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Instructional Explanations of Class Teachers and Primary School Mathematics Teachers about Division

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Abstract

This study is a qualitative research which was conducted in order to reveal the instructional explanations of class teachers and primary school mathematics teachers working in state schools about division. A semi-structured interview form with three open-ended questions about division, prepared for this purpose, was examined by the experts. The semi-structured interview form inluded three questions asking the teachers to solve the long division operations of 3385: 13 = ?, 1238: 12 =? and 102102: 12 =? using the mathematical table of digits with a descriptive language as if they were telling the primary school students the solutions. While the first two questions were suitable with the 5th grade learning outcomes, the third question was suitable with a high level learning outcome. The main purpose of asking the 3rd question was to evaluate the instructional explanation of the teachers in a problem of different difficulty. The study group consisted of 34 teachers, 16 of whom were primary school mathematics teachers and 18 of whom were class teachers, working at central primary schools in a province located in Eastern Anatolia region of Turkey. The content analysis of the data showed that not all of the teachers could interpret the operation of division regarding the concept of digit accurately, and their division was result and reasoning oriented. However, it was found that few teachers made generalizations in a similar way. It was also seen that teachers who were at problem-solving level according to Kinach's (2002b) comprehension level framework could not make sense of the logic underlying the division. In addition, the reason why zero (0) was moved to the quotient and when the divisor sought in remaining number should be completed by the teachers could not be clarified because they did not know the logic of the division.

Keywords: Mathematical Knowledge, Instructional Explanations, Operation of Division, Maths Teachers, Class Teachers

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INTRODUCTION

Changes in the field of science and technology require individuals to fulfill certain expectations. Some of these expectations are being able to generate information, to use the generated information functionally, to solve problems, to think critically, to have various communication skills, to empathize and to be beneficial. Accordingly, it is expected that the curriculum should take individual differences into account, help students gain some value and skills, elaborate learnings outcomes and explanations with a spiral approach at different subject-class levels and include intended learning outcomes rather than transferring information from the curriculum (MEB, 2018).

Mathematical knowledge is divided into two categories, conceptual knowledge and operational knowledge, by mathematics educators. The basic point in conceptual knowledge is meaning. It depends on explanation and association of different information by individuals using their knowledge. Operational information consists mostly of transactional and memorized information. It is also based on rules, symbols, and operations used to answer mathematical questions. While there is no obligation to understand the logical reason in operational knowledge, the existence of conceptual information makes sense of operational knowledge. This meaning depends on how much it is supported with conceptual knowledge. Understanding is the level of associating prior knowledge with new and different knowledge. An associated understanding can increase memorial skills and make it easier to remember information. In this way, learning can be facilitated and an individual's attitudes and beliefs can develop in a positive way (Olkun & Toluk Uçar, 2009).

It was observed that although pre-service teachers know the rules and methods for mathematics teaching after completing undergraduate education, their conceptual knowledge and instructional explanations did not develop (Toluk Uçar, 2011). Mathematics teachers are expected to have a quality content knowledge, domain-specific pedagogical knowledge and knowledge of students' cognitive development (Shulman, 1986; Ball, 1990a; Carpenter, Fennema & Franke, 1996; Ma, 1999). The conceptual and operational knowledge level of a teacher has an indicative effect on students' misconceptions, prejudices and understanding. A good instructional explanation in this direction can keep students away from memorization and lead them to a process-based on structuring and interpretation rather than a rule and action-oriented memorization process. However, if the instructional explanations of a teacher remain at the operational knowledge level, it may influence the students negatively (Toluk Uçar, 2011).

In recent years, investigating academic knowledge of the teacher has gained importance as it has been understood that the teacher's professional knowledge is important in learning-teaching process (Yesildere and Akkoc, 2010; Bütün & Baki, 2019). Shulman (1986) sorted teacher's knowledge into three parts as subject matter knowledge, general pedagogical knowledge and curriculum knowledge. While subject matter knowledge is what the teacher has learned theoretically, general pedagogical knowledge is based on the pedagogical part of the knowledge that deals with way of teaching a subject. Curriculum knowedge, on the other hand, is an educational plan pre-determined by the Ministry of National Education (2005) and includes the subject-related limitations depending on the children's level of developmental and readiness. While the teacher's subject matter knowledge mainly has academic style, the basis of general pedagogical knowledge includes the knowledge the teacher should have in order to teach a subject. This knowledge involves all activities that the teacher conducts considering the teaching principles in order to interpret and present a subject appropriate for the students' level. For providing this, the teacher should know analogies, forms of presentation, examples and materials (Newsome, 1999). According to Fennema and Franke (1992), a mathematics teacher's subject matter knowledge helps him/her to establish a relationship between mathematical concepts and daily life practices. The teacher's general pedagogical knowledge skills are directly correlational with the subject matter knowledge. Uşak (2005) defined subject matter knowledge as the teacher's basic knowledge of subject, concept and content. Pedagogical content knowledge provides teachers disciplined thinking skills as well as allowing them to reconstruct students' thoughts and actions using their own cognitive knowledge (Monte-Sano, 2011; Staley, 2004). It was stated that there is a close relationship between subject matter knowledge and pedagogical content knowledge, and the teacher should have both to teach a subject (Türnüklü, 2005; Eroğlu & Tanışlı, 2015).

Considering that the pre-service teachers' pedagogical content knowledge is related to their knowledge of mathematics teaching, it can be said that this knowledge reveals the teacher's content knowledge. Thus, teachers' instructional explanations reflect their conceptual knowledge (Toluk Uçar, 2011).

The presentation way of a subject is the most important step of mathematics teaching knowledge. The way of presentation should be appropriate to the students' cognitive levels, and subjects should be handled in parallel with their affective (interest, attitude, etc.) domains. Teachers ought to take the students' levels apart from their cognitive and affective characteristics into consideration while explaining a mathematical concept. In addition, using a clear and comprehensible language in instructional explanations may enable the course to be taught more efficiently (Charalambos, Hill & Ball, 2011).

Division is the most difficult and complex operation to be taught and learnt in terms of semantic structures (Anghileri, 1989; Kouba, 1989) and conceptual understanding (Steffe, 1988). There are various reasons for regarding it as a complex operation such as considering it only as the opposite of multiplication (Kaasila, Pehkonen, & Hellinen, 2010), interpreting it only as equal sharing (Bryant, 1997) and its containing abstract meanings such as measurement, ratio, multiplicative comparison (Ambrose, Baek, & Carpenter, 2003). While obtaining conceptual knowledge during the operation of division, it is necessary to associate between the divisor and the division concepts as well as to share the whole equally (Bryant, 1997). Division has two different meanings as partitive division and measurement division (Fischbein, Deri, Nello, & Merino, 1985). While partitive division is used when the number of groups is known but the number of objects / individuals in each group is not known, the grouping (measurement) is applied when the number of individuals / objects in each group is unknown (Fischbein et al., 1985; as cited by Sitrava, Özel, Işık, 2020). It is crucial for students to realize the association between the number of groups and the objects / individuals in the groups and to distinguish these concepts in terms of making sense of the operation of division. The children's modelling the operation of division unconsciously during preschool period has led curiosity about whether the educators use it consciously. As a result of the studies conducted in this context, it was seen that the pre-service teachers' knowledge about division was operational, and they did not have the conceptual knowledge underlying the operation of division (Baki, 2013; Ball, 1990b; Tekin-Sitrava, 2018).

Silver (1986) stated that the most common problem faced by students regarding the operation of division was that they were not taught how to associate conceptual and operational knowledge. Incorrect instructional explanations may lead to incomplete or incorrect information for children. Furthermore, it is a known fact that students have difficulties in understanding because mathematics has an abstract structure (Çiltaş & Işık, 2002).

In this study, the understanding levels proposed by Kinach (2002b) were taken as a basis to examine the teachers' instructional explanations. In this framework, 5 levels of understanding which are content, concept, problem-solving, inquiry and epistemic were taken into consideration. At content level, the statements remain superficial while the individual tries to explain the existing rules and procedures with meaningless expressions. At concept level, the individual is competent in explaining the concepts, features and using different meanings. At epistemic level, the individual can provide logical expressions underlying his definitions. At problem-solving level, the individual is successful in deductive inference, mathematical modeling and analytical strategies such as generating results. Finally, at inquiry level, the individual is capable of posing a different problem or creating new knowledge.

The purpose of the study

The aim of the study was to examine the current instructional explanations of the class teachers and primary school mathematics teachers regarding the operation of division. This study was carried out to reveal the instructional explanations of primary school mathematics and class teachers through instructional theory and approaches, the students' level of understanding the rationale behind the operation of division and division itself.

The findings of the study included the importance of revising and reorganizing the current mathematics curriculum of the Ministry of National Education, the activities about this subject and the courses such as mathematics teaching, mathematics curriculum, field education and practice-oriented teaching practices taught in education faculties.

