Mathematical Thinking as a Predictor of Critical Thinking Dispositions of Pre-service Mathematics Teachers*

Halil Coşkun Çelik [i]
Siirt University

Furkan Özdemir [i]
Siirt University

Abstract

The aim of this study was to examine the relationship between mathematics thinking sub-dimensions (higher order thinking, reasoning, mathematical thinking and problem-solving) and critical thinking disposition. In addition, this study investigated whether mathematical thinking sub-dimensions are significant predictors of critical thinking disposition. This research was designed as a correlation type relational survey model. The study group consisted of 181 (73 female and 108 male) pre-service mathematics teachers studying at the Faculty of Education of a state university in the South East Anatolia Region of Turkey. Data were collected through personal information form, "mathematical thinking scale", and "California critical thinking disposition inventory". The analysis of data was made with multiple regression analysis. As a result of the study, it was found that there is a significant relationship between critical thinking and higher order thinking, reasoning, mathematical thinking skills and problem-solving sub-dimensions. In addition, the predictors of critical thinking disposition were found to have reasoning, mathematical thinking skills and problem-solving sub-dimensions. Results also illustrated that reasoning; mathematical thinking skills and problem-solving sub-dimensions was a significant predictor of critical thinking dispositions. In this context, the study was discussed in the context of the development of critical thinking of pre-service mathematics teacher. Research results were discussed in the context of the development of mathematical thinking and critical thinking dispositions of pre-service mathematics teachers.

Keywords: Mathematical Thinking, Problem Solving, Predictors of Critical Thinking, Reasoning, Regression Analysis.

DOI: 10.29329/ijpe.2020.268.6

---

*This manuscript was presented at the 2nd International Conference on Language, Education and Culture (ICLEC-2019) is as verbal statement.

[i] **Halil Coşkun Çelik,** Assoc. Prof., Department of Mathematics and Science Education, Siirt University Education Faculty, ORCID: 0000-0003-0056-5338

Correspondence: hcoskun.celik@gmail.com

[ii] **Furkan Özdemir,** Research Assist Dr., Mathematics and Science Education, Siirt University Faculty of Education, ORCID: 0000-0001-9116-1480

INTRODUCTION

Mathematics education allows us to survive in our complex life by not providing important support such as thinking, linking between events, reasoning, making predictions, problem-solving as well as the function of teaching the numbers, operations and gaining the calculation skills in daily life (Umay, 2003). In its loosest sense thinking signifies everything that, is "in our heads" or that "goes through our minds." (Dewey, 1910). Thinking is the power/capacity or ability that can be learned as a cognitive process in an attempt to acquire knowledge (Husnaeni, 2016). Thinking is very important for everyone in everyday life, and mathematical thinking is essential to understanding the world around us. Therefore, teachers and teacher candidates need to plan and implement their teaching activities in a way that will improve their students' mathematical thinking, problem-solving, reasoning and critical thinking skills dispositions. However, the students who do not develop their thinking skills will not be able to use the knowledge in their repertoire because the greatest mental activity will be to memorize and try to remember what they have memorized.

Mathematical Thinking

Mathematical thinking is the use of mathematical techniques, concepts and processes to solve problems directly or indirectly (Henderson, et al., 2002). Mathematical thinking is a dynamic process that makes it easier for us to understand complex structures by combining our ideas (Mason, Burton & Stacey, 2010). The individual tries to solve problems throughout his life at school, at work and in daily life (Blitzer, 2003) and for this he needs mathematical thinking. Morever, mathematical thinking, making logical inferences in mathematics; use the ways of thinking in order to solve the mathematical problem, use the mathematical creativity power to properly combine the ways of thinking for mathematical questions and to protect and understand the mathematical ideas (Duran, 2005).

When the literature is examined, it is seen that different researchers try to reveal the components of mathematical thinking. For example, Tall (2002) states that mathematical thinking includes components such as abstraction, synthesizing, generalizing, modeling, problem solving, and proof. Stacey, Burton and Mason (1985) examined the specialization, generalizing, conjecturing, justifying and convincing components of mathematical thinking. Similarly, mathematical thinking involves exploring, logically associating and expressing ideas, intuitively predicting relationships between facts and procedures, and using it to solve problems (Başaran, 2011). Mathematical thinking skills can be developed as a result of various activities such as dealing with a problem, thinking on experiences and studying a designed problem process (Hacısalihoğlu, Mirasyedioğlu & Akpınar, 2003).

Problem solving enables students to gain experience in general mathematical strategies such as abstraction, expression, symbolization, generalization, proving, and posing new questions (Busbridge & Özçelik, 1997). Problem-solving in mathematics (by the nature of mathematics) is the elimination of the problem by using the necessary information and processes by means of mental processes (reasoning) (Altun, 1995). Mathematical thinking will take place when high-level thinking skills are needed in the process of problem-solving, such as generalization, estimation, customization, hypothesis generation, and assessment of accuracy (Taşdemir, 2008; Türnüklü & Yeşildere, 2005). In mathematics teaching, especially in problem-solving process, mathematical thinking performance of teachers and prospective teachers can positively shape learners' learning. In other words, it is important for the teachers to be equipped in problem-solving in terms of educating students as individuals who can solve the problems they face, and this level of equipment and knowledge of the teachers and pre-service teachers will improve the mathematical thinking skills of the learners and their gains in problem-solving (Ersoy & Güner, 2014). This is an ideal scenario that is desirable in learning and teaching activities. Students' mathematical thinking can be improved if they experience mathematical thinking, so when mathematical thinking is high, learners can solve math problems with low math anxiety and show positive attitudes towards mathematics (Kargar, Tarmizi & Bayat, 2010).

Mathematical thinking is a basic skill which is foreseen to be gained to students in mathematics teaching. However, most learners do not care about mathematical thinking because they think that mathematical thinking is insignificant for their personal development. Therefore, it is very important for the individual to acquire the skills associated with mathematical thinking, how problem-solving provides rich opportunities for the development of mathematical thinking, and how courses can be prepared to improve mathematical thinking (Kargar et al., 2010). This skill can only be achieved by creating an effective teaching environment (Taşdan, Çelik & Erduran, 2013). It is seen that it is important for teachers and prospective teachers to have comprehensive knowledge about (Erktin, 2002) how to teach mathematics as well as thinking skills in order to organize the learning environments so that they can develop learners' math perceptions and thinking skills (Olkun & Uçar, 2004). This can happen with teachers who can think mathematically. Dealing with mathematical thinking of students will contribute significantly to the development of pre-service teachers' teaching (Taşdan et al., 2013).

The most important feature that distinguishes mathematical thinking which includes high-level thinking skills (Yeşildere & Türnüklü, 2007) from other forms of thinking is that an individual obtains a new knowledge or concept by using abstraction, estimation, generalization, hypothesis and testing, reasoning, proving and describing using mathematical knowledge and concepts previously learned (Alkan & Güzel, 2005). Students with high-level thinking skills are able to recognize the reflections of the information they have learned in real-life situations and gain the ability to understand the problems they encounter and to solve them easily and effectively. Therefore, shaping of mathematics teaching activities based on innovative learning and teaching approaches that can improve the high-level thinking skills of the learners will enable them to acquire the critical thinking skill (Runisah, Herman & Dahlan, 2016) which is a high-level thinking skill in the problem-solving process. If we can examine students' mathematical thinking processes and skills in detail and determine their thinking habits, we can accelerate the development of their existing habits towards higher-level thinking habits, and consequently improve their thinking processes (Sezer, 2019).

