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An Examination of Open-Ended Mathematics Questions' Affordances

Erhan Bingölbali [i]
Afyon Kocatepe University

Ferhan Bingölbali [ii]
Independent Scholar

Abstract

This study explores the affordances that the open-ended questions hold in comparison with those of closed-ended questions through examining 6th grade students' performance on a mathematics test. For this purpose, a questionnaire including 2 open-ended and 2 closed-ended questions was applied to 36 6th grade students. The questions were prepared in the light of four categories: (i) question with one correct outcome (closed-ended), (ii) question with multiple fixed outcomes (closed-ended), (iii) question with multiple variable outcomes (open-ended), and (iv) question with limitless outcomes (open-ended). The collected data were analysed in terms of correct, incorrect, uncategorized and unanswered categories as well as with regard to the diversity of the responses. The findings reveal that students showed lower performances for the question that requires limitless outcomes, there were a lack of generalizations or general rules in their responses and they provided more diverse responses for the open-ended questions. The findings were discussed with regard to higher-order thinking skills such as creativity and divergent thinking as they are often associated with open-ended questions and their affordances. Finally some implications are put forward and further research areas are highlighted.

Keywords: Open-Ended Question, Question With Multiple Correct Outcomes, Creativity, Divergent and Convergent Thinking, Mathematic Teaching

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[i] **Erhan Bingölbali,** Assoc. Prof., Mathematics Education, Afyon Kocatepe University, ORCID: 0000-0001-5373-9341

Correspondence: erhanbingolbali@yahoo.co.uk

[ii] **Ferhan Bingölbali,** Dr., Curriculum and Instruction, Independent Scholar, ORCID: 0000-0003-0847-1328

INTRODUCTION

Dissatisfaction with multiple choice and closed-ended questions has led educators to develop measurement and evaluation tools that can provide more insights about students' understanding, knowledge development, and thinking (Silver, 1992). This dissatisfaction has also been a driving force for the development of open-ended tasks, questions and problems that can be used in a classroom for conducting a process-oriented teaching rather than an outcome-oriented one (Pehkonen, 1997; Becker & Shimada, 1997; Nohda, 2000). Alternative measurement and evaluation approaches that arise in all fields of education, including mathematics education, are the product of endeavors in this direction. International exams such as PISA and TIMSS, in which mathematical literacy is tested, have also been influenced by these endeavors and have included alternative question types for assessment (OECD, 2017; Mullis, Martin, Ruddock, O'Sullivan, & Preuschoff, 2009). One of the prominent question types that came to the foreground in this quest has been the open-ended question or task.

In Turkey, where this study is carried out, open-ended questions or questions with multiple correct answers have been gaining more attention especially due to exams at the national level (MoNE, 2017). Although open-ended questions are often on the agenda, it appears that the term open-ended question is not clearly defined and especially the affordances that they hold are not sufficiently explored. In this study, the affordances of mathematical open-ended questions are explored through the 6th grade students' performances on open-ended questions. In this regard, in order to examine the affordances of the open-ended questions more closely, firstly the open-ended question and its definition will be presented. Later, a literature review of the studies on open-ended questions will be provided. Next, the conceptual framework that also guides the preparation of the questionnaire's items will be introduced. Method, findings, discussion, and conclusion and implications sections will follow this section respectively.

What is an open-ended question?

The terms such as open-ended problem, open-ended task and open-ended question are mainly employed to refer to the openness of an item. The term 'open' in the open-ended question (we use the term question to include problem and task as well) refers to diversity. The openness allows for different definitions of the term open-ended by nature. The related literature shows that different definitions about open-ended question are the case. For instance, Pehkonen (1997, p.8) employs 'open-ended problem' as an umbrella to include "investigations, problem posing, real-life situations, projects, problem fields (or problem sequences), problems without question, and problem variations ("what-if"-method)". Silver (1995) states that the term open problem contains different meanings and provides a list of four different descriptions for it:

- Unsolved mathematics problem for some time (e.g., Fermat's Last Theorem was unsolved until 1993)
- Problem that enables different interpretations or different acceptable answers
- Problem that enables different methods of solution
- Productive problems which allow new or following problems to be posed

As Silver (1995) also stated, while the first definition is related to unsolved problems in the history of mathematics, the other three are concerned with the mathematics learning and teaching. The second description refers to multi-answers while the third one points to multi-methods in a problem. Productive problems are considered to be open when they "naturally suggested a chain of related problems" (ibid., p.68) and they are hence prolific.