METHOD

Research Design

In this study, the instructional explanations of the class and mathematics teachers regarding the operation of division were examined thoroughly, and a qualitative research approach was employed. Creswell (1998) stated that the qualitative research approach involves questioning and interpreting social life and individual problems through unique methods. According to Yıldırım and Şimşek (2011), the qualitative research approach provides an in-depth and detailed analysis of the sample. In addition, case study was taken as the basis for the research. Case study allows the researcher to focus on a single phenomenon, person, community or institution and tries to reveal certain interactions of important dimensions (Berg and Lune, 2019). McMillian and Schumacher (2010) expressed that a case study is used to examine a situation, event or relationship between states and process with a limited number of samples. According to Çepni (2012), the most important feature distinguishing case study from other research methods is that it is the most common method employed to understand the various issues of education with questions of what, how and why. Within the scope of this research, it was aimed to examine the teachers' instructional explanations regarding the operation of division more elaborately thanks to the limited number of teachers.

Participants

The sample consisted of 16 primary school mathematics and 18 class teachers working at a provincial-central school in the Eastern Anatolia Region of Turkey. The study group included class teachers and primary school mathematics teachers to provide the diversity of the data, the chance to obtain rich data and examination of various instructional explanations available. Descriptive analysis was conducted, and the primary school mathematics teachers (MN) and class teachers (CN) were given codes (N being the element of natural numbers) for ensuring confidentiality. The codes of class teachers were C3, C4, C5, C6, C7, C8, C9, C10, C11, C12, C16, C20, C21, C22, C23, C28, C29, C32; primary school mathematics teachers' codes were M1, M2, M13, M14, M15, M17, M18, M19, M24, M25, M26, M27, M30, M31, M33, M34.

Data Collection Tool

A semi-structured interview form was used as the data collection tool. The form includes 3 open-ended questions. Three academic staffs were asked to express their opinions on these questions. Furthermore, the interview form was directed to 5 different teachers excluded in the study group. In the semi-structured interview form, the teachers were expected to solve the long division operations of "3385: 13 = ?, 1238: 12 =? and 102102: 12 =?" using the digit table in a descriptive language as if they were telling the solutions to primary school students.

Data Analysis

Descriptive analysis was used for analyzing the data obtained. Yıldırım and Şimşek (2011) stated that descriptive analysis is conducted by arranging the data according to the categories and codes determined by the researcher. The data analysis was grounded on the levels of understanding proposed by Kinach (2002b). The teachers' instructional explanations were coded through the content analysis. For ensuring validity of the study, the data were re-coded by a different expert and compared with the previous codes. The different codes emerging were rearranged at a common point by getting the experts' and researchers' opinions. Within the scope of the research, instructional explanations and results of division given by teachers were evaluated under the categories of unanswered, incorrect and correct by the researchers and experts. The instructional explanations of the teachers responding the interview form were interpreted via content analysis and impact of content knowledge on instructional explanations was examined. Firstly, it was questioned whether the teachers' answers for the operation of division were correct or not. Then, it was searched whether they benefitted from appropriate instructional explanations using the digit system of the operation of division. In order to support the findings, some of the teachers' answers were quoted exactly.

Teachers were expected to make a similar explanation to the explanations given below by using the digit values fro the numbers of division.

Model 1: Divide 3385 to 13 in a descriptive language as if you were telling the solution to the primary school students.

- Firstly, how many thousands are formed when it is divided into groups of 13, each of which is 3 thousand?
- Zero thousand. Then zero is written to the thousands digit in the quotient.
- For the hundreds digit, we have 30 hundreds. There becomes 33 hundreds with the 3 hundreds in the hundreds digit of the dividend. How many hundreds appear for each group when 33 hundreds are divided into groups of 13?
- There appear 2 hundreds. We write 2 in the hundreds digit in the division. Seven hundreds are left. Now we can move on to the tens digit for the dividend. There are 70 tens in the seven hundreds. By adding 8 tens of the dividend number the result becomes 78 tens.
- When we divide 78 tens into groups of 13, how many tens are formed?
- The answer is 6 tens. Then we write six on the digit of tens in the quotient. Tens are completely finished.
- How many ones are formed when we divide 5 ones of the divident into groups of 13?
- 0 (zero) ones is left. We write 0 in the ones digit of the quotient. Thus, the result of the long division is found as 260.

Model 2: Divide 1238 to 12 in a descriptive language as if you were telling the solution to the primary school students.

- When we divide 1 thousand into groups of 12, how many thousands are obtained?
- Zero thousand. Therefore, 0 is written to the thousands digits in the quotient.

- Let's move on to the hundreds digit. 1 thousand is 10 hundreds. It becomes 12 hundreds with 2 hundreds in the hundreds digit. When we divide 12 hundreds into groups of 12, how many hundreds will appear?

- There will be 1 hundred. 1 is written to the hundreds digit in the quotient. No hundreds are left.

- We can move to tens digit of the dividend. How many tens are formed when we divide 3 tens into groups of 12?

- 0 tens are formed. We write 0 on the tens digit of the quotient.

- Now we can move on to ones digit. 3 tens are 30 ones. A total of 38 ones are obtained with 8 units in the ones digit of the dividend. When we divide the 38 ones into 12 groups, how many ones are formed?

- The answer is 3 ones. We write 3 on the ones digit of the number in the quotient. Thus, we find the result of the long division as 103.

Model 3: Divide 102102 to 12 in a descriptive language as if you were telling the solution to the primary school students.

- When we divide 1 hundred thousand into groups of 12, how many groups of hundred thousands are formed?

- 0 group is formed. Then, we write 0 on the hundred thousands digit in the quotient.

- One hundred thousands is 10 ten thousands. There are none on ten thousands digit of the dividend. When we divide 10 ten thousands into groups of 12, how many groups of ten thousands are formed?

- 0 group is formed. Then, we write 0 on the ten thousands digit of the quotient.

- As we have finished ten thousands digit, we can move on to thousands digit. 10 ten thousands are 100 thousands. 102 thousands are obtained with 2 thousands in the digit. How many thousands are formed when we divide 102 thousands into groups of 12?

- 8 thousands are formed. We write 8 to the thousands digit of the quotient. Six thousands are left.

- Six thousands are 60 hundreds. Sixty-one hundreds are obtained with one hundred in the digit. How many hundreds of 12 are there in 61 hundreds?

- There are five hundreds. We write 5 on the hundreds digit of the quotient. Only one hundred is left.

- 1 hundred is 10 tens. There is 0 in the tens digit of the dividend. There are no numbers to add to 10 tens. How many tens are formed when we divide 10 tens into groups of 12?

- Zero tens. We write 0 on the tens digit of the quotient. Then, we can move on to the ones digit.

- 10 tens are 100 ones. 102 units are formed with 2 ones in the ones digit. How many ones are formed when we divide 102 ones into groups of 12?

- 8 units. We write 8 on the ones digit of the quotient and finish the long division. So we find the quotient as 8508 and the remainder as 6.

FINDINGS

In this section, teachers' answers to three different questions of division were classified as unanswered, incorrect and correct, and their explanations about the operation of division were examined according to the levels of understanding proposed by Kinach (2002b).

Table 1. Analysis of the answers given to the first question of division.

Code	Unanswered	Incorrect Answer	Correct Answer	f
*Non-operational instructional explanation	C9, C10, C12			3
*Lack of knowledge		C32		1
*Operational error		C16, M18		2
*Generalization-attributing a different meaning			M2, C6, C7, M15, M19, C22, M25, M30, M34	9
*Way of crosschecking			C8, C28	2
*Higher-order knowledge			M13, M26	2
*Inadequate instructional explanation			M1, C3, C4, C5, C11, M14, M17, C21, C20, C23, M24, M27, C29, M31, M33	15

When table 1 was examined, it was seen that 28 of 34 teachers merely found the correct result of the first question since their instructional explanations were inadequate, they attributed different meaning through generalization, they solved through higher-order process or crosschecking. In addition, it was understood that 3 teachers' answers were incorrect because of operational errors and lack of knowledge, and 3 teachers skipped this question without any explanations or left it unanswered.

There were no teachers at the epistemic, inquiry or concept levels, but there were teachers at content and problem-solving levels according to Kinach's (2002b) levels of understanding. Teachers at content level tried to explain the operation of division through meaningless expressions apart from being superficial in terms of procedures and rules. On the other hand, it was found that the teachers at problem-solving level used special problem-solving techniques such as deductive thinking and crosschecking, and analytical strategies in the form of mathematical modeling such as double arrow sign. Additionally, it was understood that no instructional explanations within reflected logical explanation and rationale underlying operation of division within the scope of the research.

The instructional explanations of the teachers making some inferences and generalizations without mentioning the causality of the concepts underlying operation of division are given below. It can be said that these teachers were at problem-solving level according to the levels of understanding proposed by Kinach (2002b).

The instructional explanations given below were inadequate and illustrated the teachers' correct answers as a result of their different rules or generalizations.

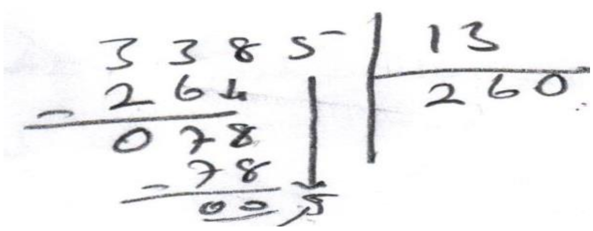

Figure 1. M2's operation of division.

