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A Study on 5th Grade Students' Achievement Levels and Misconceptions in the Concept of Angle

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Abstract

This study aims to reveal fifth grade students' achievement levels in the angles sublearning area and to examine their errors in the subject of angles. The study was designed as a survey, and the study group included a total of 254 fifth grade students attending school in central Nevşehir and in Aşkale, Erzurum. Data were collected via the "Angle Concept Level Identification Test". Students' achievement levels in the subject of angles were determined by using frequencies and percentages, and qualitative data were analyzed through descriptive and content analysis. The findings showed that the students performed best in naming angles, while merely half or approximately half of them achieved the objectives regarding angle type, measurement and drawing. Also, student errors from most to least frequent could be ordered as follows: angle measurement, angle drawing, reading the angle on the measurement tool, angle definition and angle type. Considering that the concept of angle lies at the foundation of geometry, the findings of the study suggest that building a rich concept of angle is essential to the learning of future mathematical concepts.

Keywords: Primary School, Mathematics, Geometry, Angle, Student Misconceptions

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INTRODUCTION

Mathematical concepts are the building blocks of mathematics education. Owing to the cyclical nature of mathematics, when pre and primary school children have misconceptions or deficiencies in mathematical concepts, this may lead to difficulties in the future (Bingölbali and Özmantar, 2015). Contrary to what outsiders may think, mathematics is not simply a body of abstract concepts. Rather, it is an abstraction activity. In the instructional process, concept abstraction activities provide systematic evidence for concepts via activities of human intelligence such as reasoning, discovery, inferencing or assumption. When students view mathematics as a ready packaged and polished body of information as opposed to a subject to be learned through discovery, construction and meaning-making, they fail to develop scientific processing skills and come to believe that mathematical concepts are concrete (Baki, 2014). Owing to these reasons, Turkish mathematics curriculum, which was revised in 2004, emphasizes the importance of concepts. The premise in the curriculum that conceptual meaning and learning should also be emphasized alongside operational skills reveals the importance of concepts (Dane, 2008). Geometry is one of these important concepts.

The Concept of Geometry and Its Significance

Geometry is one of the basic learning areas in mathematics curricula at all levels of education. Its connections with daily life and the questions included in national and international exams extol the importance of geometry (Sarı and Tertemiz, 2017). Geometry is a branch of mathematics which examines space and the forms that can be designed in it (shapes and objects) (Dictionnaire Larousse, 1993). Olkun and Toluk (2006) define it as a sub-branch of mathematics that enables us to examine the physical world with respect to shape, place and position. As children can see, know and understand the physical world around them, geometry education starts at an early age. It continues with higher order geometric thought, which enables students to solve geometry problems by establishing ties between geometric shapes, concepts and principles (Van de Walle, Karp and Bay-Williams, 2007). An example set of expectations from children regarding primary school geometric concepts states that children should be able to (Ontario Ministry of Education, 2005):

• define, name, construct, draw, compare and order two and three dimensional shapes,

• describe the properties and parts of two and three dimensional shapes,

• study and predict the outcomes of integrating and disintegrating parts.

These expectations, and the development of geometric thought level, become fulfilled via experience and education. During this process, maturation alone is not enough. If the categories of shapes that children encounter are limited, so will their shape concepts be. If the examples and non-examples that children encounter are inadequate, so will their mental prototypes remain (Cross, Woods and Schweingruber, 2009; Clements, 2004). Limited prototypes make the development of geometry concepts difficult (Sarı, 2015). Therefore, geometry emerges as a mathematical topic in which students suffer. Indeed, it has been an area in recent international exams such as the TIMSS and PISA where students are weak (Sarama and Clements, 2009). Similar difficulties exist also in the National Higher Education Entry Exam (NHEEE) (See NHEEE, 2018; 2019).

One of the most fundamental reasons for student difficulty in learning geometry is the emphasis placed on memorization of general procedures (i.e., calculating area by multiplying two sides or the sum of a triangle's inner angles being 180°) rather than the comprehension of concepts (Sarı, 2015; Sarı and Tertemiz, 2017). Therefore, learning geometric concepts and terminology is essential. In primary school, the terminology of geometry goes beyond simply knowing the names of shapes. Children are expected to *define* shapes in relation to their properties, *analyze* the role of these properties, and *establish logical arguments* to evaluate the results of geometric relationships (Lindquist and Clements, 2001). At the same time, the comprehension of shapes requires discovering their parts and properties. Solely seeing pictures of geometric shapes and being able to name them is not enough to construct geometric concepts (Burns, 2007; Clements, 1998). Therefore, the concept of “angle”, which is a sub-learning domain of geometry and is defined through undefined terms, is an important geometry topic within the instructional process.

The Concept of Angle and its Significance

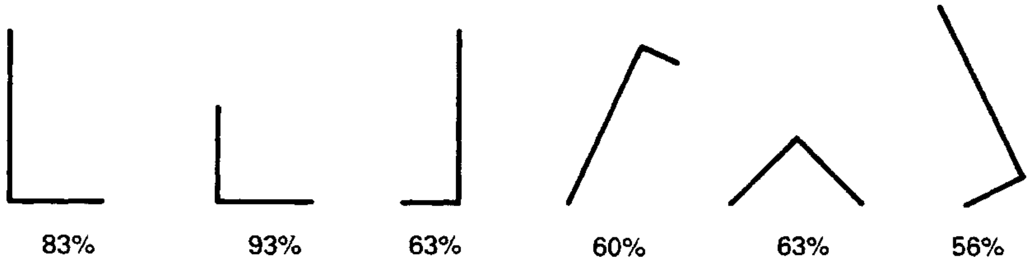
The instruction of the concept of angle starts in grade 3 in Turkish primary schools and continues into the future stages of education. The topic continues in different grade levels by the definition and naming of the elements of an angle and the classification of angles. In advanced levels, students are made to realize that a given angle is formed through rotating around a starting point and can be drawn by using standard angle measurement tools, particularly a pair of compasses (MEB, 2018).

The angle is a concept which helps with the construction of other geometry concepts and lays the foundations of geometric shapes (Clements and Burns, 2000; National Council of Teachers of Mathematics [NCTM], 2000). It has been included in mathematics curricula for longer than a century and is also used frequently in interdisciplinary programs such as physics or engineering (Moore, 2013). The concept of angle is usually perceived as abstract by students (White and Mitchelmore, 2010). The word angle may refer to multiple items such as a vertex, two arms and a vertex or a measurement of degree. However, the idea of space between two lines is hard to grasp for many primary school students as they may hold the restrictive thought that the degree of the angle they measure is its definition (Browning, Kling and Sundling, 2008). Previous research has shown that students have difficulty with angles owing to the restrictive thoughts about the concept (See Devichi and Munier, 2013; Henderson and Kieran, 2005; Keizer, 2004; Sarı, 2015; Yılmaz and Nasibov, 2012; Ubuz, 1999). For example, a study by Sarı (2015) has shown that students believe “*when the arms of the angle lengthen, its degree may also change or that the angle with longer arms will be greater*”.