Open-ended questions are defined in connection with well-structured and ill-structured problem types as well (Leung, 1997). Problems in which the given, the outcome and operations are well-defined are called well-structured problems (Reitman, 1965, cited in Leung, 1997). While the problems in which the given and the outcome are both well-defined are considered as well-structured (closed-ended), the problems in which either or both of these are not well-defined are considered as ill-structured (Reitman, 1965).

Sullivan, Warren & White (2000) employ the term open-ended to define tasks that have open goals. They stated that the tasks where the solution method is not immediately available to the student are considered to be a problem, but they prefer to use the term task as the tasks in their studies do not always have this feature. Defining the terms of open-ended and closed-ended, Sullivan et al. (2000) stated that the term closed-ended refers only to the existence of an acceptable path, response, approach or justification system, while the term open-ended refers to situations in which more than one of these exists.

Tsamir, Tirosh, Tabach, & Levenson (2010) attended to open-ended questions and related concepts through the terms of 'solution', 'methods' and 'outcome'. They stated that the term solution can be used in three different ways: i.) The process followed in a problem solution, ii.) The answer (outcome) given for a problem or iii.) Both (both process and final answer). They stated that they use the term methods for solution processes, the result (outcome) for the final answer to the problem and the solution for both method and result (outcome). Tsamir et al. (2010) used the terms of "multiple outcomes" and "multiple solution methods" while referring to the openness of the tasks and diversity of solution methods.

As can be seen from the literature review presented until now, different definitions have been presented for open-ended questions, problems and tasks. In this study, open-ended and closed-ended constructs are considered in terms of final outcomes (correct answers) of the questions, as in Tsamir et al. (2010). If a question has only one correct answer or the number of answers is fixed and determined, this question is considered as a closed-ended, and if it has more than one correct answer and the answer shows variability, it is considered as an open-ended question. If a question has a single correct answer, but asks for different solution methods as an outcome, this question is also considered as an open-ended one.

Literature review

In this study, the concept of open-ended question is considered to include open-ended problems and open-ended activities as well. While presenting the relevant literature, therefore, studies on open-ended problems and open-ended activities will also be examined.

Our own review of the literature shows that studies on open-ended questions, problems and tasks can be examined in six distinct themes. These themes are; (i) performance and opinions of students on open-ended questions (Cai, 2000; Sullivan, Warren, White, & Suwarsono, 1998), (ii) the use of open-ended questions as a teaching approach (Nohda, 2000; Becker & Shimada, 1997), (iii) the types and frequencies of open-ended questions in the textbooks (Bingolbali, 2020; Zhu & Fan, 2006), (iv) the use of open-ended questions in the professional development of pre-service and in-service teachers (Zaslavsky, 1995; Bragg & Nicol, 2008), (v) characteristics of open-ended tasks and problems, and their relationship with creativity, divergent-convergent thinking skills (Bennevall, 2016; Kwon, Park, & Park, 2006) and finally (vi) the use of open-ended questions in assessment and evaluation (Silver, 1992; MoNE, 2017; OECD, 2017). The related literature is presented in relation with these themes respectively.

With regard to the first theme, students' ways of responding to open-ended mathematics questions is one of the issues that has received attention. For example, Sullivan & Clarke (1992, p.44) conducted a study with participants at different levels using open-ended questions which they called

'good questions' (e.g., “If the circumference of a rectangle is 30 m, what's its area?”). The researchers examined how participants responded to such questions, whether the responses depended on working individually or in groups, whether age or school experience had an impact on the given responses, whether the question format and an intervening teaching increased the number of students who provided multiple correct and general rule-based responses. They also examined how participants think and justify their answers and they did this examination in several following-up stages. In the first stage, the findings showed that only a few students gave multiple correct answers to the questions, their responses were limited to one answer, and working individually or as a group did not have an effect on the number of given correct answers. The second stage findings indicate that the number of 10th grade students who provided only one correct answer to the questions was high and 10th grade students gave more of multiple correct and general rule-based answers than the students at the 6th grade level. The findings also reveal that the numbers of primary school teacher who provided only one correct answer was higher than those of 10th grade students, yet 10th grade students provided more of general rule-based responses than primary school teacher candidates did. The third stage findings revealed that an intervening teaching did not increase the number of students' correct answers, but an explicit request of multiple answers led to more correct responses. Finally, the fourth stage findings showed that when compared to written tests, more multiple correct and general rule-based answers were obtained in the interviews and the interviews hence gave more insight about the students' approach to open-ended questions.