Critical Thinking

Critical thinking is one of the primary characteristics of success. Meyers (1986) argued that students should learn to think and reason critically in order to reach their highest potential in today's society. Critical means to understand things and people around us by questioning and analyzing our own thinking processes as much as the thinking processes of others (Emir, 2012; Sumarna, Wahyudin & Herman, 2017). It is seen in the literature that there are various definitions of critical thinking. Halpern's (1999) definition is that critical thinking refers to the use of cognitive skills or strategies that increase the likelihood of a desired outcome. Critical thinking is a reasonable reply to questions that cannot be answered definitively and for which all the relevant information may not be existing (Kurfiss, 1988). He defined it as an investigation whose aim is to explore a situation, phenomenon, question, or problem to arrive at a hypothesis or conclusion about it that integrates all present information and that may therefore be convincingly justified. According to Halpern (2003) critical thinking is the strategy of evaluating the thinking process. Critical thinking is a basic skill that attaches importance to the way in which information is learned, the learning process, and the effective application of what it has learned (Adıgüzel, 2017). Although several definitions have been presented in the intervening years, most of them contain the same basic principles. Critical thinking is purposeful, logical, and objective. It is a type of thinking that thinks about problem solving, formulating inferences, calculating probabilities and making decisions. Critical thinkers use these skills appropriately, without asking, and often consciously in various settings. That is, they must be capable of critical thinking. When we think critically, we evaluate the results of our thought processes (how good a decision is or how well a problem is solved) (Halpern, 2003).

Critical thinking is a set of discrete mental processing used to establish the worth or accuracy of something as well as a set of dispositions that guide their use and execution (Beyer, 1988). Critical thinking is an active and organized mental process that aims to understand our own thinking

processes, understand the opinions of others, apply what we have learned, understand ourselves and the events around us (Cüceloğlu, 1994, 216). This definition is broad enough to cover various perspectives; therefore, critical thinking can be taught as argument analysis (Kahane & Cavender, 2006), problem solving (Mayer, 1992), or cognitive process (Rabinowitz, 1993). Other words, critical thinking is "purposeful, self-regulatory judgement that results in interpretation, analysis, evaluation, and inference, as well as explanations of the considerations which that judgment is based" (Fonga et al., 2017). In addition, critical thinking is the ability and disposition of the ability to acquire, interpret, analyze and evaluate, improve, use and apply knowledge in order to reach a reliable and valid conclusion (Eryaman, 2007).

Critical thinking skills should be section of student's learning, teachers and teacher candidates should be responsible to develop and evaluate critical thinking skills of students through teaching and learning process (Firdaus, Kailani, Bakar & Bakry, 2015). But this ability has not developed well on students. At this point, great responsibilities fall on teachers and prospective mathematics teachers. Critical thinking involves both disposition and skill (Fonga et al., 2017). Both are necessary because none by himself is enough to capture what is normally meant by referring to someone else as a 'critical thinker'. Six skills (interpretation, analysis, evaluation, inference, explanation, and self-regulation), 19 dispositions (including inquisitiveness, open-mindedness, understanding others, and so on) was identified to critical thinking (Abrami et al., 2008). Therefore, teaching critical thinking requires both cognitive and affective domains of a student's learning in one area (Mcpeck, 1981). Critical thinking characterizes as a self-adjusting process of judging what to believe or what to do in a given context (Beyer, 1988). In other words, critical thinking is a multifaceted concept that includes cognitive skills and affective tendencies (Chukwuyenum, 2013). At this point, critical thinking is different from ordinary thinking (Aslan, 2018). When the explanations given for critical thinking are considered, it is seen that the researchers emphasize that critical thinking is a mental process.

While there may be similar characteristics between critical thinking and other forms of thinking, some forms of thinking may involve critical thinking or the opposite may be the case (Palavan, Gemalmaz & Kurtoğlu, 2015). Critical thinking is to identify the connections between disciplines and to process in-depth knowledge to find creative solutions to problems (Stobaugh, 2013). Critical thinking is seen as a universal goal of higher education, but it is rarely accepted as a result (Magno, 2013). Students demonstrate critical thinking skills when assessing their own learning and learning paths (Emir, 2012). Learning to think critically is to know when to question something and what kind of questions to ask (Mcpeck, 1981). Because people who participate in critical thinking tend to provide evidence and reasoning for their views (Title, 2011). Students need to be taught how to think critical thinking skills as teachers (Karbalaei, 2012). Regardless of the academic background of the instructor or the language used to describe critical thinking, all of these approaches share a number of common assumptions: there are identifiable critical thinking skills that can be taught and learned, and students think better when they learn and apply them appropriately (Alper, 2010). When applying critical thinking in school settings, it is necessary to develop thinking skills because critical thinking people will be able to understand the logical connections between ideas, formulate and evaluate arguments, and solve problems through reasoning and systematically (Chukwuyenum, 2013). However, Recalde (2008) stated that although teachers had high critical thinking skills, they could not use these skills in teaching activities in the classroom. It does not show parallelism with this idea.

Critical thinkers, on the other hand, use reflective decision-making and thoughtful problem-solving to analyze situations, evaluate discussions, and make appropriate inferences (Stobaugh, 2013), understand the world around them and make good decisions. These skills are extremely critical to increase learners' level of university preparation, make life decisions, make educational assessments, and increase their motivation. Obviously, the benefits of incorporating critical thinking into the education program cannot be denied. Thus, students can learn critical thinking skills and apply them to improve their performance and reasoning skills (Chukwuyenum, 2013). When using critical thinking in existing teaching methods, it is more important to teach students how to think rather than what to think. Current teaching models relate to the integration of critical thinking into the core curriculum (Karbalaei, 2012). Critical thinking skills can improve the quality of mathematics learning

in a better and meaningful way, so there should be a systematic discipline for the development of these skills through mathematics learning at school (Cobb, Wood, Yackel & McNeal, 1992). In this context, it is important to transfer critical thinking skills to daily education in order to prepare students adequately for complex life situations and school assessments: In this case, the task of educators should be to create opportunities for learners to develop and advance these skills (Stobaugh, 2013).