Students who have difficulty in understanding the angle concept also have difficulty in understanding other geometry topics (Clements and Burns, 2000; Moore, 2013). The preconditions for successful mathematics education include accuracy in basic information and concepts, and the identification and eradication of misconceptions. Otherwise, building new information on existing mistakes and misconceptions would lead to a wrong learning of new concepts (Baki, 1998). Therefore, for knowledge in geometry, students should understand the concept of angle and have angle measurement skills (Clements and Sarama, 2009; NCTM, 2000).

An Overview of Studies on the Concept of Angle

The literature mostly includes overseas studies aiming to reveal the difficulties faced by students in learning about angles. Research focuses on student difficulties resulting from the different and complicated definitions of angles (Henderson and Kieran, 2005); the importance of letting students experience angles both conceptually and at an operational level, and helping them perceive angles from different aspects (Casas-García and Luengo-González, 2013); the development of angle measurement skills (Clements and Burns, 2000; Moore, 2013), the development of the topic of angles and its place in geometry (Keiser, 2004 and Henderson and Kieran, 2005), and errors and misconceptions in the measurement of angles (Yılmaz and Nasibov, 2012). For instance, Kerslake (1979) (cited in Dickson, Brown and Gibson, 1990:30) illustrated how 10-year-olds completed the right angle:



Dickson, Brown and Gibson, (1990:32) illustrate the misconceptions of a child named Kent by using his explanations. Kent says; "When I said the word angle, the look on Richard's face was as if he had never heard the word before, especially when I rotated 360 degrees around myself and told him this was an angle:

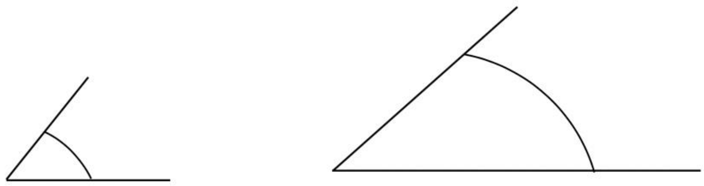
"Where are the lines?" asked Richard.

"What do you mean?"

"For the angles."

"No, it's not like that" I said.

In order to explain himself, Richard said "Angle is the distance between two lines. These angles are not the same."



As can be seen, students have misconceptions about angles such as a direct connection between arm length and degree of angle, or type of angle changing in different directions. In the Turkish context, previous research has focused on concept maps of angles (Bütüner and Gür, 2008), achievement levels revealed via testing (Fidan and Türnüklü, 2010; Yenilmez and Yaşa, 2008), increased student success in geometry after classroom exercises (Göksu and Köksal, 2016; Sarı, 2015; Sarı and Tertemiz, 2017) and misconceptions at the high school level (Ubuz, 1999). Ubuz (1999) studied the angle misconceptions of grade 10 and 11 students, and found that a failure to acquire the concept in earlier stages continued into high school, and that students did not really comprehend the properties of the concept and answered questions by relying on the shape of an angle. Considering that a misconception is a type of perception that systemically produces error (Smith, diSessa, and Roschelle, 1993), it is important to identify the difficulties and misconceptions that students may have with angles in early stages. The present study therefore aims to "reveal the achievement levels of grade 5 students in the angles sublearning area in geometry and examine their errors".

METHOD

Study Design and Model

As the study aimed to examine the achievement levels of grade 5 students in angles, it is a descriptive survey. Descriptive research aims to describe, explain and reveal those who experience, the existing, and what is experienced (Sönmez and Alacapınar, 2014). Karasar (2006) defines the descriptive survey as a scanning study conducted on an entire population of many elements or a group of samples selected from it (a sample) with the aim of reaching a general judgment about that population.

Study Group

The study group comprised 5th grade students from public secondary schools located in Nevşehir and Erzurum. The schools were selected from among those that would allow the researchers to have easy access and give consent to the study protocol. These schools were attended by students from the middle socioeconomic class. Both schools were chosen from the same socioeconomic class so as to enable an interpretation of the findings. Accordingly, a total of 254 students from the selected schools was admitted to the study. Of these, 119 were girls (47%) and 135 were boys (53%).

Data Collection Tool

Data were collected during the fall term of 2019-2020 school year. The data collection tool used in the study was developed by Sarı (2015) who reported the reliability coefficients (Kuder-Richardson (KR-20)) of the "Angle Concept Level Identification Test" as .91 and .89. The tool included 25 items in total. Sample test questions can be found in Figures 1.

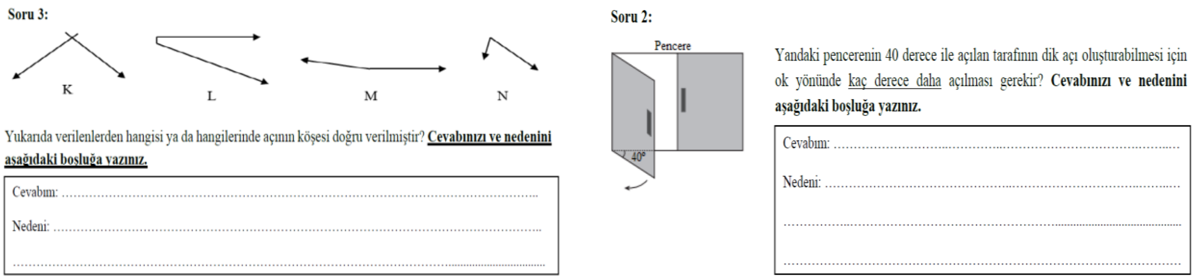


Figure 1. Samples from the measurement tool

Students answered the questions in approximately one class hour (40 mins). The study was conducted at schools during appropriate times specified by school managements. Only the researchers remained in the classroom during the process. In line with ethics principles, students were not asked to supply their names or other identifying information. Prior to the implementation, students were told that participation was strictly voluntary and they had the right to not participate.

Data Analysis

The following steps were followed in this study as quantitative data were analyzed: After the implementation of the "Angle Concept Level Identification Test" on 254 fifth grade students, their answer sheets were coded as S1, S2, S3……S254. The achievement levels of the 5th graders was identified first. In order to do so, student responses to the level identification test were categorized as correct, incorrect and unanswered for descriptive analyses. Only those answers with correct operational processes and results were accepted as correct.

The data were analyzed by using Microsoft Office Excel 2007. The findings were used to determine the overall achievement levels of 5th graders in the topic of angles. Correct answer percentages were also used to ascertain the sublearning areas and objectives where students scored best and worst. In addition, the objectives that the 5th graders achieved at 75% learning level (Özçelik, 2010; Turgut and Baykul, 2014) were identified and the findings were tabulated. In line with the problem of the study, the solution methods that the 5th graders used in angle questions were also examined. Based on a literature survey and expert views, the researchers gathered the common strategies used by students when solving angle questions and their common mistakes under the following headings:

- *Definition of Angle*
- *Type of Angle*
- *Degree of Angle*
- *Drawing Angles*

In the final stage, the 5th graders' wrong solution methods were examined via content analysis to identify common student mistakes with angles and the most frequently repeated mistakes in each question type. During error analysis, all student mistakes were examined in detail. The findings were supported with direct quotations from students.