As a follow-up study, Clarke, Sullivan, & Spandel (1992) examined whether students’ failure to provide multiple correct and general rule-based answers to open-ended mathematics questions is the case for other subjects as well. To this end, they asked open-ended questions to the 7th and 10th grade students for the subjects of Social Science, English, Science and Mathematics. The findings were examined in terms of parameters such as student grade level, gender, the nature of the problem, if the clues pointing to the openness in the questions were explicit and if these variables were related to the subjects. The findings show that the tendency to provide a single correct answer to the questions is common to all subjects and hence is not only limited to mathematics. It is also found that when multiple correct answers are explicitly requested, students provide more correct answers in all the subjects. The findings further show that in all the courses except English, more correct answers increase with the years spent at school and female students provide more correct answers.

In another study, Sullivan, Warren, & White (2000, p.8) explored 8th grade student performances through questions selected from different learning areas in terms of four types of questions: (i) closed no-context (“A rectangle has an area of 2 m^2. It is 40 cm wide. How long is it?”), (ii) closed contextual (iii) open-ended no-context and (iv) open-ended contextual (“A rectangular rug has an area of 3 m^2. What might be the length and width of the rug?”). The findings showed that “in one case, the open-ended tasks were easier; in another, there was little difference; and in the third case, the open-ended tasks were more difficult” (p.15).While presenting questions in specific contexts was sometimes helpful for students’ high performance, in other cases it was not. It was also stated that both context and open-endedness affected the answers to the questions.

Cai (1995) posed an open-ended question with more than one correct answer to 250 American and 425 Chinese sixth grade students. Although the question was presented in a different form, it can be rephrased as follow: ‘Some blocks are grouped as two, three and four and each time one block left over. How many blocks are there?’. It was found that 54% of Chinese students and 56% of American students gave correct answers to this question, in which correct answers can be derived from the algebraic expression of $1 + 12n$ ($n = 0,1,2,\ldots$). This question has answers such as 13, 25, 49 etc., and it was found that the students mostly provided 13 as an answer and the number of Chinese students who provided different responses other than 13 was higher. It was also observed that the proportion of American students who gave more than one correct answer was only 1% (2) and that of Chinese students was 3% (7).

The second theme is concerned with the teaching approaches such as the open approach method (Nohda, 2000) or open-ended approach (Becker & Shimada, 1997). The open-ended teaching approach in which open-ended problems played a central role emerged in Japan in the 1970s (Nohda, 2000; Becker & Shimada, 1997; Inprasitha, 2006). In the open approach method, since the problems are both solved by different methods and they have multiple correct answers, it is also called an open-ended approach (Lin, Becker, Ko, & Byun, 2013). The studies show that when this approach is used as a teaching method, it leads to the development of both conceptual and procedural understanding in prospective teachers (Lin et al., 2013), and also contributes to the development of communication, connectivity, mathematical thinking and conceptual understanding (Munroe, 2015). In the study of Boaler (1998) with students from two different schools, one of which is based on traditional approach and the other is based on open-ended teaching environment; it was revealed that students' mathematical understandings developed procedurally in the school where traditional education was carried out, but conceptually in the school where open-ended activities were implemented. Al-Absi's study (2013) likewise showed that the use of open-ended activities had a positive effect on the development of students' achievements while the study of Viseu & Oliveira (2017) showed that with the use of open-ended activities in the classroom, mathematical communication shifted from teacher-centered to student-centered one.

From the point of the view of textbook research, some studies have been conducted to examine whether the questions, problems or tasks in the textbooks of different countries or the same country are open-ended or not (e.g., Bingölbali & Bingölbali, 2020). Analysis of some of the textbooks from different countries show that an average of over 90% questions in the mathematics textbooks are closed-ended and very few open-ended math questions are provided (Zhu and Fan, 2006; Han, Rosli, Capraro, & Capraro, 2011; Yang, Tseng, & Wang, 2017). For example, Glasnovic Gracin (2018), who analysed 6th, 7th and 8th grade mathematics textbooks in Croatia, found that more than 97% of the tasks in the textbooks were closed-ended. Similar findings were obtained by Bingolbali (2020) and it was found that only 8% of the questions in the three textbooks (6th, 7th and 8th grades) at the elementary school level had multiple correct answers.

The issue of the higher-order thinking skills such as creativity and divergent-convergent thinking is another theme through which open-ended questions have received attention (Bennevall, 2016; Klavir & Hershkovitz, 2008; Mann, 2006; Kwon et al., 2006). For example, Bennevall (2016) has examined the relevant literature to identify the examples of tasks that have fostered creativity skills in mathematics teaching, identified different types of open-ended tasks and discussed how these tasks are useful for the development of creativity skills. Kwon et al. (2006) prepared and implemented a program based on an open-ended teaching approach using open-ended problems to improve divergent thinking in the elementary school students. The findings reveal that the experimental group students performed better in each of the components of fluency, flexibility and originality, which are components of divergent thinking, compared to the control group.