M2: By starting from the left of the dividend, we write the multiples of 13 to form a pattern to see how many times it is in it. That's how we do it. As it is the last remainder, we add zero to the quotient.

C6: The largest digit of the number 3385 is 3 in thousands digit and it is said that there will be 3 thousands, specifying that the figure, first to start from 1000s. Since there are no 13 in 3, we look at the figure to the right of the thousands of digits. We ask whether there is 13 in 33 and write 2 to the quotient and multiply. We write 26 under the multiplication result – 33, and the remainder becomes 7. We say the next digit is the tens. We got 8, so we have got 78, now we're gonna find how many 13 there are in 78. When we find 6 with rhythmic counting, we write it to the quotient and multiply. Then we find 0 by subtracting 78 from 78. There is no 13 in 0, and the last remaining digit is 5. 13 is not in 5 again. As a result, we add zero to the quotient since we cannot find the number that divides twice and finish the division.

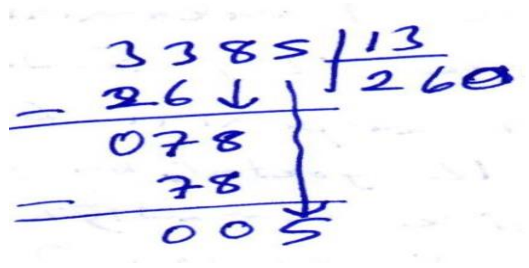

Figure 2. M15's operation of division.

M15: When dividing 3385 into 13, we first look at how many 13 are in 33 because the divisor includes 2 digits. When we count 13 each, we see that there are two 13 in 33. Then, we multiply 2 by 13 and write the result - 26 below 33. We subtract 26 from 33. Then we take 8 down from the top next to the remaining 7 and obtain 78. We find how many 13 there are in 78. Again, we count 13 each and find that there are six 13. We write 6 to the quotient. We multiply 6 by 13 and write it under 78. We find 78-78 = 0. We take 5 down and see if there is 13 in it. Since there is no 13 in it, we write zero to the quotient. The remainder is 5, and quotient is 260. We look at whether there are 13 for each number we take down, but if you don't have a divisor in it, we shouldn't forget to add zero to the quotient.

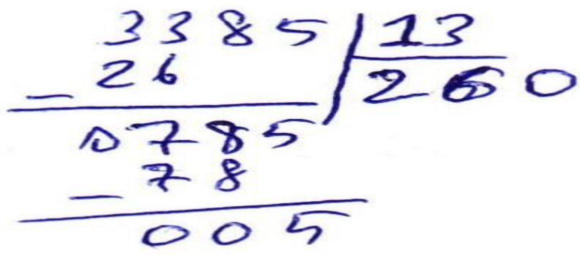

Figure 3. M19's operation of division.

M19: During operation of division, firstly, it is checked whether there is a divisor in the first digit on left side of the dividend. If there is none, then the next digit on its right is taken and re-examined. If it includes divisor, number of divisors in it is written to the quotient. How many 13 are there in 33; that is, how many groups are there if we write 33 in 13 groups? The number is written to the quotient. Then the number written to the quotient is multiplied by divisor, and written to the left of dividend. Subtraction is done by ignoring the right side. Then, the unoperated numbers in the dividend are taken down, and the result is achieved by redoing the operational steps in the same way. If zero is the result of subtraction and if the number taken down is less than the divisor, a zero is added to the right of the quotient for each digit in the remainder.

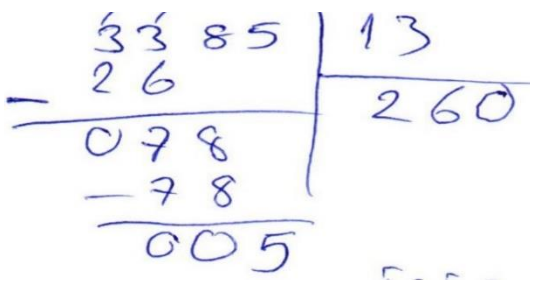

Figure 4. C21's operation of division.

C21: Firstly, operation of division is a process of sharing out. It is the short way of substraction. If we distribute from a bigger part to several people at a time, it becomes less. It is necessary to know rhythmic counting and multiplication tables well to be able to divide. The division is always started from the largest digits. Since there is no 13 in 3, 13 is searched in 33 because the number 3 is less than 13, so the next digit is added. After operation of substraction, the remaining numbers are taken down one by one and division continues. Since there is no 13 in the last number taken down, the remainder is smaller than the divisor. Thus, division cannot be continued, and division is finished by writing 0 to the quotient.

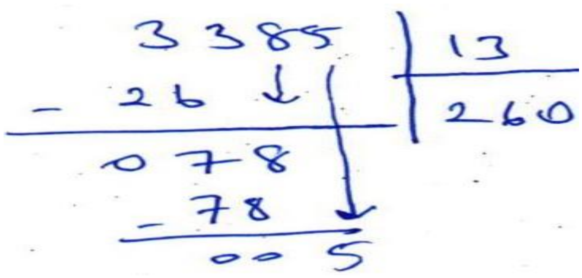

Figure 5. M34's operation of division.

M34: We start division from the leftmost digit. If the leftmost value is not divided by divisor, the division is performed by passing to the digit on the right. In the first operation, there is no 13 in 3, so we move 1 digit to the right and ask how many 13 there are in 33. Once we find it, we multiply it by the divisor and write it under the dividend and subtract them. The other numbers of the dividend are taken down. This is continued. Even if the remainder is 0, we take it down because there are unoperated numbers left in the dividend. Since there is no 13 in 5, we add a zero to the quotient as if it had digit value.

In the first question, two teachers found incorrect results because of the operational error. The following instructional description is an example.

C16: Unlike other operations, in division we start from the left. We see that the leftmost number is 3, and we ask if there is 13 in 3? No. We ask for 33 now. Is there 13 in 33? Yes. Having written 2 to quotient, we multiply 2 by 13. It is 26. 33-26 = 7. So we solved it accurately because the remainder can never be equal to or greater than the divisor. Now, 8 is taken down. What's our new number? It is 78. There are also five 13 in 78. 13 times 5= 65. We write it under 78 and substract it. The result is eight.

The following instructional explanations were considered unanswered and inadequate.

C9: I bring an apple to the class and ask "what would we do if we had 4 apples and wanted to share this with your friends?". That's how I start the lesson. Normally, we would share one by one, but knowing division makes it easier. I show how to divide an apple into 4. I give a few more examples *and open the relevant videos from the EBA (Education Information Network) or schoolistic so that they can see and perceive them visually. I help them understand and concretize case studies in the classroom.*

C12: Initially, I start operation of division by dividing 2-digit numbers with some objects. Students realize that sharing is a division. Then I explain operation of division. I explain them that division should be started from the largest digit value. I ask if there is 13 in 3. After receiving the answer "no", I ask how many 13 there are in 33? and get their answer. After the answer is received, the number in quotient is multiplied by 13 and subtraction is conducted. If there is no 13 in remainder, the other number is taken down. After finding the number of 13 in the number, the same operations are reimplemented. Operation goes like this until the remainder becomes less than the divisor. The students are told that the remainder has to be smaller than the divisor to finish the division. If not, the operation should be repeated.

The instructional explanation below was considered as incorrect due to the teacher's lack of knowledge.

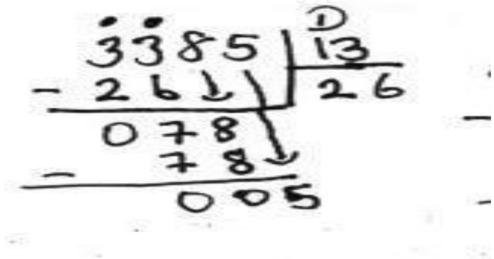

Figure 6. C32's operation of division.

C32: Is there 13 in 3? No. As you can see, there is no 13 in 3 because 3 is smaller than 13. We're checking if there's 13 in 33 because there's not in 3. There exists in 33 because it is greater than 13. In order to find out how many 13 there are in 33, we do operations of 1 times 13= 13; 2 times 13= 26; 3 times 13 = 39. Since 39 is greater than 33, we understand that there are two 13 in 33. 2 times 13= 26. 33-26= 7. Now we're taking 8 down. There are six 13 in 78. 6 times 13= 78. 78-78= 0. We're taking five down. We cannot continue divison as 5 is smaller than 13. This is a long division operation.

The following instructional explanation was regarded inadequate, and it included control through crosschecking.

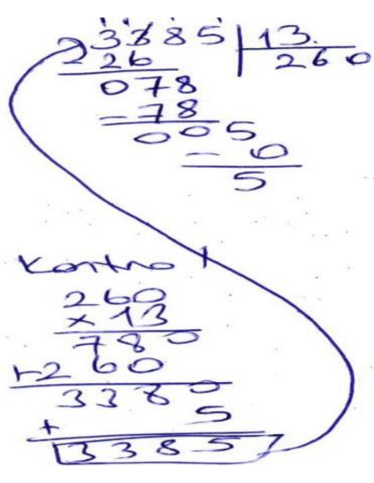

Figure 7. C28's operation of division.