Prior to data analysis, questions testing the angle-related knowledge and skills of the 254 students were read and coded separately by the two authors, who are experts in mathematics education, and four experienced classroom teachers, who are pursuing a doctoral degree. Similarities and differences between the themes were identified. Differences were later considered again to finalize the themes.

FINDINGS

This section presents findings obtained from the analysis of the answers that fifth graders supplied to the different question types on angles. Below are students' responses in relation to the dimensions specified in the method.

Student Positions in the Concept of Angle:

Table 1 shows the achievement levels of fifth graders in the concept of angle and the analyses of their responses to questions about angles.

Table 1. Analysis of student answers in the angle level identification test (N: 254)

Knowledge and Skills Tested in Relation to the Objectives in the Angles Sub Learning Area	Correct answers		Errors		Total	
	N	%	N	%	N	%
Definition of Angle						
Basic concepts (arms of angle, rays)	70	28	184	72	254	100
Basic concepts (the vertex of angle)	85	33	159	67	254	100
Naming the angle	206	81	48	19	254	100
Type of Angle						
Identifying the right angle	133	52	121	48	254	100
Type of angle	133	52	121	48	254	100
Degree of Angle						
Measuring an angle with a protractor	85	33	169	67	254	100
Measuring angles with unusual shapes and positions	134	53	120	47	254	100
Drawing Angles						
Drawing an angle by identifying its vertex	109	43	136	57	254	100
Drawing angles according to the type of angle given	84	33	170	67	254	100

*Objective achievement level was set at 0.75 (Özçelik, 2010; Turgut and Baykul, 2014)
**In cases where the level identification test included more than one item about a given objective, the one with the highest achievement level was selected.

An overall view of Table 1 highlights that the general achievement level of students in angles was rather low. When the findings are examined with respect to the dimensions of the study, student responses showed that they performed best in naming the angle (82%) in the dimension of basic concepts; their achievement level was low in the dimension of angle definition and their mistake level in the rays forming the arms of the angle was 72% and in the vertex being a point was 67%. Regarding the dimension of type of angle, half of the students (52%) were successful in "identifying the right angle" and "type of angle". In the degree of angle dimension, half of the students (52%) performed successfully in measuring angles in unusual shapes and positions, and 67% made mistakes when measuring an angle with a protractor. In the final dimension regarding students' angle drawing levels, approximately half of the students (42%) performed well in drawing an angle by identifying the vertex of the angle, but most (67%) made mistakes in drawing angles based on a given type of angle.

Analysis of Student Errors with the Concept of Angle:

Based on the findings obtained from the Angle Level Identification Test, student mistakes with the concept of angle were put into dimensions and examined in detail. These dimensions are classified in Figure 2.

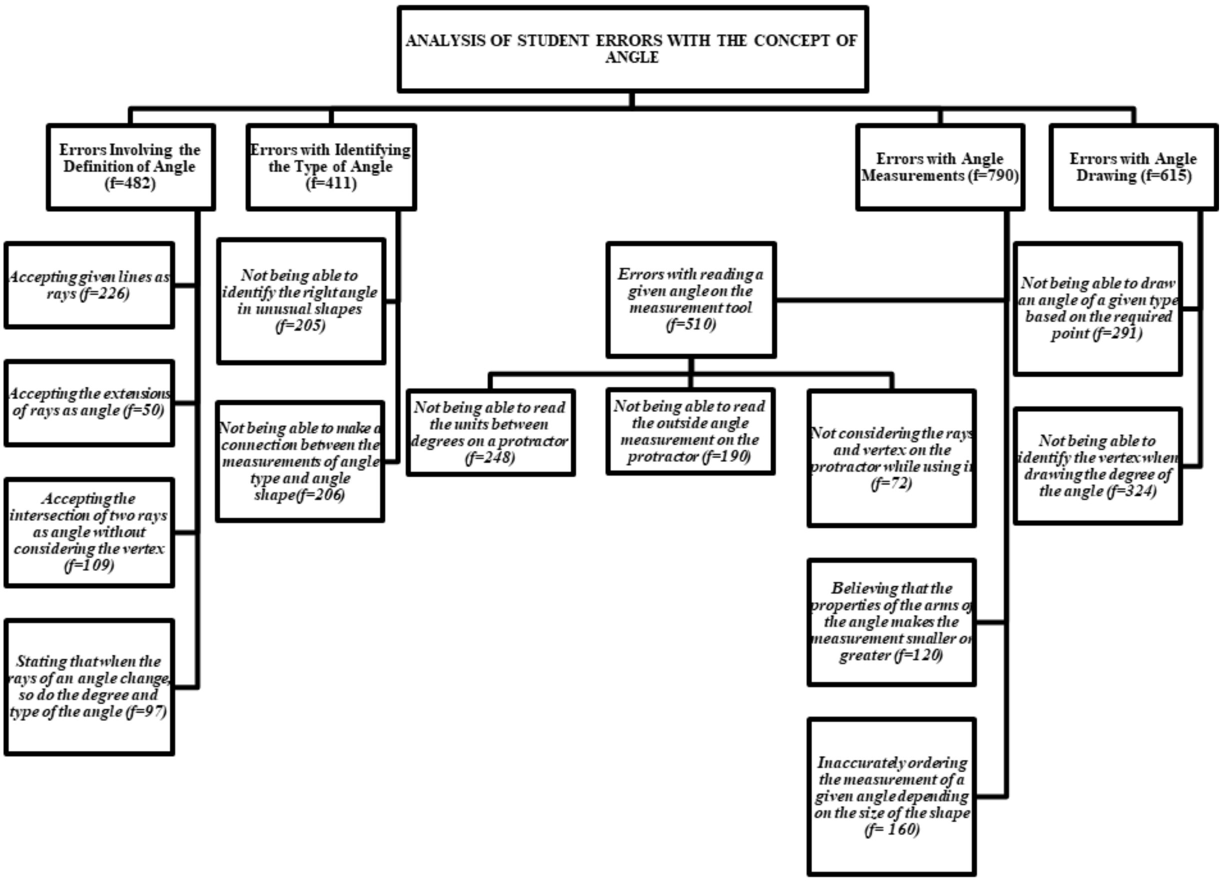


Figure 2. Student errors with the concept of angle

As shown in Figure 2, when student errors with the concept of angle are ordered from the highest to lowest frequency, the type of error with the highest frequency (f=790) included errors involving the measurement of angles. This was followed by errors involving angle drawings (f= 615), reading a given angle on the measurement tool (f=510), definition of angles (f=482) and identifying the type of angle (f=411). Based on Figure 3, when the errors under the four headings were considered in detail, the following situation emerged.