Another area in which the open-ended questions, tasks and problems received interest is concerned with the professional development and understanding of in-service and prospective teachers. For example, Pehkonen (1999) revealed that the majority of teachers could not make a satisfactory description of what open-ended tasks are. In a professional development research in which closed-ended questions were turned into open-ended questions, Zaslavsky (1995) found that the use of open-ended questions provides awareness to teachers in terms of student differences, errors as being a part of the teaching process and importance of collaboration and active participation in producing solutions. Bragg & Nicol (2008) examined prospective teachers' experience of developing open-ended problem and problem posing, and showed that this experience enabled the candidates to examine their single correct answer-based mathematics perspective and their pedagogical approaches, and provided awareness about good learning practice such as problem posing.

Finally, the use of open-ended questions has drawn attention in terms of the theme of measurement and evaluation as well (Silver, 1992; Silver & Lane, 1993; Morgan, 2003; Mullis et al.,

2009; OECD, 2017; MoNE, 2017; TIMSS, 2015). For example, in the published math test of the TIMSS exam for 4th and 8th grade students, the following question was posed to 4th grade students "Cihat is rounding the numbers to the nearest 100. Write a number that is less than 200 and that Cihat can round to 200. " (TIMSS, 2015). With this open-ended question, students are given the freedom to choose any number between 150-200. In research studies and exams, more complex open-ended questions are addressed to the participants in order to obtain more in-depth insight from students about many mathematical skills such as problem solving, reasoning, connectivity and communication (Silver & Lane, 1993; Morgan, 2003). All these reveal how functional open-ended questions, problems and tasks are for the whole teaching process.

Conceptual framework

Both the definition of the open-ended question presented above and the literature review section suggest that there is an ambiguity with regard to conceptualization of open-ended question. There is hence a need for a conceptual framework in order to conceptualize what an open-ended question can stand for. With a few exceptions (Reitman, 1965, cited in Leung, 1997), the related literature reveals that there is no systematic approach in this regard. In fact, as the literature review section shows, many different meanings have been attached to open-ended questions (Silver, 1995). In this part of the study, the conceptual framework developed by Bingolbali (2020) will be presented for the conceptualization of the open-ended question. This conceptual framework has been used in the analysis of textbooks and in this study; it is used to develop the data collection tool as well.

In the conceptual framework, the number of correct answers for the questions is an important indicator for conceptualizing them. The conceptual framework consists of two main categories: (1) Questions with one correct answer, (2) Questions multiple correct answers. The question of 'A rectangle is 3cm wide and 8 cm long. What is its area?' has a single correct answer, while the question of 'What can be the lengths of a rectangle that has an area of 24 cm^2?' has multiple correct answers. The questions with multiple correct answers are divided into two sub-categories: (2.1) questions with finite correct answers and (2.2) questions with infinite (limitless) correct answers. When the question 'What can be the lengths of a rectangle that has an area of 24 cm^2' is examined in the set of natural numbers, this question has the finite correct answer, yet when it is examined in the set of rational or real numbers, it has limitless correct answers. In addition, the questions with (2.1) finite correct answers are divided into two parts: (2.1.1) questions with multiple fixed outcomes and (2.1.2) questions with multiple variable outcomes. The answers for the question of 'Find positive integers which can be divided by 3 and smaller than 20' are finite, determined and fixed. However, if the question is posed as 'Provide a positive integer less than 20 that can be divided into 3 without a remainder?', the answers will vary and the question is considered as having multiple variable correct outcomes.

As can be seen from the outcomes of the presented questions, the conceptual framework is based on the number of answers that can be given to a question. Since this study focuses on open-ended questions, questions with one correct answer or multiple yet determined answers are considered as closed-ended, whereas questions that allow the answers to be multiple and variable are considered as an open-ended one. In this regard, the questions with one correct answer given in the category (1) and multiple fixed answers in the category (2.1.1) are considered as closed-ended, while the questions in the categories of (2.1.2) and (2.2) are considered as open-ended.

METHODOLOGY

This study adopts qualitative research approach (Strauss & Corbin 1998) and it is designed as a case study to examine the affordances of mathematical open-ended questions through the responses given by 6th grade middle school students to open-ended and closed-ended mathematics questions. The case study is limited to a group of 6 grade students and their answers to open-ended and closed-

ended questions. In what follows, we present how we carry out this study by providing the details about participants, data collection tool and data analysis respectively.

Participants

As the data collection tool consists of questions related to the learning outcome of 'interprets and calculates the arithmetic average of a data group' in the sixth grade mathematics teaching curriculum, the study was carried out specifically with 6th grade students. A purposive sample was hence chosen and a total of 36 6th grade students from two different classes in an elementary school located in the central region of Turkey took part in the study. The mathematics achievement level of the participants was evaluated as above-average and good by their teachers.

