Prospective Mathematics Teachers' Opinions about Mathematical Modeling Method and Applicability of This Method

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Abstract

The aim of this study is to identify prospective secondary mathematics teachers' opinions about the mathematical modeling method and the applicability of this method in high schools. The case study design, which is among the qualitative research methods, was used in the study. The study was conducted with six prospective secondary mathematics teachers who were taking a "teaching practice" course. In the "Teaching Practice" course, mathematical modeling method was introduced to these selected prospective teachers and activity examples appropriate to this method were presented to them. Then, the prospective teachers prepared examples similar to the activity examples that were presented to them, and they implemented these examples in the schools where they served their internship. The semi-structured interview and observation forms were used as data collection tools in the study. An attempt was made to identify prospective secondary mathematics teachers' opinions about mathematical modeling method and the applicability of this method via interviews, whereas an attempt was made to identify their efficacy in application the mathematical modeling method via observations. Descriptive analysis and content analysis methods were used in analyzing the data. In view of the study, it was found that many of the prospective teachers correctly understood what mathematical modeling meant, but they were not able to fully implement this method in classrooms. When the prospective teachers' opinions about classroom applications of the mathematical modeling method were examined, it was observed that the reasons for the experienced difficults are the fact that there was not enough time and classroom management was difficult. As for the positive aspects of mathematical modeling, the prospective teachers stated that the mathematical modeling set forth the applicability of mathematics in daily life. Furthermore, all prospective teachers stated that they consider featuring problems that involve this method in their own courses in the future, but some of them stated that they would not be able to use it since its application is difficult and time consuming.

Keywords: Mathematical Model, Mathematical Modeling, Prospective Mathematics Teacher

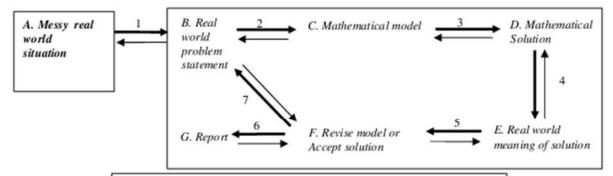
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Introduction

Today, where science and technology are rapidly developing, in order for individuals to cope with these developments, they must have equipment with which they can interpret the information and utilize it in their daily lives. The quality of education given to students, and the presented problem solving experiences, hold importance for raising individuals with these qualities. In this respect, among the aims of the high school mathematics education program in our country are developing the mathematical knowledge, thinking, skills and attitudes that students will need in discovering today and the future, and solving daily life problems that they encounter and associating these with other disciplines (Ministry of National Education [MEB], 2011). However, since mathematics is given in the classrooms as an abstract pile of rules that is unrelated to daily life and that must be memorized, most students do not know how they may encounter the subjects which they study in mathematics in daily life and the functions of these subjects in daily life, and they regard mathematics as only a course that is given in school. Featuring real life problems in mathematics courses can ensure overcoming such difficults and fulfilling the objectives of the teaching program. For this reason, it is important for students to solve mathematics problems, which help them understand the relationship between daily life and mathematics, in mathematics teaching (Kaiser, 2005). Mathematical modeling has a significant role in forming the relationships between mathematics and daily life problems. This is because mathematical modeling is the process of solving real life problems (Özer-Keskin, 2008). Models and modelling perspective requires a process of modelling in which the students go under the cycles of model construction, evaluation and revision to develop a mathematical model in the face of a given problem (Chan, 2009). Mathematical modeling has been viewed from various perspectives in the mathematics education literature (Greer, 1997; Gravemeijer, 1999; Lesh and Doerr, 2003; Van den Heuvel-Panhuizen, 2003) A focus on mathematical modeling involves three important shifts in the approach to the teaching and learning of mathematics, namely, in (1) the nature of the quantities and operations that are useful, (2) the use of contexts that will elicit the creation of useful systems (or models), and (3) the development and refinement of such models in ways that are generalisable (Doerr and English, 2003)