Errors Involving the Definition of Angle

Errors with angle definition were considered under the following headings: accepting given lines as rays, accepting the extensions of rays as angle, accepting the intersection of two rays as angle without considering the vertex, and believing that when the rays of an angle change, so do the degree and type of the angle. The students were asked three questions about the definition of angle. A total of (f=482) errors were identified when student errors in the three questions about angle definition were analyzed. The analysis showed the following errors:

Accepting given lines as rays: The most common error type about angle definition (f=226) was accepting given lines as rays.

Sample student responses:

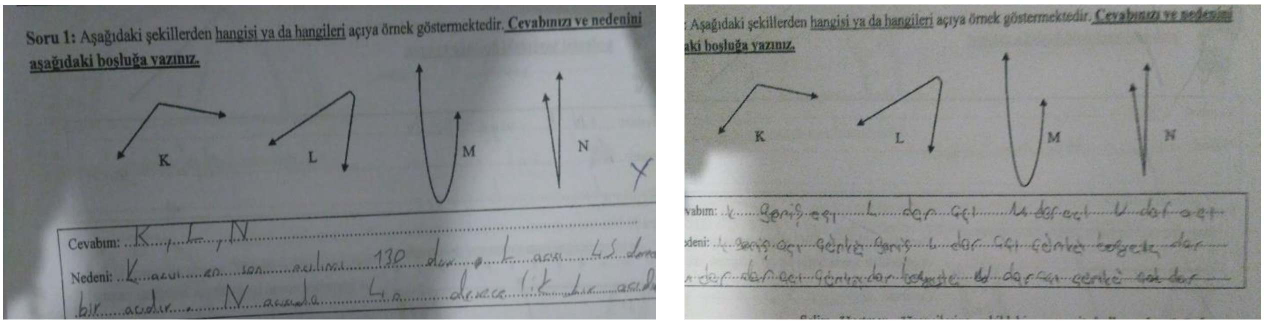


Figure 3. Sample answers accepting lines as rays

Figure 3 shows that students who supplied a wrong answer to the question also thought the shape “M” was an angle. In this situation, students generalized the intersecting point of the two rays and took lines as examples of angles.

Accepting the extensions of rays as angle: Another error type in angle definition was accepting the extensions of rays outside the vertex of the angle also as an angle. The number of errors in this sub heading was (f=50).

Sample student responses

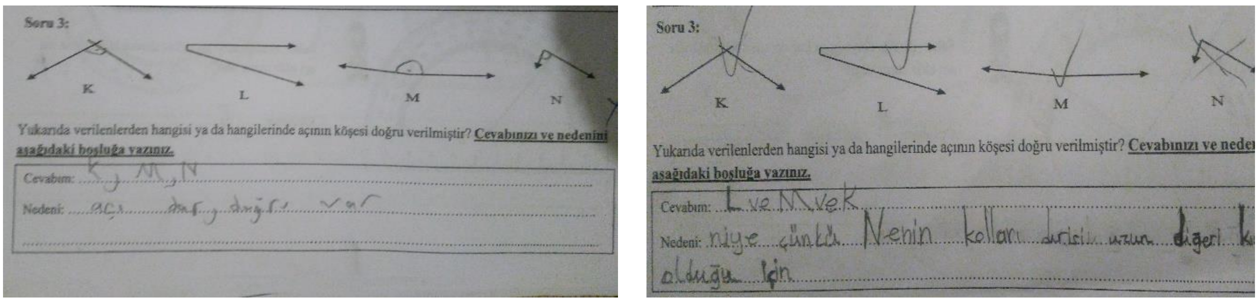
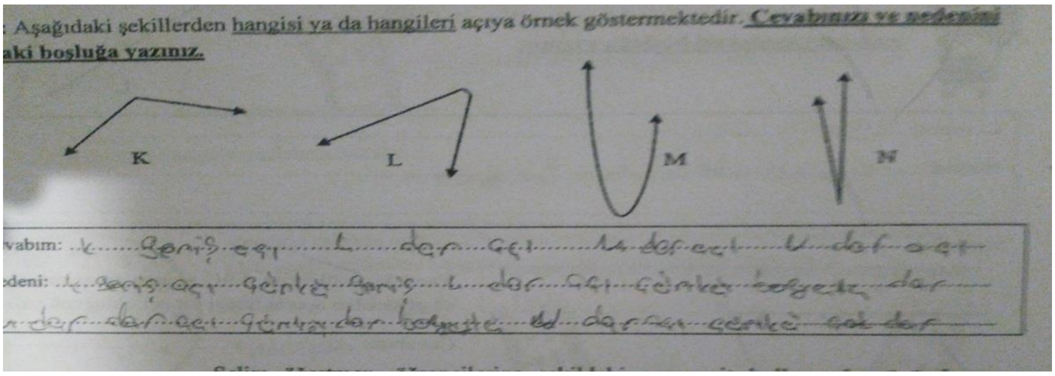


Figure 4. Sample answers accepting the extensions of rays as angle

According to Figure 4, students who marked the shape “K” as an angle gave the wrong answer by thinking that the lines of rays beyond the points of intersection were also angles.

Accepting the intersection of two rays as angle without considering the vertex: The number of students who made this particular error was (f=109). Students accepted the intersection of two rays as an angle without considering the vertex (Figure 5).

Sample student responses:



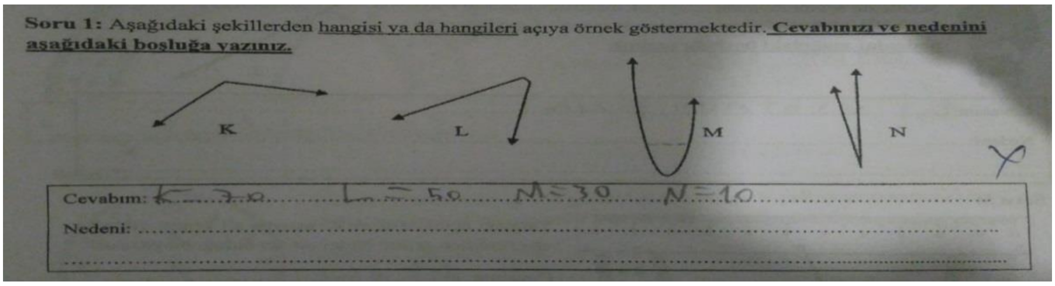
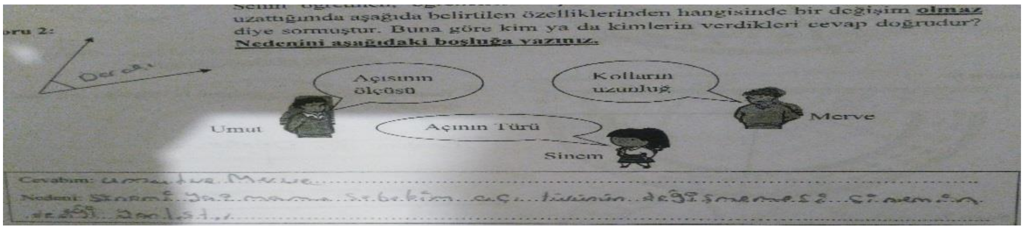


Figure 5. Sample answers accepting the intersection of two rays as angle without considering the vertex

Figure 5 shows that students who thought that all of the shapes were angles also considered shape “L” to be an angle as they held the misconception that the intersection of rays is an angle.