Beyer (1988) offers the significant thinking operations in three levels of complexity; a) Thinking strategies, b) Critical thinking skills, and c) Information processing skills. Thinking strategies are large, inclusive, complex processing such as problem solving, decision making, and conceptualizing. He suggested these skills because of constantly used in most academic subjects and frequently identified by experts as significant thinking operations. Critical thinking enables us to think on thinking. It is a thinking that helps to solve problems or make judgments (Gündoğdu, 2009) and even to find the problem (Halpern, 2003). Critical thinking is a thinking that helps to solve problems or make judgments and even to find the problem (Gündoğdu, 2009). Solving a problem requires mathematical thinking. Therefore, individuals are expected to develop mathematical thinking skills during the problem-solving stage (Abrami et al. 2008; Ersoy & Güner, 2014). Appropriate problem-solving strategies and stages used in problem solving process contribute to the development of problem-solving skills. In this process, expressing the strategies used by learners enables them to reflect their critical thinking skills and tendencies (Ersoy & Güner, 2014). Thus, problem solving triggers learners' critical thinking skills and mathematical thinking skills develop at this point. Therefore, examining the relationship between mathematical thinking and predicted as a predictor of critical thinking emerges as a necessity and the information to be obtained becomes an important process that may affect the development of these skills. On the other hand, in teacher education, it is stated that the relationship between critical thinking and various thinking skills such as reflective thinking and creative thinking is effective in the development of related skills (Yüksel, Uzun & Dost, 2013). Thus, a denominator falls to the level of the relationship between mathematical thinking and critical thinking skill, which is more extensive than thinking. Because mathematical thinking, which includes skills such as high-level thinking disposition, reasoning, and problem solving, is also associated with critical thinking, which is a higher-level and more advanced thinking skill. A consistent effort to encourage higher-order thinking skills not only promotes student critical thinking during the current instructional period, but also has a long-term effect, by becoming an integral part of these students' thinking habits (Aizikovitsh-Udi & Cheng, 2015). Critical thinking in mathematics is the process of critical thinking about mathematical evidence, mathematical reasoning and mathematical problem-solving (Krulik & Rudnick, 1995). Without precise critical thinking skills, incorrect information can adversely affect life decisions. In order to tackle these changes, students need to be equipped with thinking skills to examine information and make logical decisions (Stobaugh, 2013). Schools are searching new and innovative ways to teach critical thinking skills using the principles and definitions of existing critical thinking: Critical thinking is the ability to participate in purposeful thinking in order to eliminate personal and social biases (Karbalaei, 2012). However, if everyone can think critically, the problem they face will be a simpler and easier solution.

Critical thinking, which is related to mental processes such as problem-solving, problem building and reasoning, is very important for mathematics education (Yüksel et al., 2013) and there is a general consensus that this thinking is especially important in problem-solving. Learners should be able to develop critical thinking skills to succeed in the era of globalization (Belecina & Jose, 2018). In mathematics teaching, critical thinking skills can be used, applied and developed with effective cognitive methods. Critical thinking can develop creative problem-solving options by encouraging students to look for new strategies while solving mathematical problems (Belecina & Jose, 2018).

Critical thinking is now increasingly being addressed by educators, cognitive psychologists, behaviorists, philosophers, and content analysts (Şenşekerci & Bilgin, 2008). Heng and Sudarshan (2013) showed that teachers should deepen their knowledge about students' constructions of mathematical knowledge and strategies students used to solve problems, and that students should investigate their understanding with a critical approach in teacher-student interaction. Demonstrating critical thinking skills in the classroom and then observing their effectiveness will be of interest to

educators, as it is understood that it is difficult to teach critical thinking (Whitten & Brahmasrene, 2011).

Mathematical thinking and critical thinking are among the basic skills that are foreseen to be gained in the 21st century, especially in mathematics. In this process, teachers play a central role in gaining this skill. Teachers/pre-service teachers need to be able to create an effective teaching environment and have the competencies to make arrangements to improve students' mathematical thinking skills and critical thinking. Critical thinking, which is a high-level thinking skill, includes cognitive skills and affective tendencies.

In the field of mathematics education, it is necessary to determine whether pre-service teachers have a tendency to think critically, to provide them with critical thinking skills and how mathematical thinking approaches can be improved. Jacobs, Lamb, and Philipp (2010) showed that teachers find it difficult to comprehend students' mathematical thinking, but these skills can be improved. Mathematical thinking skills are crucial for learning mathematics and achieving learning goals (Toheri & Winarso, 2017). As explained above, problem-solving and reasoning skills are of high priority among the important components of mathematical thinking in mathematics education (Suzuki, 1998). Therefore, the information obtained from a study about the effect of mental components such as problem-solving and reasoning or other variables that are not covered in this study will be of high importance between the common denominators of mathematical thinking and critical thinking.

On the other hand, examining and developing the pre-service teachers' current status regarding the sub-dimensions of mathematical thinking (high-level thinking disposition, reasoning, mathematical thinking skills and problem-solving) in effective teaching activities and in the process of undergraduate education, it is considered that it is important to equip them with other thinking skills such as critical thinking, especially in terms of high level thinking, problem-solving and reasoning components. Because the related assessments may have significant effects on the pre-service teachers' academic achievement and the development of mathematical thinking and critical thinking tendencies.

In other words, mathematics thinking and critical thinking tendencies are needed to be developed in order for pre-service teachers to teach mathematics to their students more effectively and permanently. In this context, this study supports the need to identify and develop the sub-dimensions of mathematical thinking that are thought to have an impact on the pre-service teachers' critical thinking disposition. In addition, revealing the meaningful effects of the development of mathematical thinking on critical thinking may create supportive effects for the pre-service teachers to perform more effective teaching in classroom settings. On the other hand, although there are studies on the different thinking tendencies of critical thinking (problem-solving, epistemological belief and creative thinking) in the literature (Kutluca, 2018), no research examining the relationship between the sub-dimensions of mathematical thinking and critical thinking disposition has been found. In this context, the main starting point in this study was the relationship between the sub-dimensions of mathematical thinking and critical thinking disposition. Considering the combined effect of the relevant sub-dimensions is considered important for the development and teaching of critical thinking. In the light of the explanations, it is thought that the findings obtained from this study will contribute to the literature on the subject. Also, Ovayolu (2010) explained of students in Turkey in the PISA 2006 mathematics test scores obtained in terms of higher-level thinking processes have indicated that relatively low. Therefore, it is foreseen that our current study will fill an important gap in the development of high-level thinking skills of students.

Purpose

The aim of this study was to examine the relationship between mathematics thinking sub-dimensions (higher order thinking, reasoning, mathematical thinking and problem-solving skill) and critical thinking disposition. In addition, the extent to which the sub-dimensions of mathematical

thinking predicted the critical thinking disposition were examined. In this context, the following questions were sought:

1. What is the level of pre-service mathematics teachers' critical thinking disposition and sub-dimensions of mathematical thinking?

2. Do mathematics thinking sub-dimensions of pre-service mathematics teachers significantly predict their critical thinking dispositions?

METHOD

Research Model

This research was designed as a correlation type relational survey model. In this study, it was aimed to determine the relationship between the mathematical thinking and critical thinking disposition variables of pre-service mathematics teachers (Karasar, 2012). Pre-service teachers' critical thinking dispositions dependent variable (predicted) and the sub-dimensions of mathematical thinking (high-level thinking disposition, reasoning, mathematical thinking skills, problem-solving) were taken as independent variables (predictor).

Population and Sampling

In order to develop students' mathematical thinking skills, various responsibilities are imposed on teachers (Alkan & Güzel, 2005). Therefore, the responsibilities of the pre-service teachers in using mathematical thinking in their professional lives will be even higher. For this reason, the study group of the research consists of all pre-service mathematics teachers (N=214) studying in the mathematics and science education department of a public university in the Southeastern Anatolia Region in 2018-2019 academic year. As all pre-service mathematics teachers were tried to be reached, sampling method was not used and the whole universe was tried to be reached. However, 181 (73 female and 108 male) pre-service teachers were reached because there were pre-service teachers who were not present in the class that day and did not want to participate in the study. The average age of the pre-service teachers included in the research was 21.4±2.9 years and the overall academic success average was 72.6±7.6 out of 100 points. The reason for the selection of pre-service mathematics teachers; researchers conduct courses in these classes and it is important to reveal the tendency of critical thinking with mathematics courses taken by pre-service teachers.