Data Collection Tool

A questionnaire consisting of four questions was employed as a data collection tool. One of the questions was designed with a single correct answer and the other three with multiple correct answers. The questions were developed in line with the learning outcome of 'interprets and calculates the arithmetic mean of a data group' as a part of data processing learning area in the 6th grade curriculum. All questions were context-based. As shown in Table 1, the first question is closed-ended. The second one has multiple correct answers and the number of answers to this question is finite and fixed. The second question is also closed-ended. The third question has also multiple correct answers, however since the answers will vary, this question is considered as having multiple variable correct answers. The fourth one has infinitely (limitless) accurate answers.

Table 1 Data Collection Tool Items

No	Question Type	Question
1	Question with one answer	The arithmetic average of the ages of Ali, Bilge and Cemile is 16. Ali is 12 and Cemile is 20 years old. How old is Bilge?
2	Question with multiple fixed answers	The average age of three siblings named as Ahmet, Zeynep and Kemal is 3. What are all possible cases for their ages?
3	Question with multiple variable answers	The average age of three siblings named as Ayşe, Emin and Yusuf is 15. What could be an example for their ages?
4	Question with limitless answers	Aylin, Betul and Emre are playing a game by randomly writing a positive integer on the cards. If the arithmetic mean of the integers is a positive even integer, what could be the numbers written by these three?

In parallel with the conceptual framework, since the number of correct answers to the first question is one and to the second question is determined and fixed, they are regarded as closed-ended. Since the third and fourth questions have multiple and variable correct answers, they were considered as open-ended ones.

Data Collection and Analysis

The questionnaire was administered to students after the topic of average mean was covered at the school. The administration took about 15 minutes. The participants were volunteers and they were ensured that their responses were not going to be graded.

Qualitative content analysis was employed in the analysis of students' answers to the questions. All the obtained data were put together and subjected to preliminary examination first. As a result of the iterative examination, codes and categories were determined for the analysis and this process enabled us to figure out how to analyze the data. After this process, it was decided to analyze the obtained data in two stages. In the first stage, the answers were analyzed using four categories: i.) Correct, ii.) Incorrect, iii.) Uncategorized, iv.) Unanswered. Only these categories were used for the first question. In the second stage, the answers given for the other three questions and their distinct features were taken into consideration and hence analyzed separately. In the third and fourth questions,

each response given by the participants was evaluated as a category and analyses hence were carried out on individual responses. In the second question, since the responses to be given are fixed, the analysis was carried out over the number of correct answers.

After the preliminary examination of the data and determination of the categories, the data was analyzed by both researchers. Since the questionnaire items allowed the responses to be classified quantitatively, there was a close agreement of 98% between the two researchers regarding the classification of students' responses. A common decision was reached for all responses when there was a disagreement.

FINDINGS

The findings will be presented in four parts, respectively, for each question.

Findings Related to the First Question

The first question posed to the students is closed-ended and has only one correct answer. The results show that 78% of students answered this question correctly. When the incorrect answers and uncategorized answers are evaluated together, it seems that 22% of the students did not answer this question correctly.

Table 2 Findings for Question-1

Category	Frequency/percentage (n=36/%)
Correct	28 (78%)
Incorrect	6 (16.5%)
Uncategorised	2 (5.5%)
Unanswered	-

With regard to the incorrect responses, the findings show that the incorrect responses such as 8, 10, 14, 26 were given by the students as an answer for the first question.

Findings Related to the Second Question

The second question has multiple correct answers, yet all possible correct answers are fixed and their numbers are finite. All 7 correct answers in the category A in Table 3 are the answers that can be given for this question. However, as some students use zero in their answers, the answers in the category B were also accepted as the correct answer. The second question was hence evaluated over 12 answers in total. Other possible situations arising from the change of the numbers in each answer were not taken into account in the analysis.

Table 3 Correct Answers for Question-2

Category-A Responses	Category-B Responses
7, 1, 1	9, 0, 0
6, 2, 1	8, 1, 0
5, 3, 1	7, 2, 0
4, 4, 1	6, 3, 0
5, 2, 2	5, 4, 0
4, 3, 2	
3, 3, 3	

The student answers were analyzed using the number of correct answers based on Table 3. The numbers given with the letters A and B in Table 4 indicate how many correct answers were given together from the relevant category. For example, A7 shows that 7 correct answers from the category-A are given together, and B2 indicates that only two correct answers from the category-B are given

together. Table 4 shows that 5 students gave all 7 correct answers in the category-A, which corresponds to 14% of all students. The number of students who gived any of two correct answers in the category-A as the correct answer is 9 (e.g., (3,2,4) (5,3,1)); the number of students who provided only one correct answer is 6 (e.g., (3,3,3)). Compared to the category-A, the number of students responding from the category-B was very low. The maximum number of correct answers given in the category-B at one time is 2, and the number of students responding in this way is only 2. Five students each gave only one correct answer from the category-B.