Stating that when the rays of an angle change, so do the degree and type of the angle: The number of this particular error was (f=97). Here, the students stated that when the rays/arms of an angle change, so do its degree and type.

Sample student responses:



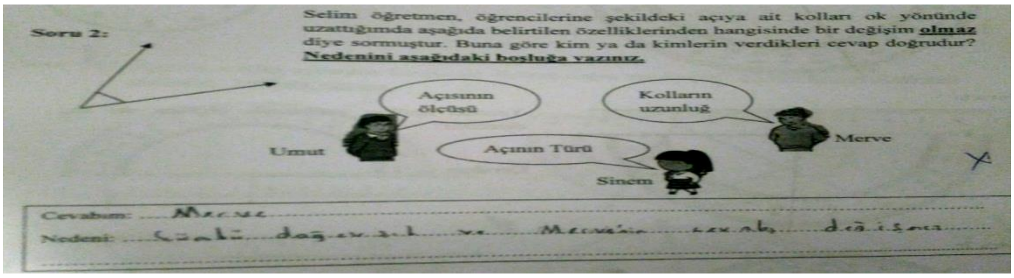


Figure 6. Sample answers stating that when the rays of an angle change, so do the degree and type of the angle

The student statements given in Figure 6 show that students who think that the degree and type of a angle changed with the arms of the ray had misconceptions about the concept of angle.

Errors with Identifying the Type of Angle

Errors with identifying the type of angle included not being able to identify the right angle in unusual shapes and not being able to make a connection between the measurements of angle type and angle shape. Students were asked 3 questions about identifying the type of angle. An analysis of student responses revealed a total of (f=411) errors. These errors were grouped under the following headings:

Not being able to identify the right angle in unusual shapes: This particular error was committed by (f=205) students. In this error type, students could not see the right angle in unusual shapes.

Sample student responses:

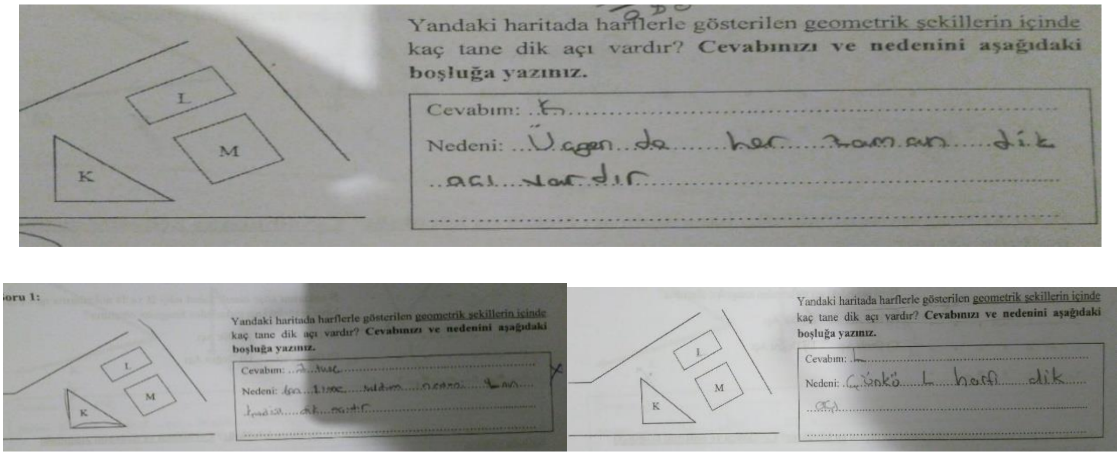
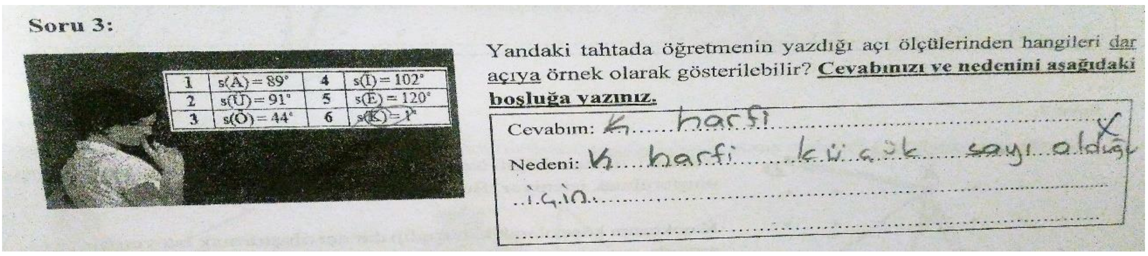


Figure 7. Student responses that failed to identify the right angle in unusual shapes

Figure 7 shows that students who could not accurately answer the question which asked them to spot the right angle generalized this type of angle to the “L” shape, and thus failed to spot the right angles in different sizes and shapes.

Not being able to make a connection between the measurements of angle type and angle shape: The number of errors in this question type was (f=206). In this error type, students gave wrong answers as a result of failing to make a connection between the degrees and shapes of various angle types (Figure 8).

Sample student responses:



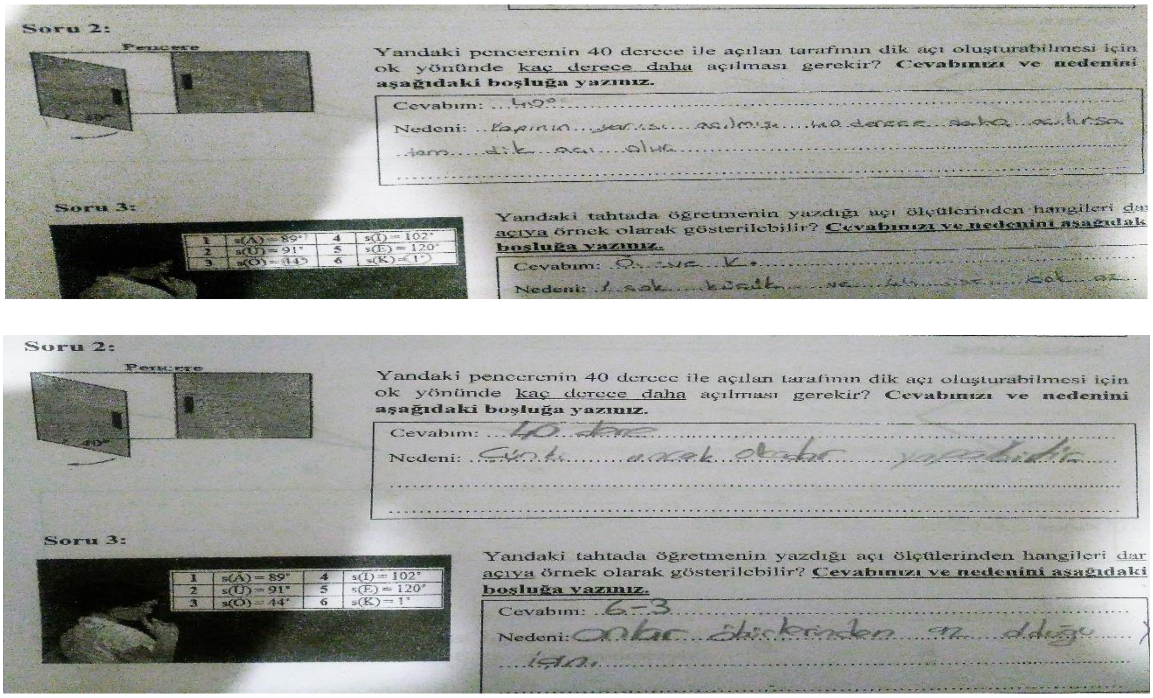


Figure 8. Student responses that could not connect angle measurement and shapes

As shown in Figure 8, students who made an error answered question 2 by merely looking at the shape of the angle without remembering that the right angle is 90 degrees and, similarly, they answered question 3 wrong because they believed that as 89 degrees is close to 90 degrees, the shape is not an acute angle.

Errors with Angle Measurements

The number of student mistakes in questions about angle measurement was (f=790). The sub headings of this error type included errors with reading the angle on the measurement tool; not being able to read the units between the degrees on the protractor, not being able to read the outside angle measurement on the protractor, and not considering the rays and vertex given on the protractor when doing a measurement with it. Additionally, there were errors such as believing that the properties of the arms of the angle makes the measurement smaller or greater, or inaccurately ordering the measurement of an angle depending on the size of the shape.

Errors with reading a given angle on the measurement tool: The number of errors on questions aiming to test the objectives related to angle measurement was (f=510). The basic reasons for these errors included failing to read the units between degrees on a protractor, not being able to read the outside angle measurement on the protractor, not considering the rays and vertex when doing a measurement with a protractor. The analysis showed that student errors could be grouped under the following subheadings:

Not being able to read the units between degrees on a protractor: The number of student errors in this error type was (f=248). Figure 9 shows sample student errors resulting from not being able to read the units between degrees on a protractor.

Sample student responses:

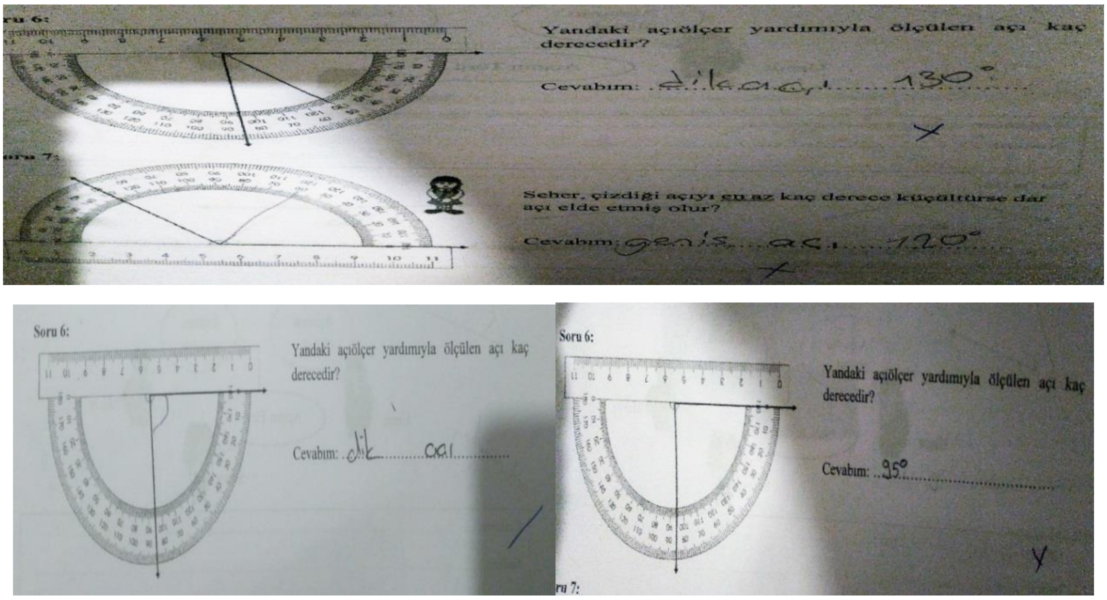


Figure 9. Responses of students who were unable to read the units between degrees on a protractor

Figure 9 shows that while some students who made errors in questions involving angle measurement with a protractor simply guessed the answer by looking at the shape as they could not measure the units between degrees, others made wrong measurements upon misreading the units between degrees.

Not being able to read the outside angle measurement on the protractor: It may be seen in Figure 10 that the students who gave wrong answers to the questions about angle measurement (f=190) did so because they were unable to read the outside angle degree on the protractor.

Sample student responses:

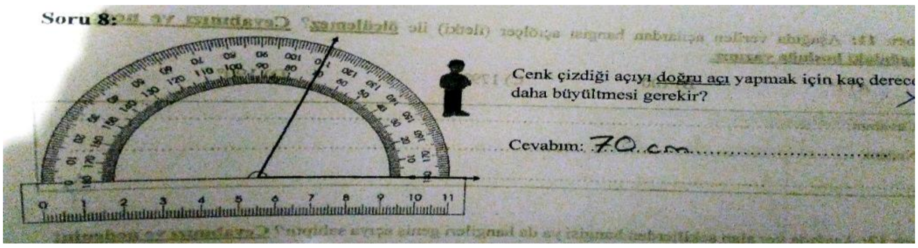


Figure 10. Responses of students who were unable to read the outside angle on a protractor

Figure 10 shows that students incorrectly answered a question which required them to turn a 70 degree angle into a right angle because they were unable to measure the outside angle on a protractor.

Not considering the rays and vertex on the protractor while using it

It may be seen in Figure 11 that the students who inaccurately answered the questions aiming to test angle measurement objectives (f= 72) made these errors by not considering the rays and vertex shown on the protractor when doing a measurement.