Data Collection Tools

The data collection tools used in this research consist of two parts. In the first part, there are gender, age and academic achievement points of participants, and in the second part there is a scale of mathematical thinking and a critical thinking scale.

Mathematical thinking scale: This scale was developed by Ersoy and Başer (2013) in order to measure the learning dispositions of the participants in cognitive dimension. The scale consists of 25 items (e.g; Knowledge should be used effectively for mathematical thinking. Mathematical thinking does not help in solving my daily life problems) answered on a rating scale from "1 (totaly agree)" to "5 (never disagree) under four sub-dimensions. He stated that the four dimensions of the scale explained 61.86% of the total variance. 6 items (5-9-17-18-19-25) in the higher order thinking sub-dimension of the scale, and 4 items in the reasoning sub-dimension (1-2-3-4), 8 items in the mathematical thinking skill sub-dimension (6-7-8-20-21-22-23-24) and in the problem-solving sub-dimension there are 7 items (10-11-12-13-14-15-16). Higher scores on the scale indicate that the level of mathematical thinking is relatively high, and low scores indicate that the level of mathematical thinking is relatively low. In addition, the Cronbach's Alpha coefficient, which is used to test the reliability of the scale, was calculated as .86. Since this value is greater than .70 (Leech, Barrett & Morgan, 2005), it can be said that the scale is good level reliable.

California critical thinking disposition inventory: The scale which is used to determine the critical thinking disposition of the people was created within the framework of a project organized by the American Philosophy Association and translated into Turkish by Kökdemir (2003). The scale, which has 51 items, has six dimensions in six-grade likert type ("Totally Agree=6", "Agree=5"…"Never Agree=1"). He reported that the total variance ratio explained by the six sub-dimensions of the scale was 36.13%. The high score obtained from the scale shows the disposition to think highly critically, and the low point indicates a low critical thinking disposition. The scores below 40 are low, 40-50 points are intermediate and above 50 points show high level of critical thinking for each subdimension. On the other hand, Cronbach's Alpha reliability coefficient of the scale was calculated as .85. Since this value is greater than .70, it can be said that the scale is good level reliable.

Data Analysis

SPSS 21.0 package program was used to analyze the research data. Critical thinking dispositions and mathematical thinking levels of pre-service teachers were calculated with the help of arithmetic mean and standard deviation values. On the other hand, whether the mathematical thinking sub-dimensions predict the pre-service teachers' critical thinking dispositions was examined by multiple linear regression analysis. At this point, the assumptions about multiple linear regression analysis were tested first. For the extreme value analysis, Mahalanobis distance values were determined to be between .02 and .99. When the values at $p<.001$ were taken as an extreme value, it was seen that the data did not contain the extreme value. It was determined by calculating the skewness and kurtosis coefficients that the variables showed normal distribution and it was determined that the related coefficients varied between +2 and -2 in all variables. Accordingly, it was accepted that the data showed no significant deviation from the normal distribution. The correlations of each independent variable with the dependent variable were investigated and all independent variables were linearly correlated with critical thinking disposition. Another assumption that must be tested to implement multiple regression analysis is that there is no multiple connection problem between independent (predictive) variables. Multicollinearity is a high level of relationship between independent variables (Büyüköztürk, 2018). Pearson Product Moment Correlation Coefficient were calculated to determine whether there was multicollinearity between the independent variables. Since the calculated correlation coefficient values are between .02 and .65, it can be said that there is no multicollinearity problem. If the tolerance value is lower than .20 and the variance magnification factor (VIF) value is greater than 10, there is a multicollinearity problem (Tabachnick & Fidell, 2013). In this context, it can be said that there is no multicollinearity problem in the multiple regression analysis process, since the smallest calculated tolerance value for the data pattern is .42 and the highest VIF is 2.35.

FINDINGS

The mean and standard deviation values of the pre-service mathematics teachers' mathematical thinking sub-dimensions and critical thinking disposition levels were found, the relationships between variables were calculated by Pearson Product Moment Correlation Coefficient. The findings are presented in Table 1.

Table 1. Descriptive statistics and correlation coefficient values related to mathematical thinking and critical thinking levels (n=181)

Variables		Mean	S.D.	CT	HTT	R	MTS	PS
ELD		3.94	.48	1				
MT	HTT	3.89	.47	.329**	1			
	R	3.94	.51	.351**	.352**	1		
	MTS	3.56	.42	.461**	.385**	.404**	1	
	PS	3.43	.46	.390**	.322**	.280**	.459**	1

**p<.01, [CT: Critical Thinking; MT: Mathematical Thinking; HTT: High-level Thinking Tendency; R: Reasoning; MTS: Mathematical Thinking Skill; PS: Problem-Solving]

When Table 1 is examined, it is seen that the average score of critical thinking disposition of pre-service mathematics teachers is 3.94. In addition, in terms of sub-dimensions of mathematical thinking scale, the highest score average of the pre-service teachers is in the reasoning dimension (3.94), the lowest average score is the problem-solving dimension (3.43). On the other hand, between the critical thinking disposition and the high-level thinking dimension of mathematical thinking (r=.329; p<.01), reasoning dimension (r=.351; p<.01), mathematical thinking skill dimension (r=.461; p<0.01) and the problem-solving dimension (r=.390; p<.01) is found to have a positive, moderate, significant relationship.

Multiple linear regression analysis was used to determine whether the sub-dimensions of mathematical thinking were a significant predictor of critical thinking dispositions of pre-service teachers. According to the results of the analysis, ANOVA F value which testes the significance of the model as a whole was obtained as F=17.737 (p<.01). This shows that the model of predicting critical thinking by mathematical thinking is acceptable. The coefficient of determination is R^2=.287. Regression coefficients, significance levels, partial and bivariate correlations are shown in Table 2.

Table 2. Results of multiple linear regression analysis on the power of critical thinking disposition to predict mathematical thinking

Variables	B	Standard Error	β	t	p
Constant(CT)	1.155	.340		3.400	.001
HTT	.113	.073	.111	1.546	.124
R	.141	.068	.149	2.081	.039
MTS	.313	.089	.272	3.513	.001
PS	.196	.076	.187	2.567	.011

Dependent variable: CT. R=.536, R^2=.287, F(4, 176)=17.737, p=.000

The obtained regression model shows that the dimensions of mathematical thinking, high-level thinking tendency, reasoning, mathematical thinking skills and problem-solving dimensions have a moderate significant relationship with pre-service teachers' critical thinking disposition levels (R=.536, R^2=.287, p<.01). Together, these variables explain 28.7% of the total variance in critical thinking. Accordingly, the 72% change in the level of critical thinking can be explained by different variables not included in the regression model.

According to standardized regression coefficients (β), the relative importance of predictive variables on critical thinking disposition; MT, PS, R, HTT. According to the results of the t-test on the significance of regression coefficients, reasoning (B=.141, p<.05), mathematical thinking skills (B=.313, p<.05) and problem-solving (B=.196, p<.05) variables are significant (important) predictors in explaining pre-service teachers' critical thinking levels. Considering the positive relationship found, the increase in reasoning, mathematical thinking skills and problem-solving success is thought to improve the increase in the critical thinking of pre-service teachers positively. On the other hand, the high-level thinking tendency variable has no significant effect. The regression-equation (mathematical model) for the prediction of critical thinking is given below.