Table 4 Findings for Question-2

No	Responses from category-A	(n=36)	No	Responses from category-B	(n=36)
1	A7	5	1	B1	5
2	A6	1	2	B2	2
3	A4	1			
4	A3	1			
5	A2	9			
6	A1	6			
7	General Rule	2			
8	Incorrect	5			
9	Uncategorized	9			
10	Unanswered	1			

Note. The answers of each student are classified under one category. However, there were cases when a student response (A2 + B1) was addressed under two categories (A2 and B1).

The second question findings also show that 14% of the students (5) provide incorrect answer (e.g., stating that siblings may be 1 year old). In addition, 9 student responses were not categorized. The answers in this category contained both correct and incorrect answers, or contained unclear statements such as "it may be 1,2,3, ..., 9 for all, or the age of all may be the same". Only 1 student left this question unanswered, so students' overall effort to solve the question was quite high. The number of students answering this question with a general rule is only two. For example, one of the students gave the following response containing a general rule: "All of them can be 3 years old. The ages of two may be the same. They may all have different ages". Although not presented in the Table 4, it should be stated that four students in total gave answers from both the category A & B together.

Findings Related to the Third Question

The third question posed to the students enables multiple variable correct answers to be provided as responses. Findings show that 78% of the students correctly answered this question which allows variable answers to be presented. Although only one example was requested from the students, one student gave two correct answers and three students gave three correct answers. Twenty-four students gave a single correct answer as requested.

Table 5 Findings for Question-3

Category	Frequency/percentage (n=36/%)
Correct	28 (78%)
Incorrect	4 (11%)
Uncategorised	3 (8%)
Unanswered	1 (3%)

Since the third question enables multiple variable correct answers, the variety of answers given to the question has also been analyzed. As can be seen in Table 6 below, students gave 16 different answers to this question. Among the answers, (15, 15, 15) was the most common one. Only one student responded at the general rule level, and this student stated that every triple number would be the answer, with a total age of 45.

Table 6 Further Analysis of Question-3 Findings

No	Responses	Frequency	No	Responses	Frequency
1	15, 15, 15	10	9	20, 16, 9	1
2	15, 10, 20	5	10	17, 23, 5	1
3	20, 5, 20	3	11	11, 8, 26	1
4	14, 15, 16	3	12	10, 5, 30	1
5	5, 15, 25	2	13	35, 5, 5	1
6	29, 3, 13	1	14	1, 15, 29	1
7	0, 26, 19	1	15	3, 2, 40	1
8	13, 15, 17	1	16	General rule: Any numbers with a total of 45-age.	1

The findings of question-3 also point to the fact that the question holds the opportunities of multiple variable correct answers which we consider an important feature of the open-ended question.

Findings Related to the Fourth Question

The fourth question addressed to the students enables infinitely correct responses to be provided as answers. Findings show that the least correct answer was given for this question in the whole test. While 56% of the students answered the question correctly, 22% left this question unanswered. 44% of the students failed to answer this question correctly. The answers of the four students were not categorized as they provide such vague responses as 'the sum of two positive same number = even number'. When the answers of the students who gave the correct answers were subjected to further analysis, it was seen that 9 of these students gave one correct answer, 3 gave two correct answers, 2 gave three correct answers and 5 gave more than three correct answers. The number of students who provided a general rule as an answer is only 1, which corresponds to 3% in total. In this regard, this student provides "(1,2,3); (3,4,5); (5,6,7) etc. They all go up by two" as a rule.

Table 7 Findings for Question-4

Category	Frequency (n=36/%)	Category	Frequency
Correct	20 (56%)	One correct response	9
Incorrect	4 (11%)	Two correct responses	3
Uncategorised	4 (11%)	Three correct responses	2
Unanswered	8 (22%)	More than three correct responses	5
		General Rule	1

Note. The numbers in the right column of the table were obtained from the analysis of 20 correct answers in the left column. In addition, the examples given to illustrate the general rule have not been evaluated additionally under other categories.

All correct and incorrect answers to the fourth question are given together in Table 8 below. Table 8 shows that different answers were given to this question in 45 categories. The findings reveal that the answers are generally different from each other (only one answer in 36 categories) and the number of the same answers given by different students is at most 4.