Sample student responses:

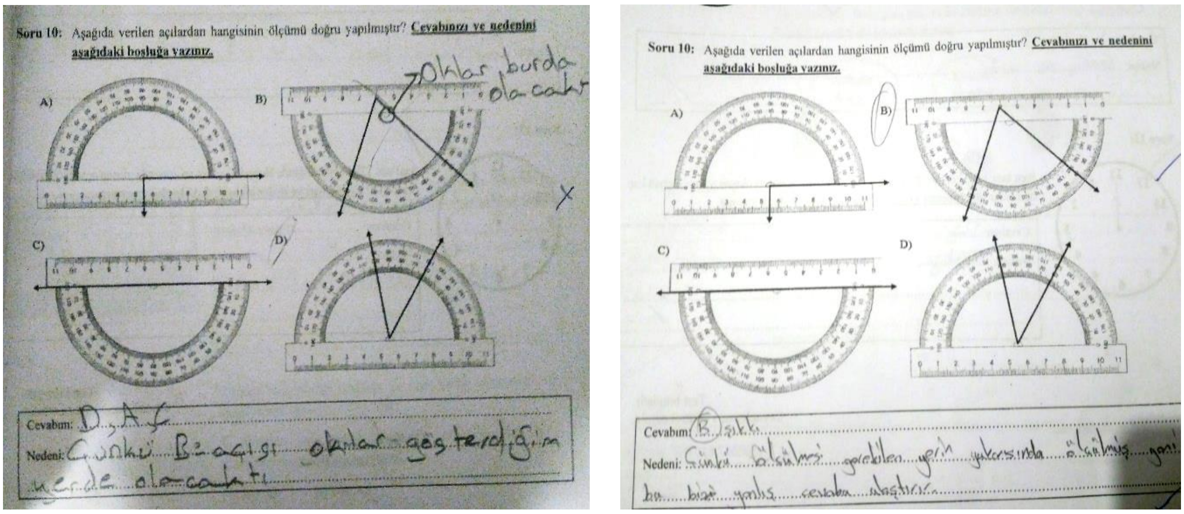


Figure 11. Responses of students who did not consider the rays and vertex given in the measurement tool

Figure 11 shows that students wrongly answered a question involving a measurement on the protractor because they did not consider the given rays and vertex.

Believing that the properties of the arms of the angle makes the measurement smaller or greater: Students who responded inaccurately to the questions aiming to measure angle measurement objectives (f=120) could not decide if the angle measurements were small or big, as they equated long or short angle arms to big or small measurement (Figure 12).

Sample student responses:

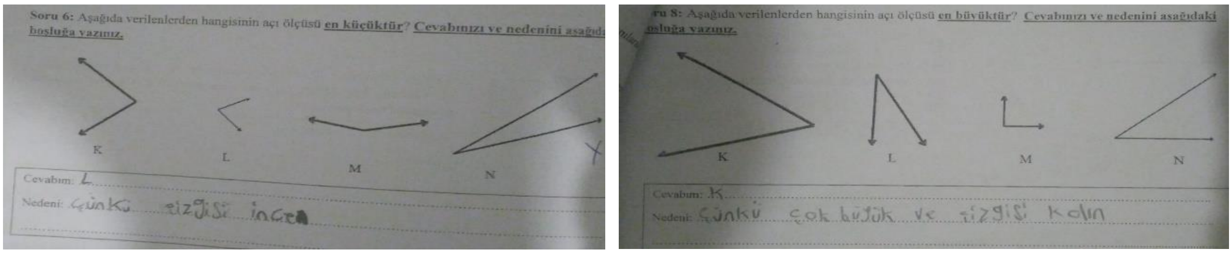
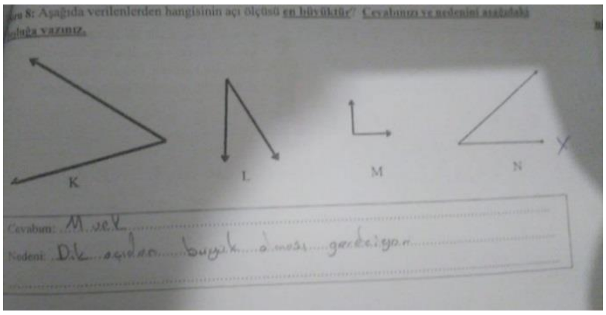


Figure 12. Student responses that stated angle measurement would be big or small based on the arms of the angle

Figure 12 shows that students who responded inaccurately to questions which asked them to identify the size of different types of angles made these errors by not considering the angle measurement, but basing their answers on the rays of the angle and on whether the arms are long/short and thin/thick.

Inaccurately ordering the measurement of a given angle depending on the size of the shape: Students who gave wrong answers to the questions aiming to test angle measurement objectives (f=160) made errors with ordering a given angle measurement depending on the size or shape of the angle (Figure 13).

Sample student responses:



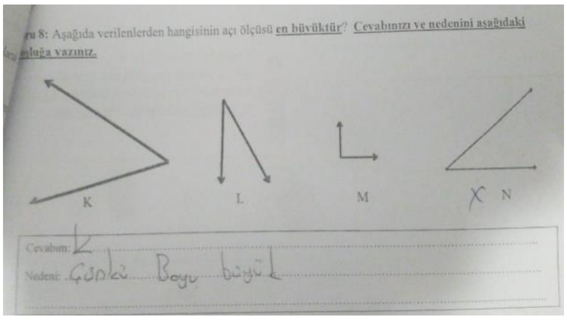


Figure 13. Responses of students who misordered the measurements of given angles based on the size of the shape of the angle

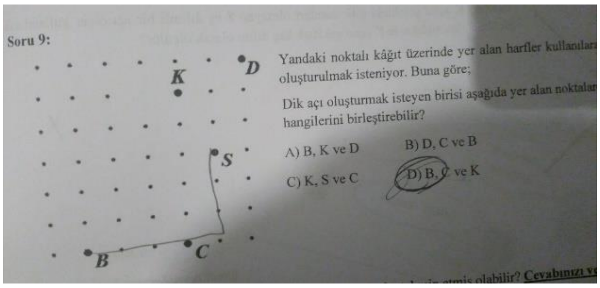
Figure 13 shows that as students who made errors in their answers tried to identify angle size, they decided that "K" had the highest angle measurement and were unable to discover that the right angle "M", which seemed to be the smallest, was actually the greatest angle.

Errors with Angle Drawing

Errors with angle drawing included not being able to draw an angle of a given type based on the required point and not being able to identify the vertex when drawing the degree of the angle. Six questions were asked to test angle drawing skills. There was a total of (f=615) student errors in these six questions. The analysis showed that student errors and samples could be gathered under the subheadings below:

Not being able to draw an angle of a given type based on the required point: In this error type, a total of (f= 291) errors were detected. As students were not able to draw angles of a given type based on a certain point, they produced faulty drawings.