CT=1.155+ .113*HTT + .141*R + .313*MT + .146* PS

DISCUSSION, RESULT AND SUGGESTION

In this study, the relationships between mathematical thinking and critical thinking dispositions of pre-service mathematics teachers were examined. In addition, it has been determined whether the sub-dimensions of mathematical thinking are significant predictors of critical thinking disposition. The results of the research showed that mathematical thinking was a significant predictor of explaining the pre-service teachers' critical thinking disposition.

Critical thinking needs to be high in order for teachers to perform effectively and to be successful in classroom teaching activities. In this context, teachers need to be able to create teaching

environments that can improve the critical thinking skills of learners (Yüksel et al., 2013), to be able to apply the new learning and teaching methods in their classrooms, and to be able to make lesson plans about the activities of these methods. Therefore, it is an important responsibility for pre-service mathematics teachers to teach and develop critical thinking to their students in the future. Therefore, it is thought that the pre-service mathematics teachers should have a high critical thinking disposition due to the nature of mathematics and the department they read. In this study, it was found that the pre-service teachers' critical thinking dispositions were relatively above the middle level. When this result is evaluated in terms of critical thinking dispositions, it is found that there are similar findings in the literature. For example, Deringöl (2017) concluded that pre-service elementary school teachers and mathematics teachers had high levels of critical thinking. Similarly, Ocak, Eymir and Ocak (2016) found that pre-service teachers had a positive level of critical thinking disposition. However, there are different study findings that do not agree with this finding. Yüksel et al. (2013) found that pre-service mathematics teachers' critical thinking dispositions were low. Similarly, Aliustaoğlu and Tuna (2015) found low levels of critical thinking disposition of pre-service teachers studying in different departments. In addition, Türnüklü and Yeşildere (2005), who included pre-service mathematics teachers in their research, found their critical thinking dispositions positively, but they were not high enough. Additionally, Aktaş and Ünlü (2013) found that the critical thinking skills of teacher candidates of elementary mathematics were medium level but not high enough. The different results related to the levels of critical thinking may be due to the sample (preservice mathematics teachers) included in the research or data collection tool applied. It is thought that performing the same study with other participants or with different data collection tool(s) may give different results.

When the findings obtained from the study were evaluated in terms of sub-dimensions of mathematical thinking, it was found that the highest mean score of the pre-service teachers was in the reasoning dimension and the lowest mean was in the problem-solving dimension. Mathematical reasoning is a prominent ingredient of mathematical thinking (Alkan & Taşdan, 2011), and relevant components play essential role in the structuring of mathematical knowledge. The results obtained from the research findings are consistent with this explanation. Morever, pre-service teachers think that reasoning comes first in the process of mathematical thinking and that problem-solving takes place after the components of high-level thinking tendency and mathematical thinking skills. This finding is expected to be due to the characteristics of the sample, its size and the nature of mathematics. When the relevant literature is examined, it is seen that there are limited studies on this subject. Arslan and İlkörücü (2018) applied the mathematical thinking scale in their studies and reported that the highest average of pre-service mathematics teachers was in the mathematical thinking skill sub-dimension, the lowest average was in the reasoning dimension and then the problem-solving dimension. It can be said that these findings do not overlap with some aspects of our research findings. In order to support this, various study findings that determine the level of mathematical thinking are included. In this process, it was seen that Arslan and İlkörücü (2018) found the mathematics thinking levels of pre-service mathematics teachers above the middle level. Toheri and Winarso (2017) emphasized that mathematical thinking skills of pre-service teachers still need to be improved, especially in terms of proving mathematical expressions, completing contextual and open-ended problems, and representing mathematical expressions in various forms through problems requiring high thinking skills. Zhu, Yu, and Cai (2018) stated that specialist teachers relied on previous teaching experiences to understand students' mathematical thinking, but that non-specialist teachers did not trust their previous teaching experiences (they might be incomplete in understanding mathematical thinking) because they did not know how to do so.

Critical thinking skills encourage students to think independently, to solve problems in the context of school or daily life (The National Council of Teachers of Mathematics [NCTM], 2000). Problem-solving is a high-level special method knowledge (McCormick, 1996). In other words, critical thinking skill is a necessary tendency needed in problem-solving (Husnaeni, 2016) and students with advanced skill gain problem-solving skills. In the early stages of the problem-solving process, students are expected to understand the problem and realize what they really need to solve before solving the problem. Performing these steps supports the development of critical thinking skills (Türnüklü & Yeşildere, 2005). Critical thinking in mathematics learning is a process of cognitive or

mental action that is based on mathematical reasoning in an effort to acquire mathematical knowledge (Husnaeni, 2016). Similarly, Mansoor and Pezeshki (2012) stated that critical thinking involves reasoning. This is supported by the fact that critical thinking states that there is reasonable reasoning about what to believe and what to do (Title, 2011). To have critical thinking skills by knowing and using the principles of reasoning. Furthermore, high-level thinking skills include some cognitive activities such as reasoning, critical thinking, problem-solving, making a judgment, coping with uncertainty, etc., which require the systematic organization of basic thinking skills (Çakır & Senemoğlu, 2016). The results of this study show that there is a significant relationship between critical thinking and mathematical thinking, high-level thinking tendency, reasoning, mathematical thinking skills and problem-solving dimensions. Findings from the studies support this finding (Arslan & İlkörücü, 2018; Kölemen & Erişen, 2017). Arslan and İlkörücü (2018) found that there is a significant relationship between the reasoning and mathematical thinking levels of pre-service mathematics teachers. Kölemen and Erişen (2017) found that students had a strong positive relationship between problem-solving and critical thinking skills. On the other hand, Prayitno (2018) stated that the critical thinking process can be used in mathematical problem-solving reflects the relationship between critical thinking and problem-solving and this is consistent with the findings of our study.

The dimensions of mathematical thinking, high level thinking tendency, reasoning, mathematical thinking skills and problem-solving variables together contributed 28.7% to explain the total variance in critical thinking. This ratio shows that there are different variables that may have an impact on critical thinking. Considering that this study was conducted on pre-service mathematics teachers, it can be said that academic achievement which may affect critical thinking disposition and neglected within the scope of the study may be effective. A strong critical thinking pedagogy that supports students' critical knowledge, skills, and dispositions can increase students' academic achievement (Karbalaei, 2012). Many researchers have stated that the development of critical thinking skills can improve mathematics achievement (Chukwuyenum, 2013; Özcan, 2017; Özelçi, 2012). For example, Özelçi (2012) reported that "academic achievement" had a significant effect on the prediction of critical thinking attitude. Özcan (2017) found that critical thinking skills were related to mathematics achievement and predicted mathematics achievement. For this reason, it can be said that activities that improve students' critical thinking skills (effective use in educational environments) can improve mathematics achievement. Similarly, Jacob (2012) found a meaningful linear relationship between critical thinking and mathematics achievement of university students, and concluded that mathematical achievement would develop positively when critical thinking skills were properly supported. In addition to academic achievement, it is seen in the study findings that there are other factors that predict critical thinking. For example, in his study, Riccio (2015) found that variables such as time spent in graduate education, course satisfaction and whether a student felt supported were important determinants of students' improvement in critical thinking, it also found that learning in the connectedness component (in the course, in the classroom, with the trainer, and in the group) was the best predictor of whether students' critical thinking skills were developed. Unlike these studies, Facione (1990) found that students' critical thinking self-confidence was not a significant predictor of critical thinking.