C28: We ask if there is 13 in 3, which is the first digit of the division?. Then we look for 13 in 33. We try to find a number close to 33 by multiplying numbers by the divisor 13. We find the number 2. Then the number 13 is multiplied with 2, and 26 is written under 33. It is subtracted. The remainder is 7. Then the next numbers are taken down from left to right. Firstly 8 is taken down. We obtain 78 with the number 8 next to 7. In 78, the number of 13 is searched. We ask by which number we multiply 13 to find 78 or around 78? We find the number 6 by trial. 6 times 13= 78. 78-78 = 0. The next number to be taken down is 5. Is there 13 in 5? When the answer "no" is received, "no" is replaced with 0, multiplied and subtracted. After finishing the division, the divisor and quotient are multiplied to crosscheck.

The instructional explanation below was insufficient and contained higher-order knowledge.

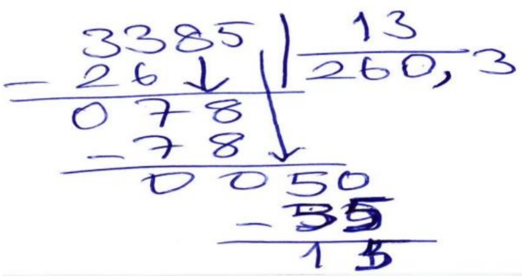

Figure 8. M13's operation of division.

M13: We determine whether or not the divided number is in the divisor. We divide the numbers that can be divided by using the divisor. If the number in the last digit of the divisor does not include as many as the number of divisors, we add a zero to the quotient, and we add a zero to the rightmost of the number. Then, we finish the operation.

The following instructional explanations were inadequate.

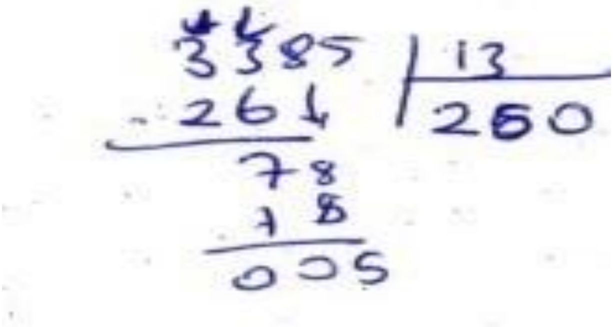

Figure 9. M30's operation of division.

M30: Division is a process of repetitive subtraction. When dividing large numbers, a method other than this is required. Repeated division is performed starting from the last digit.

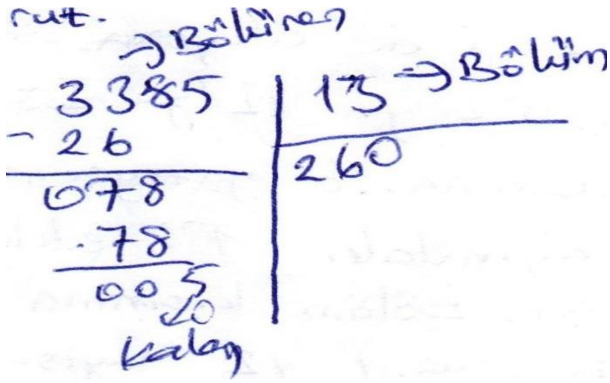

Figure 10. C3's operation of division.

C3: Dear children, now we'll solve an example of long division of a 4-digit number to a 2-digit number. When dividing, let's remember that there are dividend, divisor, quotient and remainders. We start division with the digit with the largest digit value in the dividend. The most important thing to consider in operation of division is that we should know multiplication well and take the numbers downwards correctly.

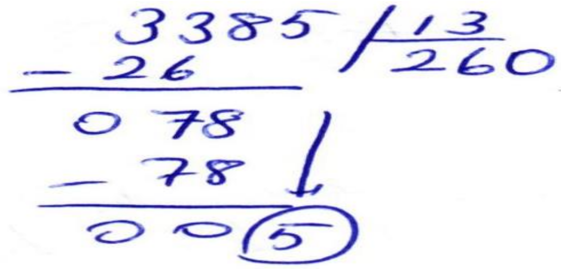

Figure 11. M14's operation of division.

M14: Since 3 is a number smaller than 13, there cannot be 13 in 3. There are two 13 in 33. 13 times 2= 26, 33-26 = 7. Since there is no 13 in 7, we take 8 down. There are six 13 in 78. 13 times 6= 78. We'll take 5 down. Since there is no number next to 5, 5 is the remainder. 5 is alone because it's less than 13. Since we have taken 5 down, we add 0 to the quotient.

Table 2. Analysis of the answers given to the second question of division.

Code	Unanswered	Incorrect Answer	Correct Answer	f
*Instructional without explanation	C9, C10, M17			3
*Operational error		M26, C23		2
*Generalization-attributing a different meaning			M2, C6, C7, M15, M19, C21, C22, M25, M30, M34	10
*Way of crosschecking			C8, C28	2
*Insufficient instructional explanation			M1, C3, C4, C5, C11, C12, M13, M14, C16, M18, C20, M24, M27, C29, M31, C32, M33	17

In table 2, the instructional explanations of none of the 28 teachers who found the correct result of the second operation of division could not be at different levels of understanding except for the content and problem-solving levels. The teachers' instructional explanations were at the level of rules - procedures and were completely superficial. Some instructional explanations consisted of meaningless expressions and generalizations. In addition, 10 of these teachers provided explanation at the level of inference in the form of mathematical modeling such as taking two digits down at the same time and the double arrow. According to Kinach's (2002b) levels of understanding, there weren't any teachers making explanations at concept, epistemic and inquiry levels.

(M26) Teacher 26 misinterpreted the question due to misperception.

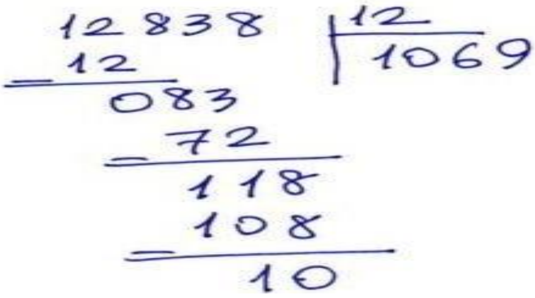

Figure 12. M26's operation of division.

M26: There is one 12 in 12. The remainder is 0. 8 is taken down. Since there is no 12 in 8, 0 is written to the quotient. Then, 3 is taken down. There are six 12 in 83, it is written to quotient and substraction is implemented. The remainder is 11. 8 is taken down. 12 is looked for in 118. It is 9 times. The operation goes like this. The remainder is 10.

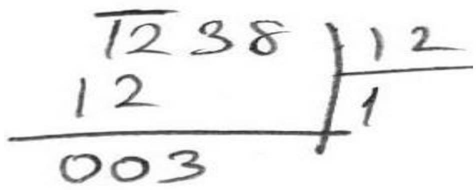

Figure 13. M17's operation of division.

M17: Since we can't look for 12 pens in a pen, we use the number on the next digit. There is one 12 pens in 12 pens. There are 12 pens in total. The division is continued through this.

Teacher 17's operation of division was regarded in the category of unanswered. In addition, the following instructional explanations were found to be inadequate in the form of rules with various generalizations.

C6: It should be started with the largest digit value - thousands digit. In thousands digit, there is the number 1. Then, we emphasize that there is one thousand. There's no 12 in 1. There's one 12 in 12. Write 1 to the quotient, subtract the result from the dividend and find 0. There's no 12 in 0. 3 goes down. Since there is no 12 in 3, zero is written in the quotient, then 8 is taken down. There are three 12 in 38. 38-36= 02 so it is said that the quotient is 103.

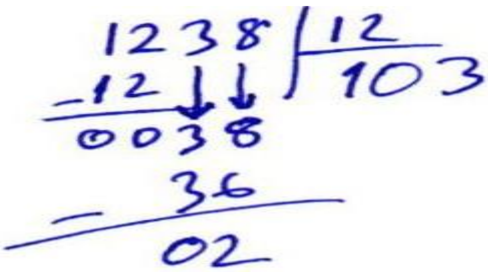

Figure 14. M15's operation of division.

M15: There is one 12 in 12. Therefore, we write 1 in he quotient. Multiplication of 1 by 12 is 12. It is substracted by writing below the dividend. It is 00. When we take the number 3 down and obtain 03, we write 0 to the quotient as there is no 12 in it. We take the number 8 down. Now our number is 38. There are three 12 in 38. 3 is written to the quotient. 3 times 12= 36. It is written under 38 and subtracted, the remainder is 2, quotient is 103.

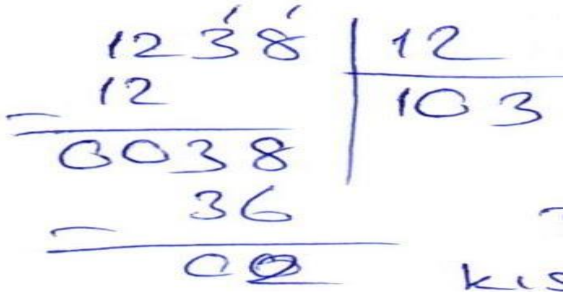

Figure 15. C21's operation of division.