Sample student responses:



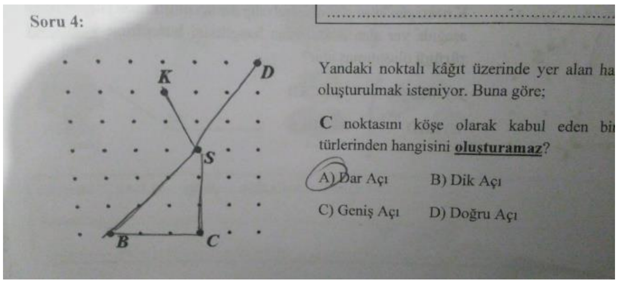


Figure 14. Responses of students who could not draw an angle of a given type based on a required point

Figure 14 shows that students who made errors in their answers could not identify the points when drawing angles of a given type, and drew random angles that resembled the required one.

Not being able to identify the vertex when drawing the degree of the angle: The total number of errors in this category amounted to (f=324).

Sample student responses:

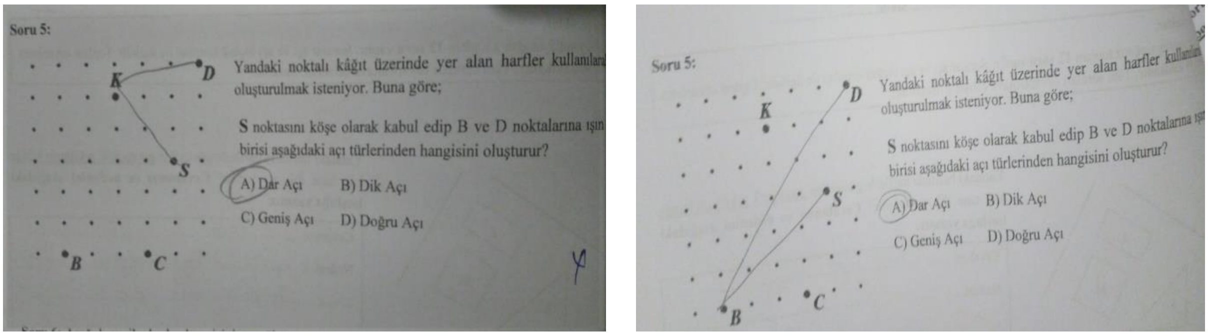


Figure 15. Responses of students who could not identify the vertex when drawing the angle measurement

Figure 15 shows that students who answered incorrectly could not draw the required angle as they could not identify the vertex given in the questions.

DISCUSSION, CONCLUSION AND RECOMMENDATIONS

The present study aimed to examine and reveal 5th grade students' achievement levels and errors in the concept of angle. In line with this aim, its findings were gathered under two main headings: Students' achievement levels in the concept of angle and their errors with the concept.

An examination of students' achievement levels in the concept of angle revealed that they had achieved the knowledge and skills mentioned in the objective of angle naming. However, merely half or one-third of the students achieved the remaining objectives in the angle concept.

The findings have shown that students were insufficient in objectives involving practical upper level skills such as measuring and drawing angles. Similarly, they also seem to be insufficient in questions involving comprehension and analysis skills, which include recognizing the properties of the angle concept and establishing connections between them. On the other hand, they performed better in lower order cognitive knowledge level objectives. To illustrate, students performed better in angle naming and angle type identification objectives than others.

Regarding student errors with the angle concept, they seemed to have many errors with angle definition, angle type identification, angle measurement and drawing. The most common mistake with angle definition involved the rays forming the angle. Students who lacked a conceptual understanding of rays accepted curved shapes as rays. They took the intersection of two rays as an angle and held the misconception that longer angle arms meant different angle measurement and type. Güngörmüş (2002) stated that students were unable to associate the concepts of ray and line segment with their existing concepts, which led to misconceptions. Similarly, Kiriş (2008), who also obtained corroborative results, emphasized that students develop misconceptions when they cannot associate angle-related concepts (such as ray, vertex, line and line segment) with daily life and fail to reach conceptual understanding. Similar results to those of the present study were found also in a study by Yılmaz and Nasibov (2012) on student misconceptions. They also emphasized that students should learn geometry not through memorization but through definitions or evidence. As a solution to potential errors and misconceptions, Fyhn (2007) reported that they could be avoided by teaching angles in relation to physical activities (such as various arm and leg angles) and daily life.

Frequent errors and misconceptions about angle type identification included the failure of students to distinguish unusual angle types. While students could easily classify usual or prototype angle examples and types, they had difficulty with identifying and classifying unusual shapes. Devichi Munier (2013) also reported that students generalized the right angle prototype to other angle types and could not distinguish between angle types of different measurements. This might be attributed to the limited shape categories that students are subjected to in the instructional process. Clements (2004) states that if the shape categories students encounter in the instructional process are limited and the examples and non-examples are inadequate, their mental prototypes will stay that way too. This makes it hard for students to construct a rich geometry concept (Sarı, 2015).

Another topic in which students in the present study had errors and misconceptions was angle measurement. The achievement of curriculum objectives in this topic was also rather low. Students particularly had errors and misconceptions such as not being able to identify the degree of the angle, believing that longer or shorter arms may change the degree of an angle, and not being able to read the degree on a protractor due to deficient knowledge about the tool. This may suggest that the instructional process does not give students enough experiences with using a protractor to measure or draw an angle. It can be emphasized that, as also mentioned by Browning, Kling and Sundling (2008), experience in angle measurement can facilitate making meaning of angle concepts (such as point, ray, line and line segment).

The final subject in which students held misconceptions and made errors was angle drawing. Students displayed poor angle drawing skills. This may be attributed to not knowing how to use a protractor and deficient learning. Students had a particular problem with drawing angles based on a given type and determining the vertex while drawing an angle.

The early stages in the instructional process are important for the learning of geometry concepts and its terminology. The terminology of geometry in primary school goes beyond simply knowing the names of shapes. Therefore, children must *establish logical arguments* in order to *define* shapes in relation to their contexts, *analyze* the role of their properties and evaluate the results of geometric relationships (Lindquist and Clements, 2001). Considering student deficiencies in defining the properties of shapes and analyzing and establishing logical arguments, the present study also concluded that it is essential to enable a rich construction of the geometry concept via discovery of the parts and properties of shapes through the instructional process. Seeing pictures of geometric shapes and using them in the instructional process without discovering their properties restrict students' mental prototypes of the shapes. Instead, drawing geometric shapes, learning about their properties and analyzing them would be essential for the provision of a rich terminology in geometry. Munier and Merle (2009) emphasized in their study that geometry and mathematics should be learned by interacting with concepts via manipulative tools and discovering with models.

The study has several limitations. To begin with, students' existing thought patterns as they solved the questions on the tests are unknown. The study used their responses alone in order to evaluate their achievement levels and classify their error types. Future studies may be built on student responses to the questions to reveal their thought patterns. In addition, the study only concerns itself with the angle and angle measurement sublearning domains of geometry. It would be of utmost importance for curriculum developers and implementers to determine students' achievement levels, errors and misconceptions in the basic concepts of geometry such as point, line, ray and line segment by expanding the scope of the study. In the present study, the alarmingly low level of achievement displayed by students in measuring and drawing angles was attributed to lack of experience. It would be important to explore the actual reasons for this via classroom observations and interviews.

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