In the study, the predictive power of the sub-dimensions of mathematical thinking on the critical thinking disposition was examined. The results of the analysis showed that reasoning, mathematical thinking skills and problem-solving dimensions of mathematical thinking were significant predictors of explaining critical thinking disposition. The positive relationship between dimensions of reasoning, mathematical thinking skills and problem-solving dimensions of mathematical thinking shows that the increase in these dimensions can lead to a positive increase in the critical thinking levels of pre-service teachers. In other words, reasoning, mathematical thinking skills and problem-solving are the most effective predictors of the critical thinking disposition. It is the responsibility of educational institutions to develop and support students' critical thinking skills (Karbalaei, 2012). The primary and main tool used to understand or solve the math problem is reasoning (Napitupulu, 2017). In the secondary school mathematics curriculum, emphasis is placed on developing some basic skills (problem-solving, mathematical process skills, affective skills,

psychomotor skills, etc.) for effective learning and use of mathematics. Reasoning is also one of these basic skills (Ministry of National Education [MEB], 2013: 1). Students' mathematical thinking, problem-solving and reasoning skills are integrated processes. Developing one aspect of these skills is thought to improve the other (Başaran, 2011). Mathematics curriculum based on constructivist teaching, mostly based on inductive reasoning (Özcan, 2017). On the other hand, there are courses covering problem-solving, reasoning and mathematical thinking processes in the undergraduate program of primary school mathematics teachers. These courses can have important contributions to increase students' critical thinking dispositions. Indeed, incorporating critical thinking skills into the curriculum helps to continue educating educated individuals; prepares students for college, future careers and life situations; prepare exams and standards to fulfill their tasks and enable students to develop their success potential positively (Stobaugh, 2013). In other words, equipping students with critical thinking skills enables them to reason effectively, make rational decisions and solve problems (Stobaugh, 2013). Therefore, critical thinking skills should be applied and developed in the teaching and learning process in order to educate students who think quality in the future (Firdaus et al., 2015). Continuous studies should be carried out using various methods and appropriate learning approaches to develop critical thinking skills (Rajendran, 2010) and should be a fundamental agenda integrated with the curriculum of mathematics education. In support of this idea, Karbalaei (2012) emphasized the importance of critical thinking as an active part of the curriculum, regardless of educational level. Therefore, the above-mentioned explanations support the finding that reasoning, mathematical thinking skills and problem-solving dimensions of mathematical thinking are significant predictors of explaining the critical thinking disposition. In addition, the lack of any study on the relationship between related dimensions and critical thinking has left our findings in a situation of incomparability. On the other hand, the findings obtained from this study can be an important guide for the researchers in the related field. Accordingly, it can be said that it may be more positive to develop pre-service mathematics teachers' critical thinking with a focus on reasoning, mathematical thinking and problem-solving

Higher-order thinking is to think at a higher level than memorizing facts or telling something as you are told (Thomas & Thorne, 2014). In the 21st century, the training of students with high-level thinking skills is an important requirement and teachers have an important role to play in achieving this goal. In order to gain high level thinking skills; the efforts of teachers to involve students in non-routine problem-solving process, facilitate critical thinking and creative thinking development and encourage them to form their own knowledge (Apino & Retnawati, 2016) may be effective in the development of students' mathematics achievement. On the other hand, teachers who are not interested in improving their students' high-level thinking skills will cause inconsistencies in teaching activities involving elements of high-level thinking skills and will be more likely to apply traditional teaching methods (Alhassora, Abu & Abdullah, 2017). However, this is a very undesirable scenario in teaching activities. In the present study, it was concluded that mathematical thinking was not a significant predictor of explaining the critical thinking disposition. This finding may be influenced by the lack of or limited courses related to high-level thinking in the mathematics undergraduate program, the lack of self-confidence of the pre-service teachers in the relevant field, and the lack of self-confidence in mathematics or especially in mathematical problem-solving. High-level thinking skills indicate the need for unusual thinking processes or thinking that requires more complex and unusual effort (Apino & Retnawati, 2016). For the formation of high-level thinking, the problem needs to be identified. The student has to think mathematically so that he tries to solve the problem by establishing the relationship between the concepts for the solution of the determined problem (Ersoy & Başer, 2013). This content is a reflection that high-level thinking can have meaningful relationships with mathematical thinking and critical thinking disposition. However, such a result could not be reached in our study.

To develop students' critical thinking knowledge, skills and dispositions, educators can develop instructional pedagogy with relevant and appropriate learning activities / approaches that encouraging critical thinking skills (Karbalaei, 2012). Appropriate approaches to learning make critical thinking easier (Magno, 2013). In this context, it may be suggested that in subsequent studies on this subject, researchers may examine the effect of constructivist learning-based teaching activities

on critical thinking disposition. Furthermore, apart from factors such as reasoning, problem-solving, mathematical thinking skills, it is suggested that more comprehensive researches should be done to determine the effect of components such as mathematical association, mathematics self-efficacy perception and mathematical thinking styles on critical thinking disposition.

REFERENCES

Abrami, P. C., Bernard, R. M., Borokhovski, E., Wade, A., Surkes, M. A., Tamim, R., & Zhang D. (2008). Instructional interventions affecting critical thinking skills and dispositions: A stage 1 meta-analysis, *Review of Educational Research, 78*(4), 1102–1134. Doi: 10.3102/0034654308326084

Adıgüzel, A. (2017). Curriculum development process in education. In B. Oral & T. Yazar (Ed.), *Curriculum development and evaluation in education* (pp. 175-201). Ankara: Pegem Academy.

Aizikovitsh-Udi, E., & Cheng, D. (2015). Developing critical thinking skills from dispositions to abilities: mathematics education from early childhood to high school. *Creative Education, 6,* 455-462. Doi:10.4236/ce.2015.64045

Aktaş, G. S., & Ünlü, M. (2013). Critical thinking skills of teacher candidates of elementary mathematics. *Procedia-Social and Behavioral Sciences, 93,* 831-835. Doi: 10.1016/j.sbspro.2013.09.288

Alhassora, N.S.A., Abu, M.S., & Abdullah, A. H. (2017). Inculcating higher-order thinking skills in mathematics: Why is it so hard? *Man, in India, 97*(13), 51-62.

Aliustaoğlu, F., & Tuna, A. (2015). Investigation of critical thinking dispositions of teacher candidates (Kastamonu University sample) *The Journal of International Education Science, 2*(4), 131-142.

Alkan, H., & Güzel, E. B. (2005). Development of mathematical thinking in the student teachers. *Gazi University Journal of Gazi Educational Faculty, 25*(3), 221-236.

Alkan, H., & Taşdan, B. T. (2011). Mathematical thinking through the eyes of prospective mathematics teachers at different grade levels. *Inonu University Journal of the Faculty of Education, 12*(2), 107-138.

Alper, A. (2010). Critical thinking disposition of pre-service teachers. *Education and Science, 35* (158), 14-27.