C21: There are some simple rules we need to learn about division. For example, we start the division from the largest digit. While we continue, if we take down two numbers at the same time after subtraction, we need to write 0 to the quotient. That is, there is no 12 in 0. We take 3 down. There's no 12 in 3. We take down the second number - 8. Since we take down two numbers at the same time, we add 0 to the quotient.

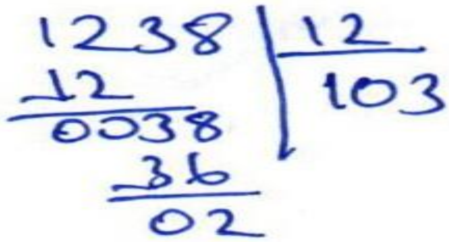

Figure 16. M25's operation of division.

M25: If 2 consecutive digits are taken down, we should write 0 to the quotient and continue.

M2, C6, C7, M15, M19, C21, C22, M25, M30 and M34 expressed the second operation of division with a similar instructional explanation and a memorization-based generalization. This may be the result of teachers' general educational background. We explored that the digit system and logic to be explained were replaced by rote learning and fabricated information. Moreover, 2 teachers stated that the operation of division should be controlled by crosschecking. Although the teachers found the correct result, the following excerpts show that none of the teachers could make necessary and sufficient instructional explanation.

C8: I would like to emphasize that it is difficult to separate the number of 1238 into groups of 12 to find out how many groups there would be in total, and to explain that division makes it easy. It is asked how many 12 there are in 1. After receiving the answer, it is asked how many 12 there are in 12. With rhythmic counting, it is found as 1. The operation is continued with the number 3 that is in the tens digit. Since there are zero 12 in 3, it is said that zero should be added to the quotient. Then, 8 is taken down. It is said that there are three 12 in 38. The remainder is 2. If it is asked why we don't add 0 although there is no 12 in 1, it can be stated that the first digit cannot be 0. Crosschecking is implemented.

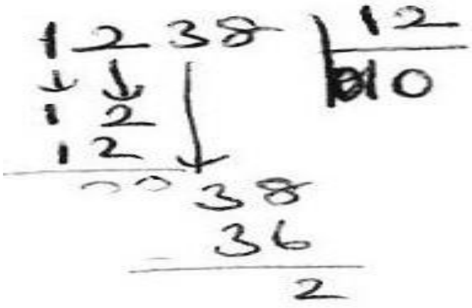

Figure 17. M1's operation of division.

M1: The operation starts from the largest digit value. Let's look at the thousands digit. Is there 12 in 1? No. Let's look again by taking the next digit. Is there 12 in 12? Yeah, there is one. We write it to the quotient. There is no remainder. Let's go on. Is there 12 in 3? No. There's 0. 0 is written to the quotient. Now let's check with the number next to 3. Is there 12 in 38? Yes, because 38 is greater than 12. Let's count rhythmically: 12, 24, 36, 48. Then, we write 3 to the quotient. 3 times 12= 36. We had 38. We have 2 as the remainder.

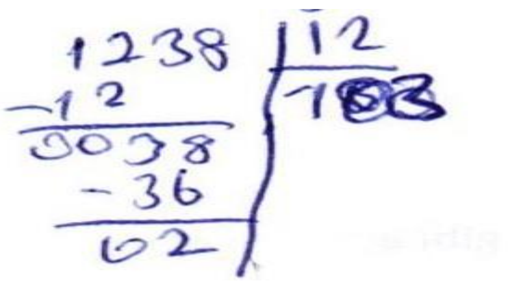

Figure 18. C3's operation of division.

C3: We will divide the number 1238 by the number 12. We start our division from the thousands digit. 1 is not divided by the number 12. That's why we take the thousands and hundreds digit together. There are 1 in the thousands digit and 2 in the hundreds digit. It is divided by 12. Then we look at tens and ones digit. We divide the number 38 by the number 12. We check the multiples of 12 and find 36. When we subtract 36 from 38, a long division is finished.

C11: Is there 12 in 1? Children say no. Then let's ask, is there 12 in 12? There is 1. 1 is written to the quotient. Then, 3 goes down. Is there 12 in 3? No. There is 0. Zero is written to the quotient. Then 8 goes down. It is asked, is there 12 in the number 38?. There are 3 times. 3 is written to the quotient, it is multiplied by the divisor and written under the dividend. The remainder is found via subtraction. As it is smaller than the divisor, it remains so.

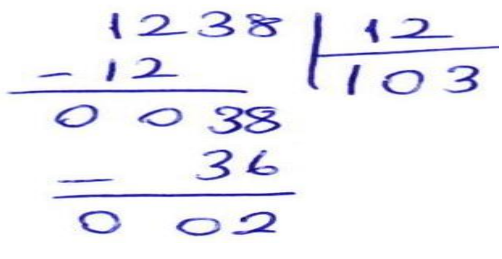

Figure 19. M14's operation of division.

M14: Since the number 12 has 2 digits, it is divided by at least 2 digits. As the number 12 is equal to 12, there is 1 time. Since our operation continues, we take 3 down. Since there is no 12 in 3, *we also take 8 down. Now we look for 12 out of 38. There are three 12 in 38. When we multiply 12 by 3, it becomes 36. Having written the numbers below and below, we make subtraction. It remains 2. Since there is no 12 in 2, we finish the long division. 2 is our remainder.*

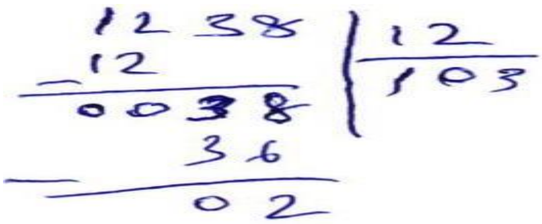

Figure 20. M19's operation of division.

M19: When we take down one of the numbers above that are not operated during division, we take one more digit down if we do not have any divisors in the number we obtained. While doing this, we write 0 to the right of the quotient. The operation of division goes on till no number that is not operated in the dividend is left.

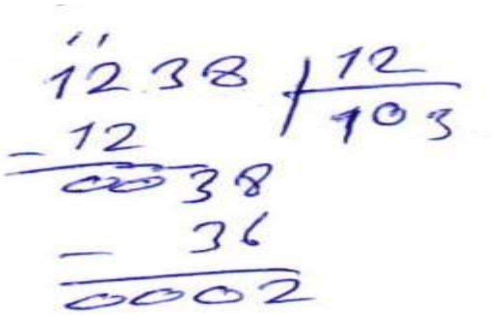

Figure 21. M33's operation of division.

M33: There is no 12 in 1. There is one 12 in 12. 1 times 12= 12. 12-12= 0. 3 is taken down. There is no 12 in 3. 8 is taken down. Since 2 numbers are taken down at the same time, 0 is written to the quotient. There are three 12 in 38. 3 x 12= 36, the remainder is 2.

Table 3. Analysis of the answers given to the third question of division.

Code	Unanswered	Incorrect answer	Correct Answer	f
*Non-operational instructional explanation	C4, C5, C6, C7, C9, C10, C11, C12, M19, C20, C21, C23			12
*Lack of knowledge		C3		1
*Operational error		M33		1
*Generalization-attributing a different meaning			M2, M15, C22, M25, M34	5
*Way of crosschecking			C8	1
*Higher-order knowledge			M13	1
*Insufficient Instructional Explanation			M1, M14, C16, M17, M18, M24, M26, M27, C28, C29, M31, C32, M30	13

In order to analyze the teachers' instructional explanations better, the analysis of the third question prepared by increasing the number of digits by 2 was shown in table 3.

(M33) S/He answered the third operation of division incorrectly due to an operational error.

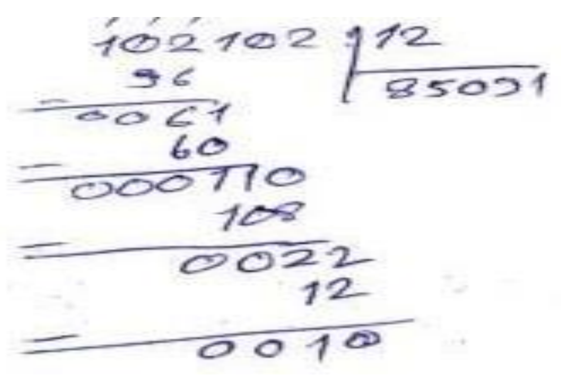

Figure 22. M33's operation of division.

M33: There is no 12 in 1. There is no 12 in 10. There are eight 12 in 102. 12 times 8= 96. If it is subtracted from 102, it remains 6. 1 is taken down. There are five 12 in 61. 5 times 12= 60. If we subtract it from 61, it remains 1. 1 is taken down. There is no 12 in 11. Since 2 numbers are taken down, 0 is added to the quotient. There are nine 12 in 110. 9 times 12= 108. It remains 2. 2 is taken down. There is one 12 in 22. 22-12= 10.

Clasroom Teacher 3 answered the question incorrectly due to lack of knowledge.