Table 8 Further Analysis of Question-4 Findings

No	Responses	Frequency	No	Responses	Frequency
1	1, 2, 3	4	21	6, 2, 8	1
2	5, 6, 7	4	22	2, 0, 6	1
3	3, 4, 5	3	23	3, 6, 9	1
4	11, 12, 13	3	24	5, 4, 3	1
5	2, 4, 6	3	25	8, 10, 12	1
6	13, 14, 15	2	26	3, 8, 1	1
7	7, 8, 9	2	27	11, 3, 4	1
8	15, 16, 17	2	28	99, 1, 140	1
9	9, 10, 11	2	29	17, 18, 19	1
10	12, 20, 16	1	30	10, 1, 1	1
11	20, 10, 40	1	31	20, 2, 2	1
12	20, 20, 20	1	32	5, 5, 2	1
13	10, 20, 30	1	33	15, 10, 15	1
14	40, 50, 30	1	34	50, 50, 30	1
15	90, 60, 30	1	35	1, 1, 1	1
16	10, 30, 50	1	36	2, 2, 2	1
17	10, 10, 10	1	37	3, 3, 3	1
18	20, 40, 90	1	38	4, 4, 4	1
19	60, 80, 120	1	39	5, 5, 5	1
20	60, 40, 35	1	40	6, 6, 6	1
41		0, 8, 4			1
42	2, 4, 6, 8, 10 all positive even integers	1	43	Siblings can write the same numbers.	1
44	Any positive integer that is a multiple of 3	1	45	(1, 2, 3); (3, 4, 5); (5, 6, 7) etc. they all go up by two.	1

There were no students who stated that the answer to the fourth question was directly infinite. However, as stated above, the rate of the students who answered this question at the general rule level was 3%. It should be stated that the student answer in some categories (e.g., 22,35,43) does not satisfy the conditions of the problem.

DISCUSSION

Four questions were posed to sixth grade students about the arithmetic average calculation and interpretation in order to examine the affordances of the open-ended questions through their performances. The findings revealed that the success rate of students was 78% for the one-answered closed-ended question. In the second closed-ended question which required multiple determined answers, there were a small number of students (14%) presenting all of the specific answers. In the third open-ended question, the correct answer rate for this question was 78%. In the last open-ended question with infinitely correct answers, the correct answer rate was 56%. When the questions are examined in terms of the number of correct answers; the first question enables a fixed single correct answer, the second one has 12 specific answers, the third one enables multiple variable answers, and the last one allows infinite correct answers. In the first question, one correct answer was given; and in the second question, a maximum of 7 correct answers were given. Although the only one correct answer is given in the third question, the answers in 16 different categories mean that the answer variety is high for it. In the last question, the answers were classified in 31 different correct categories. The third and fourth questions enable the affordances of variable responses to be provided.

These findings reveal some similarities and differences compared to the related studies in the literature. The study of Sullivan & Clarke (1992) showed that when the open-ended questions are compared with the closed-ended ones, they are both easy, difficult and have no difference in difficulty. In this study, similar to the study of Sullivan & Clarke (1992), the students provided an equal percentage of correct answers for the closed-ended first question and the open-ended third question. However, in the last question that required limitless answers, the number of the correct answers that the students provided was lower. On the other hand, when compared to the work of Cai (1995), the

findings show that many more students presented different correct answers for the open-ended questions. It should be noted that this difference might be due to the cognitive demands and difficulty levels of the questions used in the studies.

The affordance that the open-ended questions provides in terms of different answers is a valued aspect emphasized in the related literature. It is especially stated that the open-ended questions with multiple correct answers are functional for the development of creativity and divergent thinking skills (Bennevall, 2016; Klavir & Hershkovitz, 2008; Mann, 2006; Kwon et al., 2006; Imai, 2000). The open-ended questions enabling multiple correct answers are related to divergent thinking, and closed-ended questions are associated with convergent thinking (Foster, 2015). The research has shown that the open-ended problems that require multiple correct answers and different solution strategies are effective in developing divergent thinking (Kwon et al., 2006). It can be stated that the multi-answer questions presented in this study, especially the third and fourth questions, provide students with opportunities in terms of both correct answers and different solution strategies and thus have the potential to lead divergent thinking. The emergence of the answers in 31 different correct categories for the fourth question and 16 different categories for the third question means that the open-ended questions with variable answers contain opportunities for divergent thinking.

