study group were mostly located at Level 3. When the solutions of the pre-service teachers were examined, it was seen that those in the comparison group directly took action and came to the conclusion without showing the relationships expressed in the question, while those in the study group turned the problem statement into a mathematical form and expressed it in writing and then reached the solution. None of the pre-service teachers answered the third sub-question.

In the Hale the Bike Rider question, only the third sub-question was answered by pre-service teachers and it was seen that the comparison group demonstrated more performances at Level 0 and Level 1, while the study group demonstrated more performances at Level 2 and Level 3. It is obvious that the pre-service teachers in the study group used formulas with necessary assumptions, variables and relations in their solutions and interpreted the mathematical results. On the other hand, the pre-service teachers in the comparison group tried to reach the solution directly without using any solution model and without making any evaluation on the basis of the relations obtained from the data.

In the first sub-question of the Mount Fuji Climbing question, there were more pre-service teachers performing at Level 1 in the comparison group, while more pre-service teachers performing at Level 3 in the study group. From the solutions while it was seen that the pre-service teachers in the comparison group tried to reach direct conclusions about the situation with the solutions given to them, the pre-service teachers in the study group effectively read and understood the given problem and after establishing a relationship between the data and interpreting them, they were engaged in mathematization. About half of the pre-service teachers in the comparison group did not answer the second sub-question. On the other hand, most of the remaining pre-service teachers performed at Level 3. For the same question, it was observed that a small number of the pre-service teachers in the study group did not answer the question and that the majority of the pre-service teachers who answered the question performed at Level 3. When the solutions to the second sub-question were examined, it was seen that the pre-service teachers read and understood the question and they tried to find a solution by using and evaluating the assumptions and relationships among the data. In the third sub-question, while there was no pre-service teacher performing at Level 3 in the comparison group, the majority of the pre-service teachers performed at Level 1. The majority of the pre-service teachers in the study group on the other hand performed at Level 3. When the solutions of the pre-service teachers in the comparison group were analyzed, it was seen that they tried to reveal direct results about the given problem situation while the pre-service teachers the study group formed a solution model by establishing relationships and making interpretations about the problem situation.

In the first sub-question of the Drip Rate question, half of the pre-service teachers in the comparison group performed at Level 2 and the majority of the remaining half at Level 3. In the same question, the majority of the pre-service teachers in the study group performed at Level 3. When the solutions produced by the pre-service teachers were examined, it was seen that while the pre-service teachers in the comparison group did not state a definite result by making comments without using the formula given in the question, the pre-service teachers in the study group came to a conclusion by using the formula given in the question and making associations on the basis of the formula. In the second sub-question, the majority of the pre-service teachers in both the comparison group and the study group performed at Level 3. However, it was seen that some of the pre-service teachers in the comparison group used the given formula, but made the wrong association in the formula as a result of misinterpretation and thus they reached wrong solutions. It was observed that the pre-service teachers in the study group correctly evaluated the formula given in the solution of the problem and reached the right conclusion through the associations they made.

DISCUSSION AND SUGGESTIONS

Today, the importance of mathematical literacy and therefore mathematization has begun to be recognized more. Thus, both learners and teachers are expected to develop themselves in this regard (Kabael & Barak, 2016). In the current study, the contribution of mistake-handling activities to the development of pre-service elementary school mathematics teachers' competency of mathematization in the context of mathematical literacy is described.

When the results of the current study, which is thought to contribute to the development of mathematical literacy, are examined, it is seen that according to the scoring of the answers they gave to the PISA questions, the pre-service teachers in the study group received more full points than the pre-service teachers in the comparison group and demonstrated more performances at Level 3. In addition, the results revealed that the pre-service teachers in the study group answered more questions than the pre-service teachers in the comparison group. When the answers given by the groups to the questions are examined in detail, it is seen that the solutions of the pre-service teachers in the study group are more explanatory and include more mathematical expressions than those of the pre-service teachers in the comparison group. When the answers of the pre-service teachers given to the PISA questions are examined considering their scoring as a full, partial and zero point, it is seen that the pre-service teachers in the study group received more full points than partial and zero points, while the pre-service teachers in the comparison group received more partial and zero points. When both the PISA scoring and mathematization level results are examined, it is seen that the study group performed higher in seven questions out of a total of eleven questions directed to the pre-service teachers, while both groups performed close to each other in three questions. In the remaining one question, the comparison group performed better than the study group. Similarly, Breen et al. (2009) revealed that from the six-stage literacy level specified in PISA, 23% of the pre-service teachers are at Level 3 and below, 39% at Level 4, and 38% at Levels 5 and 6. Tarım et al. (2017) concluded in their study that the pre-service teachers have an average level of mathematical literacy. However, in a study conducted on the basis of activity design, it was noted that the mathematical literacy levels of pre-service primary teachers improved (Canbazoğlu & Tarım, 2020). In another experimental study, it was concluded that the mathematical literacy levels of high school students subjected to realistic mathematics applications were significantly different when compared to those of the students subjected to standard education applications (Sumirattana, Makanong & Thipkong, 2017).

When the mathematization levels of the pre-service teachers are examined, it is seen that the study group performed more at Level 3, while the comparison group performed mostly at Level 1. According to these results, it is seen that the pre-service teachers in the study group read the given questions more effectively, paid more attention to the root of the question, reached solutions more carefully and performed better in developing different solutions compared to the pre-service teachers in the comparison group. It can be concluded that development of the abilities of reading questions effectively, creating different solutions and paying attention was positively correlated with the engagement of mistake-handling activities by the pre-service teachers in the study group. It is seen that the positive changes occurred in how the pre-service teachers in study group perceive the root of a question and ways of solving a question as a result of their engagement with mistake-handling activities for three weeks. It can be thought that this change and development helped the pre-service teachers to understand, interpret and question the PISA questions they have solved. In fact, questioning and interpretation are among the important skills that mistake-handling activities will develop in pre-service teachers (Aksu et al., 2016; Borasi, 1987; Gedik & Konyalıoğlu, 2019; Heinze & Reiss, 2007; Santagata, 2005). In some previous studies, it was found that when mistake-handling activities were included in the implementation process, pre-service teachers' perceptions of and attention to questions and solutions improved positively (Bilgili et al., 2020; Lucero & Elmore, 2017; Özkaya & Konyalıoğlu, 2019). Yılmaz and Tekin-Dede (2015) observed that pre-service teachers actively validated throughout the process and were able to correct the mistakes they detected. Thus, they concluded that the pre-service teachers had performed extensive mathematization. As a result of their study investigating the mathematization of high school students in the context of mathematical literacy, Mariani and Hendikawati (2017) concluded that the group subjected to realistic mathematics education had better mathematization performance than the group subjected to traditional mathematics education.

In light of the results obtained in the current study, it is thought that such studies will be more effective if mistake-handling activities are applied in a longer time period and pre-service teachers are engaged in both horizontal and vertical mathematization activities. When the importance of mathematization competency is taken into consideration within the context of mathematical literacy, it can be suggested that mistake-handling activities should be applied to pre-service teachers for them to develop their other competencies influential on mathematical literacy.

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Ethical Statement: We act in accordance with scientific ethical principles and rules from all stages of the study, including preparation, data collection, analysis, and presentation of information; we have cited all data and information not obtained within the scope of this study and we have included these sources in the bibliography; we have not made any changes in the data used and that we comply with ethical duties and responsibilities by accepting all the terms and conditions of the Committee on Publication Ethics (COPE). At any time, we declare that we consent to all moral and legal consequences that may arise in the event that a situation contrary to this statement we have made regarding the study is detected.

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