C3: We will divide a 6-digit number by a 2-digit number. In 102102, we will look for 12 digit by digit. First we'll check the hundred thousands digit, ten thousands digit and thousands digit. It is not divided as there is 1 in hundred thousands digit. The number is 10 because there is 0 in the hundreds digit. 10 is not exactly divided by 12. That's why we get 2 in the thousands digit. Our number is 102. There are eight 12 in 102. We write 8 to the quotient and 6 to the remainder. We add 1 on the hundreds digit because 6 is not divided by 12. Our number is 61. We divide 61 by 12. We add 5 next to 8 to the quotient, and write 1 to the remainder. Since 1 cannot be divided by 12, we add the number 0 next to 1. We get the number 10. Since the number 10 cannot be divided by 12, we add 2 to the ones digit and divide 102 by 12. The quotient is 858, and the remainder is 6.

Instructional explanations given below are examples of the teachers' insufficient explanations consisting of stereotypical expressions without revealing the meaning of division.

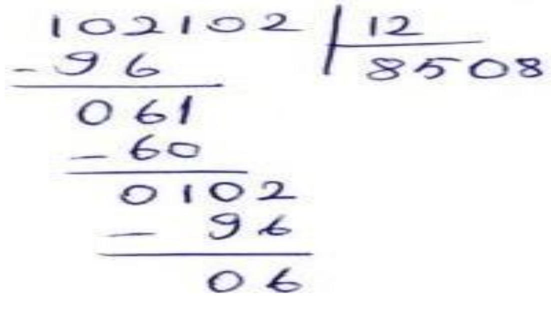

Figure 23. M14's operation of division.

M14: We use 3-digit numbers because 1-digit and 2-digit numbers cannot divide 12. There are eight 12 in 102. We continue our operations until we get a single digit by multiplying. Just like our other examples, we do operation of long division in this way.

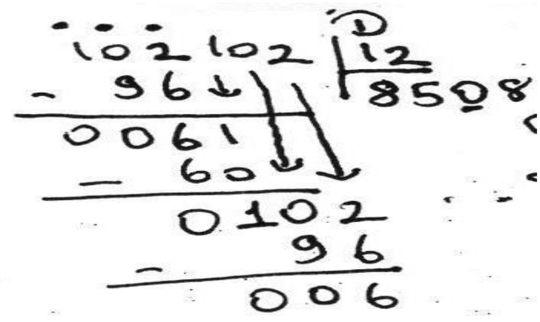

Figure 24. C32's operation of division.

C32: There is no 12 in 1. We look for 12 out of 10. As there is none, I check whether there is 12 in 102 or not. There are eight 12 in 102. 8 times 12 is 96. The remainder is 6 as a result of subtraction. Since there is no 12 in 6, we take 1 down. There are five 12 in 60. The remainder is 1. When we take 0 down, there is no 12 in 10. We write 0 to the right of the quotient. We take 2 down, there are eight 12 in 102. 8 times 12= 96. The remainder is 6 as a result of the subtraction.

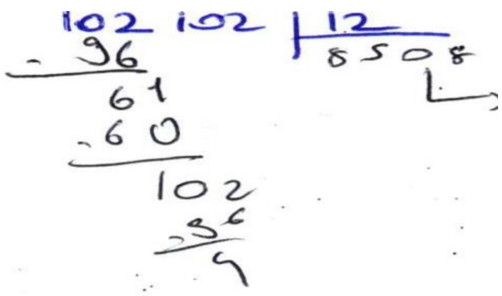

Figure 25. M30's operation of division.

M30: Because 1 digit has been taken down.

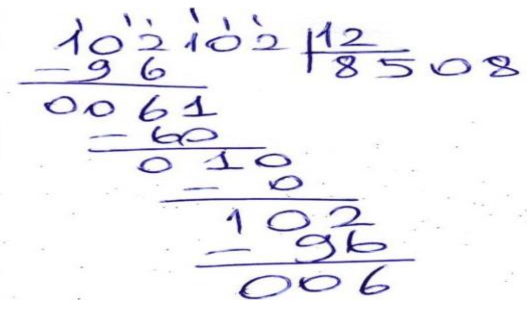

Figure 26. C28's operation of division.

C28: There is no 12 in 1. There is no 12 in 10. Then, we look for 12 in 102. By trial, we find the nearest number. There are 8 times. We subtract 96 from 102. The remainder is 6. We take 1 down. There are five 12 in 61. The remainder is 1. We take 0 down. It becomes 10 when it goes down. Since there is no 12 in 10, we add 0 to the quotient. We multiply again, and the remainder is 10. There are eight 12 in 102. Subtracting 96 from 102, we get 6.

The following instructional explanations can be given as an example of the instructional explanations at the problem-solving level proposed by Kinach (2002b).

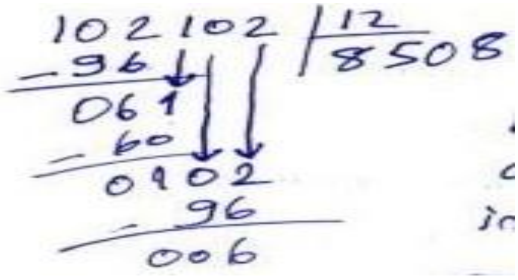

Figure 27. M15's operation of division.

M15: Since there is no 12 in 10, we include 1 more digit and look for 12 in 102. Since it is 8 times, 8 is written to the quotient. Then, we multiply, write under 102 and subtract it. There is no 12 in the remainder 6. When we take 1 down, we look for 12 in 61. There are five 12 in 61. 5 times 12= 60. We subtract it from 61 and obtain 1. There is no 12 in 1. Therefore, we take 0 down. Since there is no 12 in 10, one 0 is added to the quotient. Now, we take down the last two digits and look for times of 12 in 102. Since it is 8 times, 8 is written to the quotient. 12 times 8= 96, and 6 is the remainder. Since 6 is smaller than the divisor, division is finished here.

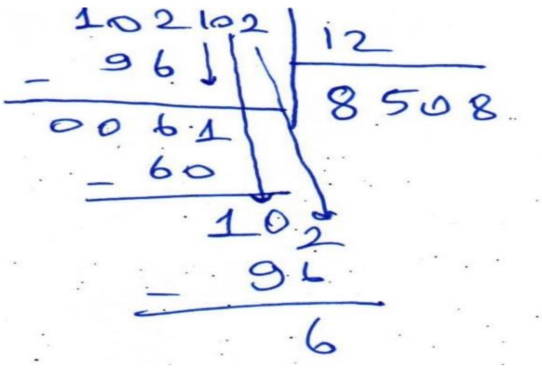

Figure 28. M34's operation of division.

M34: Is there 12 in 1? No, there isn't. Is there 12 in 10? No. If 1 digit is increased, there are eight 12 in 102. There is no 12 in the remainder 6. We take 1 down, there are five 12 in 61. Then we take 0 down. There is no 12 in 10. In this case, we take 1 more digit down. We add zero to the quotient; we seem to give it credit to pass to the next digit. There are eight 12 in 102. Since all the digits of the division have been used, the process ends and the remainder is found as 6.

The following instructional explanations were insufficient and evaluated in the category of unanswered.

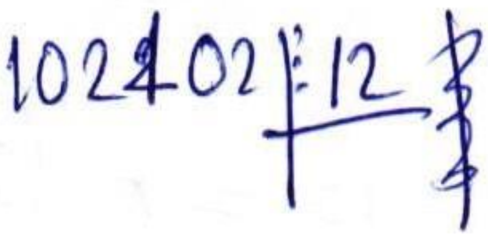

Figure 29. C23's operation of division.

C23: Likewise, subctraction is said to be a short way of division.

C20: I can't solve a 6-digit number in a descriptive language. The solution is similar to the solutions of the questions above.

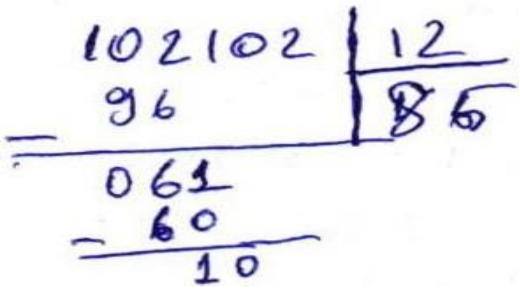

Figure 30. M19's operation of division.

M19: While dividing, the number we operate and the quotient, the digits of the number we multiply are written starting from the rightmost of the dividend, and it is subtracted. Then, the solution continues.

On the other hand, 12 teachers (C4, C5, C6, C7, C9, C10, C11, C12, M19, C20, C21, C23) did not answer the last question stating that it wasn't included in the learning outcomes of the curriculum.

C4: There is no such a learning outcome.

One of the things to be done to reveal the mathematical meaning underlying the operation of division is to separate the numbers according to the digit values and carry out the operation step by step. Here, teachers are expected to use the concept of digit. In the study, the teachers carried out the operations of division without using the digit table and revealing the mathematical meaning for the operation of division.