Apino, E., & Retnawati, H. (2016). Developing mathematical higher order thinking skills of senior high school students. *Journal of Phisycs*: *Conference Series, 812,* 1-7. Doi: 10.1088/1742-6596/812/1/012100

Arslan, Ç., & İlkörücü, Ş. (2018). Preservice mathematics and science teachers' mathematical thinking levels. *Erzincan University Journal of Education Faculty, 20*(1), 156-166. Doi: 10.17556/erziefd.310384

Altun, M. (1995). *A study on the problem-solving behaviours of the elemamtary school students in the 3th. 4th and 5th grades* (Unpublished doctoral dissertation). Hacettepe University, Ankara, Turkey.

Aslan, S. (2018). The relationship between critical thinking skills and democratic attitudes of 4th class primary school students. *International Journal of Progressive Education, 14*(6), 61-69. Doi: 10.29329/ijpe.2018.179.5

Başaran, S. (2011). *An exploration of affective and demographic factors that are related to mathematical thinking and reasoning of university students* (Unpublished doctoral dissertation). METU, Ankara, Turkey.

Belecina, R. R., & Jose M. O. J. (2018). Effecting change on students' critical thinking in problem-solving. *EDUCARE: International Journal for Educational Studies, 10*(2), 109-118.

Beyer, B. K. (1988). Developing a scope and sequence for thinking skills instruction. *Educational Leadership*, 7, 26-30.

Blitzer, R. (2003). *Thinking mathematically.* New Jersey: Prentice Hall.

Busbridge, J., & Özçelik, D. A. (1997). *Elementary Mathematics teaching. YÖK / Dünya bank national education development project, initial teacher training*. Ankara: Ajans-Türk Publishing.

Büyüköztürk, Ş. (2018). *Data analysis handbook for social sciences*. Ankara: Pegem Academy.

Chukwuyenum, A. N. (2013). Impact of critical thinking on performance in Mathematics among senior secondary school students in Lagos State. *IOSR Journal of Reasearch & Method in Education (IOSR-JRME), 3*(5), 18-25.

Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of classroom Mathematics tradition: An interactional analysis. *American Educational Research Journal, 29,* 573-604.

Cüceloğlu, D. (1994). *Think well and make the right decision (6th Edition).* Istanbul: Sistem Publishing.

Çakır, N. A., & Senemoğlu, N. (2016). Analytical thinking skills in higher education. *Kastamonu Education Journal, 24*(3), 1487-1502.

Deringöl, Y. (2017). Determining critical thinking standards of teacher candidates. *Igdir University Journal of Social Sciences,* 13, 44-65.

Dewey, J. (1910). *How we think.* Boston: D. C. Heath Publications.

Duran, N. (2005). *A research on mathematical thinking skills* (Unpublished master thesis). Hacettepe University, Ankara, Turkey.

Emir, S. (2012). Critical thinking dispositions of education faculty students. *HAYEF: Journal of Education, 17*(1), 34-57.

Erktin, E. (2002). Developing thinking skills in elementary education. *Journal of Educational Sciences, 16,* 61-70.

Ersoy, E., & Başer, N. (2013). The development of mathematical thinking scale. *Kastamonu Education Journal*, 21(4), 1471-1486.

Ersoy, E., & Güner, P. (2014). Mathematics teaching and mathematical thinking. *Journal of Research in Education and Teaching, 3*(2), 102-112.

Eryaman, M. Y. (2007). From reflective practice to practical wisdom: Toward a post-foundational teacher education. *International Journal of Progressive Education*, *3*(1), 87-107.

Facione, P. A. (1990). *The california critical thinking skills test: College level, technical report 3, gender, ethnicity, major, self-esteem and the CCTST.* Millbrae: The California Academic Press.

Firdaus, F., Kailani, I., Bakar, M. N. B., & Bakry, B. (2015). Developing critical thinking skills of students in Mathematics learning. *Journal of Education and Learning, 9*(3), 226-236.

Fonga, C.J., Kima, Y., Davisa, C.W., Hoanga, T., & Kimb, Y. W. (2017). A meta-analysis on critical thinking and community college student achievement. *Thinking Skills and Creativity 26,* 71–83. Doi: 10.1016/j.tsc.2017.06.002

Gündoğdu, H. (2009). Critical thinking and some misperceptions on teaching critical thinking. *Celal Bayar University Jornal of Social Sciences, 7*(1), 57-74.

Hacısalihoğlu, H., Mirasyedioğlu, Ş., & Akpınar, A. (2003). *Teaching mathematics: Constructivist learning and teaching in Mathematics.* Ankara: Asil Publishing.

Halpern, D. F. (1999). Teaching for critical thinking: Helping college students develop the skills and dispositions of a critical thinker. *New Directions for Teaching and Learning*, 80, 69-74. Doi: 10.1002/tl.8005

Halpern, D. F. (2003). *Thought & knowledge: An introduction to critical thinking*. USA: Lawrence Erlbaum Associates, Inc.

Henderson, P. B., Hichtner, L., Fritz, S. J., Marion, B., Scharff, C., Hamer, J., & Riedesel, C. (2002). Materials development in support of mathematical thinking. *ACM SIGCSE Bulletin, 35*(2), 185–190. Doi: 10.1145/782941.783001

Heng, M. A., & Sudarshan, A. (2013). "Bigger number means you plus!" teachers learning to use clinical interviews to understand students' mathematical thinking. *Educational Studies in Mathematics, 83* (3), 471-485. Doi: 10.1007/s10649-013-9469-3

Husnaeni. (2016). The enhancement of mathematical critical thinking ability of aliyah madrasas student model using gorontalo by interactive learning setting cooperative model. *Journal of Education and Practice, 7*(8), 159-164.

Jacob, S. M. (2012). Mathematical achievement and critical thinking skills in asynchronous discussion forums. *Procedia-Social and Behavioral Sciences, 31,* 800-804.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169-202.

Kahane, H., & Cavender, N. (2006). *Logic and contemporary rhetoric: The use of reason in everyday life* (Tenth Ed.). Belmont, California: Wadsworth.

Karasar, N. (2012). *Scientific research method.* Ankara: Nobel Academic Publishing.

Karbalaei, A. (2012). Critical thinking and academic achievement. *Medellin- Colombia, 17*(2), 121-128.

Kargar, M., Tarmizi, R. A., & Bayat, S. (2010). Relationship between mathematical thinking, Mathematics anxiety and Mathematics attitudes among university students. *Procedia Social and Behavioral Sciences, 8,* 537–542. Doi: 10.1016/j.sbspro.2010.12.074

Kölemen, C. Ş., & Erişen, Y. (2017). An investigation on the relationship between problem-solving and critical thinking skill, and academic achievement of vocational and technical high school students. *Turkey Education Journal, 2*(2), 42-60.

Kökdemir, D. (2003). *Decision making and problem-solving under uncertainty* (Unpublished doctoral dissertation). Ankara Universiy, Ankara.

Krulik, S., & Rudnick, J. A. (1995). *The new sourcebook for teaching reasoning and problem-solving in elementary school*. Needham Heights: Allyn & Bacon.

Kurfiss, J. G. (1988). *Critical thinking: theory, research, practice and possiblities.* ASHE-ERIC Higher Education Report No.2, Washington, D.C.: Association for the Study of Higher Education.

Kutluca, A. Y. (2018). The investigation of variables predicting prospective teachers' problem-solving skills. *Asian Journal of Instruction, 6*(1), 1-20.