For divergent thinking skills, which are sometimes considered to have components such as fluency, flexibility, and originality (Kwon et al., 2006), different correct responses are associated with the fluency component, and answers from different sets of responses are associated with flexibility and novel responses with the originality component (Evans, 1964: cited in Imai, 2000). The open-ended questions in our study provide students with opportunities for the fluency component, and the diversity of responses reveals that students benefit from this opportunity. As stated above, the variety of responses given especially for the third and fourth questions reveal that the posed questions contain opportunities for the fluency component. The data obtained from the fourth question that enables infinite correct responses show that the number of students who gave two or more correct answers was 10, and 5 of them gave more than three correct answers (Table 7). Given that the number of students in the study is 36, these findings reveal that students do not perform well in terms of the fluency component. Considering that the responses of the students are generally similar and their responses rarely include a general rule, it can be stated that students' responses are insufficient in terms of the flexibility and originality requirements as well.

The use of open-ended problems is associated with the open approach or open-ended teaching approach, as mentioned in the literature review (Nohda, 2000; Becker & Shimada, 1997). In this study, although questions are used instead of problems, it can be said that open-ended questions meet principles such as (i) Open (multi-solution) process, (ii) Open outcome (multiple correct answers), and (iii) enabling new questions to be generated (Nohda, 2000). For example, the third and fourth questions theoretically meet all three principles expressed by Nohda (2000) because of the outcomes they demand. The use of these questions in real classroom settings or interviews will surely provide better insights about their potentials with regard to afore-mentioned principals of the open-ended teaching.

The variety of the responses provided by the questions with multiple correct answers holds both advantage and disadvantage for the teaching. As can be seen in the fourth question, the emergence and evaluation of the answers in 31 correct distinct categories can be a workload for the teacher, but can also turn into an opportunity. When this opportunity is considered in terms of socio-mathematical norms, it is apparent that a request from students to provide solutions with different methods and different answers for a posed question can enrich the classroom culture (Yackel & Cobb, 1996; Uçar, 2016). Open-ended questions can thus be functional in terms of both creating and establishing socio-mathematical norms such as different and effective mathematical solution methods in the classroom. The findings indicate that the questions employed in the study can provide the opportunities for forming the stated norms in the classroom. Especially the promotion of responses

that require stating a general rule and the demand for different and effective solutions may provide an opportunity for the formation of socio-mathematical norms.

Open-ended questions, like the ones employed in this study, are also used for the measurement and evaluation purposes in international exams (Silver, 1992; OECD, 2017; MoNE, 2017; TIMSS, 2015). For example, the third question used in our study is similar to TIMSS-2015 question that we have provided earlier. The use of the question styles employed in this study for the measurement and evaluation purpose is hence valuable not only for formative and diagnostic assessment but also for summative assessment that often international tests such as TIMSS requires.

CONCLUSIONS AND IMPLICATIONS

In this study, the affordances of the open-ended questions were examined through students' performances on the closed-ended and open-ended questions. With regard to each question type and its feature, the findings show that open-ended questions give students the opportunity to provide different answers and this increases the diversity of students' answers. The closed-ended question that required only routine procedures revealed that some students had difficulties with it as well. The findings also revealed that students were less successful in an open-ended question that required infinite correct responses and were unable to produce general rule-based responses. The findings of the second question, which required certain multiple correct responses, also showed that the students who presented all the answers correctly were limited. The results overall reveal that the open-ended questions give students the opportunity to provide different answers by their nature, but the answers of the students are insufficient in terms of quality (e.g. generating general rules).

In addition to these results, this study is considered to have provided important ideas for further research and applications. For example, considering the student's perspective, further research on different topics with different groups of students is expected to produce more instructive and explanatory results regarding the possibilities and limitations of open-ended questions. In terms of teaching, how an instruction based on the use of open-ended questions affects student achievement, thinking style, belief and attitude is considered as an issue that should be addressed in further research. Since open-ended questions with multiple correct answers allow students to construct their own answers and solutions, it is then important to examine how a teaching based on such questions can contribute to the development of their individual autonomy (Yackel & Cobb, 1996).

The quality of teaching is closely related to the competence, belief and attitude that teachers have. It is hence necessary to conduct detailed examinations on the beliefs, skills, knowledge of teachers about open-ended questions and whether they use them in the classroom or not (Kasar, 2013). The effective conduction of teaching is also closely related to the quality of the materials offered to teachers and students, and therefore further research is needed to examine not only how well teaching materials provide opportunity for open-ended questions but also how such materials can be developed.

Finally, from the point of the measurement and evaluation, there is an increasing interest in recent years in the use of open-ended questions in various forms. It is clear, however, that a comprehensive conceptual framework that guides the preparation of open-ended questions is missing. The conceptual framework of this study, which deals with open-ended and closed-ended questions based on the number of correct answers, needs to be explored through further research in order to examine whether it can guide question preparation for the measurement and evaluation purpose.

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