CONCLUSION AND DISCUSSION

In this study conducted at central primary schools, it was found that a total of 34 teachers had operational knowledge about operation of division, but it was superficial. In addition, it can be claimed that they did not have sufficient knowledge of instructional explanation and, in this direction, that they lacked conceptual knowledge as well. This result complies with Kinach's (2002) result that instructional explanations of the preservice mathematics teachers regarding operations of addition and subtraction in integers was at the level of procedural knowledge and their insufficient conceptual knowledge affected their instructional explanations (Toluk Uçar, 2009; Hacıömeroğlu, 2013; Sitrava, Özel, Işık, 2020; Gökkurt, Şahin, Soylu, 2012). On the other hand, it was in line with Baki's (2013) result that pre-service teachers had low success and insufficient conceptual knowledge about operation of division. In addition, the study was similar to the superficial operational knowledge of Işıksal and Çakıroğlu (2005).

The findings showed that giving the rules was sufficient for instructional explanation, and that teachhers did not explain the reason appropriately (Oral, 2020), they resorted to formal tricks and some generalizations which they made in their own way (Albayrak, Şimşek 2017; Hacıömeroğlu, 2013). These findings were in parallel with Toluk Uçar's (2011) study about instructional explanations conducted with pre-service teachers. Moreover, in this study, it was observed that mathematics and class teachers did not mention about different meanings of the concept of division, and they tried to explain division with a rule and operation focus. As a result, it was understood that the teachers limited their teaching approaches. In this sense, it was in parallel with Bütün and Baki's (2009) study titled the structure of primary school mathematics teachers' field education knowledge about the concept of division.

In this study, it was observed that mathematics and class teachers were unwilling to operate according to the digit values in the operation of division, their instructional explanations were insufficient and their operational knowledge was superficial. The findings were in line with the studies of Southwell and Penglase (2005) and Thanheiser (2009). Similarly, Baki's (2013), Işık and Kar's (2012) studies carried out on pre-service teachers were in line with this study regarding lack of conceptual knowledge and result-oriented, superficiality of operational knowledge for crosschecking.

Çimen and Tat (2018) examined the problem posing skills of 8th grade students in interpreting long division. The findings of this study also revealed that the students were incapable of using the mathematical language. In line with this finding, it was seen that the students could not make sense of the operation of division sufficiently. Such results are likely to occur if the teacher does not provide sufficient instructional explanations for the operation of division at primary school level.

Instructional explanations are crucial for the teachers to teach mathematical rules and concepts better and to transfer the pedagogical knowledge of mathematics. Results of several studies revealed that there were more rules, procedures and memorization in teachers' instructional explanations (Henningsen and Stein, 1997; Kinach, 2002a, 2002b; Kılcan, 2006). There may be several reasons why the results are in this direction. According to Borko and Putnam (1996), teachers' inadequate mathematical knowledge, beliefs and attitudes towards mathematics may be among these reasons. If the teachers' content knowledge is based on disconnected information, their instructional explanations will also be based on disconnected operations. On the other hand, if the teachers' content knowledge is based on appropriate ways of thinking and logical connections, then their instructional explanations will be in this direction (Thompson, Carson and Silverman, 2007).

Researches on instructional explanations have been mostly on operations with fractions (Toluk-Uçar, 2011; Charalambos et al., 2011), addition and subtraction with integers (Baki, 2013, Kinach, 2002a, 2002b), addition and subtraction in natural numbers (Thanhieser, 2009) conducted with teachers or prospective teachers. In the studies, it has been seen that the teachers' instructional explanations were mostly predicated on the rule and procedure based on memorization but this was not accurate mathematically (Toluk and Uçar, 2011).

Arslan and Kılcan (2006) stated that as a result of insufficient mathematical content knowledge of the teachers, teacher-centered learning environments would take place where student participation do not occur, students would have misconceptions and understand the subjects incompletely. It was also stated that effective teaching depends on more than one variable, but the most important of these variables is the teacher. In this context, Çakmak (2004) stated that the number of studies focusing on teacher training, teacher qualifications and the types of knowledge to be had by teachers has been increasing. In line with the studies conducted, it was mentioned that teacher knowledge was defined differently and there were variables effective in structuring this knowledge (Fennema & Franke, 1992; Grossman, 1990; Hill, Ball & Schilling, 2008; Shulman, 1986). Different studies have been carried out in accordance with the content knowledge and pedagogical content knowledge among the existing variables (An, Kulm & Wu, 2004; Daehler & Shinohara, 2001; Kahan, Cooper, & Bethea, 2003; McDuffy, 2004; Penso, 2002). In this study, instructional explanations of primary school mathematics teachers and class teachers regarding the operation of division were presented. Although many of the teachers found the correct result, they failed to make instructional explanations revealing the mathematical meaning underlying the operation of division.

In a study conducted on prospective teachers and teachers in Australia and the USA, it was observed that the teachers had difficulty in understanding the operations by numbers and digit values, as well as conducting the operations according to the rules and this was insufficient to explain the underlying causes of mathematical operations (Southwell and Penglase, 2005; Thanheiser, 2009).

SUGGESTIONS

The Ministry of National Education expects teachers to teach within the framework of the renewed curriculum. However, this is not enough. Pre-service teachers should also be trained according to the qualifications suggested in this curriculum. A teacher who is familiar with explaining the operation of division based on stereotypical expressions finds it very complicated to teach division by separating division according to the digit values. In order to help pre-service teachers reach to the goals as suggested in the mathematics curriculum, such courses should be offered in teacher training programs. Learning and teaching environments that foster instructional explanations should be created for teachers to be more equipped. Presentations made in these environments should be monitored by instructors, feedbacks should be given to pre-service teachers step by step, and they should be allowed to see what they do. In order to achieve this goal, the number of weekly courses such as teaching math should be increased and these courses should be enriched in content. On the other hand, pre-service teachers should be followed up by faculty members who are experts in their subject areas, such as teaching practice, and appropriate feedbacks should be given to pre-service teachers in case of any problems.

REFERENCES

Albayrak, M. & Şimşek M. (2017). Yetişkinlerin davranışlarını değiştirmenin güçlüğü: bölme örneği. *Erzincan Üniversitesi Eğitim Fakültesi Dergisi.19*(1) 184-198.

Ambrose, R., Baek, J.-M. & Carpenter, T.P. (2003) Children's invention of multiplication and division algorithms. In A. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Recent research and theory*. Mahwah, NJ: Erlbaum.

Anghileri, J. (1989). An investigation of young children's understanding of multiplication. *Educational Studies in Mathematics*, *20*(4), 367-385.

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China and the U.S. *Journal of Mathematics Teacher Education*, 7, 145–172.

Arslan Kılcan, S. (2006). *İlköğretim matematik öğretmenlerinin kesirlerle bölmeye ilişkin kavramsal bilgi düzeyleri*. (Yayınlanmamış Yüksek Lisans Tezi). Sosyal Bilimler Enstitüsü, Abant İzzet Baysal Üniversitesi, Bolu.

Baki, M. (2013). Pre-service classroom teachers' mathematical knowledge and instructional explanations associated with division. *Education and Science, 38*(167), 300-311.

Ball, D. L. (1990a). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449–466.

Ball, D. L. (1990b). Prospective elementary and secondary teachers‟ understanding of division. *Journal for Research in Mathematics Education, 21*(2), 132–144.

Borko, H. & Putnam, R. (1996). Learning to teach. In D. Berliner & R. Calfee (eds.), *Handbook of Educational Psychology* (pp, 673-708). New York: Mcmillan.

Berg, B. L. & Lune, H. (2019*). Sosyal bilimlerde nitel araştırma yöntemleri*. (Çev. Ed. Asım Arı). Konya: Eğitim Yayınevi.

Bryant, P. (1997). Mathematical understanding in the nursery school years. In T. Nunes & P. Bryant, (Eds.), *Learning and teaching mathematics: An international perspective* (pp. 53-67). East Sussex, UK: Psychology Press.

Bütün, M. & Baki, A. (2019). İlköğretim matematik öğretmeni adaylarının matematiği öğretme bilgilerinin gelişimi. *Cumhuriyet International Journal of Education*, *8*(1), 300-322.

http://dx.doi.org/10.30703/cije.493676

Carpenter, T. P., Fennema, E., & Franke, M. L. (1996). Cognitively guided ınstruction: A knowledge base for reform in mathematics ınstruction. *The Elementary School Journal*, *97*(1), 3–20.

Charalambous, C. Y., Hill, C. H. & Ball, D. L. (2011). Prospective teachers'learning to provide instructional explanations: How does it look and what might it take? *Journal of Mathematics Teacher Education, 14, 441-463.* DOI 10.1007/s10857-011-9182-z

Creswell, J. W. (1998). *Qualitative inquiry and research design: Choosing among five traditions*. California: Sage Publications.

Çakmak, M. (2004). *İlköğretimde matematik öğretimi ve öğretmenin rolü*. Retrieved 10.02.2020, from http://www.matder.org.tr/ilkogretimde-matematik-ogretimi-ve-ogretmenin-rolu/

Çepni, S. (2012). *Araştırma ve proje çalışmalarına giriş* (6. baskı). Trabzon: Celepler Matbaacılık.