Leech, N. L., Barrett, K.C., & Morgan, G.A. (2005). *SPSS for intermediate statistics, use and interpretation* (Second edition). Lawrence Erlbaum Associates Publishers, New Jersey.

Magno, C. (2013). Assessing students' critical thinking and approaches to learning. *The International Journal of Educational and Psychological Assessment January, 12*(2), 19-32.

Mansoor, F., & Pezeshki, M. (2012). Manipulating critical thinking skills in test taking. *International Journal of Education, 4*(1), 153-160.

Mason, J., Burton, L., & Stacey, K. (2010). *Thinking mathematically* (Second edition). Harlow England: Pearson Education Limited.

Mayer, R. E. (1992). *Thinking, problem solving, cognition*. New York: Freeman.

McCormick, R. (1996). Instructional methodology. In P. J. Williams & A. P. Williams (Eds.), *Technology education for teachers* (pp. 63–92). Melbourne: Macmillan.

Mcpeck, J. E. (1981). *Critical thinking and education.* New York, NY: Routledge.

MEB. (2013). *Secondary school mathematics curriculum (Grades 5, 6, 7 and 8).* Ankara: Talim Terbiye Kurulu Başkanlığı. Retrieved from https://ttkb.meb.gov.tr

Meyers, C. (1986). *Teaching students to think critically.* San Francisco, CA: Jossey Bass.

Napitupulu, E. E. (2017). Analyzing the teaching and learning of mathematical reasoning skills in secondary school. *Asian Social Science, 13*(12), 167-173.

NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Ocak, G., Eymir, E., & Ocak, İ. (2016). Investigation of pre-service teachers' critical thinking tendencies in terms of different variables. *Erzincan University Journal of Education Faculty, 18*(1), 63-91.

Olkun, S., & Uçar, Z. T. (2004). A*ctivity based mathematics education in primary education (3rd edition). Ankara: Anı Publishing.*

Ovayolu, Ö. (2010). *The range of scores in competency clusters of Turkish students in Mathematics sub-test according to PISA 2006* (Unpublished master thesis). Ankara Universiy, Ankara.

Özcan, Z. C. (2017). Critical thinking skills and mathematics performance of middle school students. *Journal of Civilization Education Research, 1*(1), 43-52.

Özelçi, S. Y. (2012). *Factors affecting critical thinking attitude: A study on primary pre-service teachers* (Unpublished doctoral dissertation). Adnan Menderes University, Aydın, Turkey.

Palavan, Ö., Gemalmaz, N., & Kurtoğlu, D. (2015). Perspectives of classroom teachers on critical thinking skill and its improvement in students. *Mustafa Kemal University Journal of Graduate School of Social Sciences, 12*(30), 26-49.

Prayitno, A. (2018). Characteristics of students' critical thinking in solving mathematics problem. *The Online Journal of New Horizons in Education, 8*(1), 46-55.

Rajendran, N. S. (2010). *Teaching and acquiring higher order thinking skills: Theory and practice.* Tanjong Malim, Perak: Penerbit Universiti Pendidikan Sultan Idris.

Recalde, X. D. (2008). *Teachers' educational technology practices: Understanding teachers' critical thinking skills and teachers' cognitive demand practices in the classroom* (Unpublished doctoral dissertation). Northen Illinois University, Illinois.

Riccio, P. A. (2015). Predictors of improvement in critical thinking skills among nursing students in an online graduate nursing research course. *Universal Journal of Educational Research, 3*(9), 606-609, Doi: 10.13189/ujer.2015.030904

Rabinowitz, M. (Ed.) (1993). *Cognitive science foundations of instruction.* Hillsdale, N.J.: Erlbaum.

Runisah, Herman, T., & Dahlan, J. A. (2016). The enhancement of students' critical thinking skills in mathematics through the 5E learning cycle with metacognitive technique. *Advances in Social Science, Education and Humanities Research (ASSEHR), 57,* 101-106.

Sezer, N. (2019). *Mathematical thinking processes and skills of secondary school students' longitudinal investigation* (Unpublished doctorate dissertation). Uludağ University, Bursa, Turkey.

Stacey, K., Burton, L., & Mason, J. (1985). *Thinking mathematically.* England: Addison-Wesley Publishers.

Stobaugh, R. (2013). *Assessing critical thinking in elementary schools: Meeting the common core.* New York, NY: Routledge.

Sumarna, N., Wahyudin, & Herman, T. (2017). The increase of critical thinking skills through mathematical investigation approach. *IOP Conf. Series: Journal of Physics: Conference* Series 812, 012067. Doi: 10.1088/1742-6596/812/1/012067

Suzuki, K. (1998). *Measuring "to think mathematically": cognitive characterization of achievement levels in performance-based assesment* (Unpublished doctorate dissertation). University of Illinois at Urbana-Champaign.

Şenşekerci, E., & Bilgin, A. (2008). Critical thinking and its teaching. *Uludağ University International Journal of Social Inquiry, 9*(14), 15-43

Tabachnick, B. G., & Fidell, L. S. (2013). *Using multivariate statistics* (6th Ed.). Boston: Allyn & Bacon.

Tall, D. (2002). *Advanced mathematical thinking.* USA: Kluwer Academic Publishers.

Taşdan, B. T., Çelik, A., & Erduran, A. (2013). Examination of mathematics teacher candidates' views on mathematical thinking and improving students' mathematical thinking. Kastamonu Education Journal, 21(4), 1487-1504.

Taşdemir, A. (2008). *The influences of mathematical thinking skills on the success, problem-solving skills and attitudes of students in primary schools at science and technology course* (Unpublished doctoral dissertation). Gazi University, Ankara, Turkey.

Thomas, A., & Thorne, G. (2014). *Higher-order thinking*. Retrieved from http://www.readingrockets.org/article/higher-order-thinking

Title, P. (2011). *Critical thinking an appeal to reason*. New York, NY: Routledge.

Toheri, T., & Winarso, W. (2017). Mathematical thinking undefended on the level of the semester for professional mathematics teacher candidates. MPRA Paper No. 78486. Retrieved from https://mpra.ub.uni-muenchen.de/78486/1/MPRA_paper_78486.pdf

Türnüklü, E. B., & Yeşildere, S. (2005). A profile from Turkey: Critical thinking dispositions and abilities of pre-service Mathematics teachers of 11-13 year. *Ankara University, Journal of Faculty of Educational Sciences, 38*(2), 167-185.

Umay, A. (2003). Mathematical reasoning ability. *Hacettepe University Journal of Education,* 24, 234-243.

Whitten, D., & Brahmasrene, T. (2011). Predictors of critical thinking skills of incoming business students. *Academy of Educational Leadership Journal, 15*(1), 1-13.

Yeşildere, S., & Türnüklü, E. (2007). Examination of students' mathematical thinking and reasoning processes. *Ankara University, Journal of Faculty of Educational Sciences, 40*(1), 181-213.

Yüksel, N. S., Uzun, M. S., & Dost, Ş. (2013). Critical thinking tendencies of prospective mathematics teachers. *Hacettepe University Journal of Education, Special Issue (1),* 393-403.

Zhu, Y., Yu, W., & Cai, J. (2018). Understanding students' mathematical thinking for effective teaching: A comparison between expert and nonexpert Chinese elementary Mathematics teachers. *EURASIA Journal of Mathematics, Science and Technology Education, 14*(1), 213-224. Doi: 10.12973/ejmste/78241.