Çiltaş, A. & Işık, A. (2012). Matematiksel modelleme yönteminin akademik başarıya etkisi. *Çağdaş Eğitim Dergisi Akademik*, *2*, 57-67.

Çimen Ev, E. & Tat, T. (2018). Sınıf öğrencilerinin bölme işleminde kalanın yorumlanması konusunda problem kurma becerilerinin incelenmesi. *Eğitim ve Öğretim Araştırmaları Dergisi. 7*(4). 1-11.

Daehler, K. & Shinohara, M. (2001). A complete circuit is a complete circle: exploring the potential of case materials and methods to develop teachers‟ content knowledge and pedagogical content knowledge of science. *Research in Science Education*, 31, 267-288.

Eroğlu, D. & Tanışlı, D. (2015). Ortaokul matematik öğretmenlerinin temsil kullanımına ilişkin öğrenci ve öğretim stratejileri bilgileri. *Necatibey Eğitim Fakültesi Elektronik Fen ve Matematik Eğitimi Dergisi, 9*(1), 275-307.

Fennema, E. & Franke, M. L.(1992). Teachers' knowledge and its impact. In D.A. Grouws (ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp 147-164). New York: Macmillan.

Gökkurt,B., Şahin, Ö. & Soylu, Y. (2012). Matematik öğretmenlerinin matematiksel alan bilgileri ile pedagojik alan bilgileri arasındaki ilişkinin incelenmesi. *The Journal of Academic Social Science Studies (JASSS), 5*(8), 997-1012.

Grossman, P.L. (1990). *The making of a teacher: teacher knowledge and teacher education*. New York: Teachers College Press.

Hacıömeroğlu, G. (2013). Sınıf öğretmeni adaylarının öğretim için matematiksel bilgisi: öğrencilerin toplama ve çıkarma işlemlerine ilişkin çözümlerinin analizi. *Eğitim ve Bilim, 38*(168), 332-346.

Hill, H.C., Ball, D.B. & Schilling, S.G. (2008). Unpacking pedagogical content knowledge: conceptualizing and measuring teachers‟ topic-specific knowledge of students. *Journal for Research in Mathematics Teacher Education,* 39 (4), 372-400.

Kahan, J., Cooper, D. & Bethea, K. (2003). The role of mathematics teachers‟ content knowledge in their teaching: a framework for research applied to a study of student teachers. *Journal of Mathematics Teacher Education*, 6, 223-252.

Kılcan, S. A. (2006). *İlköğretim matematik öğretmenlerinin kesirlerle bölmeye ilişkin kavramsal bilgi düzeyleri*. (Yayımlanmamış Yüksek Lisans Tezi). Sosyal Bilimler Enstitüsü. BİBÜ, Bolu.

Kinach, B. M. (2002a). Understanding and learning-to-explain by representing mathematics: Epistemological dilemmas facing teacher educators in the Secondary mathematics "methods" course. *Journal of Mathematics Teacher Education*, 5, 153–186.

Kinach, B. M. (2002b). A cognitive strategy for developing prospective teachers. "Pedagogical content knowledge in the secondary mathematics methods course": Toward a model of effective practice." *Teaching and Teacher Education,* 18(1), 51–71.

Kouba, V. L. (1989). Children's solution strategies for equivalent set multiplication and division word problems. *Journal for Research in Mathematics Education*, 147-158.

Kaasila, R., Pehkonen, E. & Hellinen, A. (2010). Finnish pre-service teachers' and upper secondary students' understanding of division and reasoning strategies used. *Educational Studies in Mathematics*, *73*(3), 247-261.

Leinhardt, G. & Smith, D. (1985). Expertise in Mathematics Instruction: subject matter knowledge. *Journal of Educational Psychology, 77*(3), 247-271.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates Press.

Mcduffy, A. (2004). Mathematics teaching as a deliberate practice: an investigation of elementary pre-services teachers‟ reflective thinking during student student teaching. *Journal of Mathematics Teacher Education*, 7, 33-61.

McMillan, J.H. & Schumacher, S. (2010). *Research in education: Evidence-basedinquiry* (7th Edition). London: Pearson.

Milli Eğitim Bakanlığı (MEB) (2005). *İlköğretim (1-5) matematik dersi öğretim programı*. Ankara: Milli Eğitim Yayınları.

Milli Eğitim Bakanlığı [MEB] (2018). *Matematik dersi öğretim programı* (İlkokul ve Ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. sınıflar). Ankara, Türkiye: MEB.

Monte-Sano, C. (2011). Learning to open up history for students: Pre-service teachers' emerging pedagogical content knowledge. *Journal of Teacher Education, 62*(3), 260-272. doi: Doi 10.1177/0022487110397842.

Newsome, J. G. (1999). Pedagogıcal content knowledge: An introduction and orientation. In GessNewsome, J. & Ledermen, N.G. (eds.), *Examining Pedagogical Content Knowledge*. (pp.3-17). Dordrecht: Kluwer Academic Publishers.

Olkun, S. & Toluk Uçar, Z. (2009). *İlköğretimde etkinlik temelli matematik öğretimi* (4. Baskı). Ankara: Maya Akademi.

Oral, N. (2020). 5*. Sınıf öğrencilerinin doğal sayılarla bölme işleminde yaşadığı zorluklar ve bu zorlukların nedenleri*. (Yayınlanmamış Yüksek Lisans Tezi). Eğitim Bilimleri Enstitüsü, Matematik ve Fen Bilimleri Eğitimi Anabilim Dalı, Pamukkale Üniversitesi, Denizli.

Penso, S. (2002). Pedagogical content knowledge: How do student teachers identify and describe the causes of their pupils learning difficulties? *Asia-Pacific Journal of Teacher Education,* 30 (1), 25-37.

Shulman, L. S. (1986). Those who understand: Knowledge Growth in Teaching. *Educational Researcher.* 15 (2), 4-14.

Simon, M. (1993). Prospective elementary teachers' knowledge of division. *Journal for Research in Mathematics Education*, *24*(3), 233-254.

Silver, E. A. (1986). Using conceptual and procedural knowledge: A focus on relationships. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 181-189). Hillsdale, NJ: Lawrence Elrbaum.

Southwell, B. & Penglase, M, (2005). Mathematical knowledge of preservice primary teachers. In. H. L. Chick and J. L. Vincent (Eds.), *International Group for the Psychology of Mathematics Education*, l4, 209-216.

Staley, K. N. (2004). Tracing the development of understanding rate of change: a case study of changes in a pre-service teacher's pedagogical content knowledge. (Unpublished Phd Thesis). Graduate Faculty of North Carolina State University, Raleigh. http://www.lib.ncsu.edu/resolver/1840.16/3872

Steffe, L. P. (1988). Children's construction of number sequences and multiplying schemes. *Number Concepts and Operations in the Middle Grades*, *2*, 119-140.

Tekin Sitrava, R., Özel, Z. & Işık, A (2020). Sınıf Öğretmeni adaylarının bölme işleminin anlamına dair alan bilgilerinin incelenmesi. *Kastamonu Eğitim Dergisi, 28*(2).

Tekin Sitrava, R. (2018). An investigation of prospective mathematics teachers' knowledge of basic algorithms with whole numbers: A case of Turkey. *European Journal of Educational Research*, *7*(3), 513-528.

Thompson, P.W., Carson, M. & Silverman, J. (2007). The desing of task in support of teachers'development of coherent masthematical meanings. J*ornal for Mathematics Teacher Education, 10*(4-6), 415- 432.

Tirosh, D., Fischbein, E., Graeber, A., & Wilson, J.W. (1999). Prospective elementary teachers' conceptions of rational numbers. Access: http:// jwilson. coe. uga. edu/Texts. Folder/ Tirosh/ Pros.El.Tchrs.html.

Türnüklü, E. B. (2005). Matematik öğretmen adaylarının pedagojik alan bilgileri ile matematiksel alan bilgileri arasındaki ilişki. *Eğitim Araştırmaları Dergisi*, 21, 234–247.

Thanhieser, E. (2009). Preservice elementaray school teachers' conceptions of multidiğit whole numbers. *Journal for Research in mathematics Education*, 40, 252-281.

Toluk Uçar, Z. (2011). Öğretmen Adaylarının Pedagojik İçerik Bilgisi: Öğretimsel Açıklamalar. *Turkish Journal of Computer and Mathematics Education*, 2 (2), 87-102.

Uşak, M. (2005). F*en bilgisi öğretmen adaylarının çiçekli bitkiler konusundaki pedagojik alan bilgileri*. (Yayınlanmamış Doktora Tezi). Gazi Üniversitesi Eğitim Bilimleri Enstitüsü, Ankara.

Yeşildere, S. & Akkoç, H. (2010). Matematik Öğretmen Adaylarının Sayı Örüntülerine İlişkin Pedagojik Alan Bilgilerinin Konuya Özel Stratejiler Bağlamında İncelenmesi. *Ondokuz Mayıs Üniversitesi Eğitim Fakültesi Dergisi,* 29 (1), 125-149.

Yıldırım, A. & Şimşek, H. (2011). *Sosyal bilimlerde nitel araştırma yöntemleri* (8. baskı). Ankara: Seçkin Yayıncılık.