The concepts of the mathematical model and mathematical modeling are concepts that are frequently confused with each other. Mathematical model is to imitate reality using mathematical language (Bender, 1978). Mathematical concept segments such as variables, constants, functions, inequalities, formulas and graphs can be given as examples of mathematical models (Meyer, 1984). On the other hand, mathematical modeling is to simplify and abstract real life problems or to transform these problems into a mathematical form (Özturan-Sağırlı, Kırmacı & Bulut, 2010). In other words, mathematical modeling is the process of carrying the problem situations, which exist or which are built in the areas outside the mathematics world, to the mathematics world and transforming these problem situations into mathematical terms and mathematical solutions (Güzel & Uğurel, 2010; Perrenet & Zwaneveld, 2012). Modeling problems differ from routine problems as they require deep and high mathematical thinking and they are based on richer information in terms of content (Eric, 2010). According to Özer-Keskin (2008), the stages in the mathematical modeling process are as follows: understanding the problem, selecting the variables, forming the model, solving the problem and interpreting the solution in terms of real life. Each of these stages interacts with the other. Moreover, these stages do not have to follow a linear order. The kinds of mental activity that individuals engage in as modellers attempt to make the transition from one modelling stage to the next are given by the broad descriptors of cognitive activity 1 to 7 in Figure 1(Stillman, Galbraith, Brown & Edwards, 2007).



- 1. Understanding, structuring, simplifying, interpreting context
- 2. Assuming, formulating, mathematising
- 3. Working mathematically
- 4. Interpreting mathematical output
- 5. Comparing, critiquing, validating
- 6. Communicating, justifying (if model is deemed satisfactory)
- 7. Revisiting the modelling process (if model is deemed unsatisfactory).

Figure 1. Modelling Process.

The light arrows that are in the reverse direction to the modelling cycle are included to emphasise that the modelling process is far from linear, or unidirectional, and to indicate the presence of reflective metacognitive activity (Maaß, 2006).

Mathematical modeling is a principled activity that has both principles behind it and methods that can be successfully applied. The principles are over-arching or meta-principles phrased as questions about the intentions and purposes of mathematical modeling. These meta-principles are almost philosophical in nature. These methodological modeling principles are also captured in the following list of questions and answers (Dabbaghian, 2015):

- Why? What are we looking for? Identify the need for the model.
- Find? What do we want to know? List the data we are seeking.
- Given? What do we know? Identify the available relevant data.
- Assume? What can we assume? Identify the circumstances that apply.
- How? How should we look at this model? Identify the governing physical principles.
- Predict? What will our model predict? Identify the equations that will be used, the calculations that will be made, and the answers that will result.
- Valid? Are the predictions valid? Identify tests that can be made to validate the model, i.e., is it consistent with its principles and assumptions?
- Verified? Are the predictions good? Identify tests that can be made to verify the model, i.e., is it useful in terms of the initial reason it was done?
- Improve? Can we improve the model? Identify parameter values that are not adequately known, variables that should have been included, and/or assumptions/restrictions that could be lifted. Implement the iterative loop that we can call "model-validate-verify-improve-predict."
 - Use? How will we exercise the model? What will we do with the model?

This list of questions and instructions is not an algorithm for building a good mathematical model. However, the underlying ideas are key to mathematical modeling, as they are key to problem formulation generally.

The modeling process is regarded as a problem solving activity that is appropriate to the purpose of mathematics teaching (Kertil, 2008). Mathematical modeling also plays a great role in making students good citizens and ensuring that they participate in social developments (Blum & Ferri, 2009). This is because mathematical modeling increases the creativity of students, affects their attitudes towards problem solving, helps them understand mathematical concepts and ensures that they have a better understanding of the world (Blum, 2002; Blum & Ferri, 2009; Kim & Kim, 2010). In modeling activities, students typically work in small groups where they are required to develop shareable products that contain mathematical representations and explanations (English, Fox & Watters, 2005; Fox, 2006; Eric, 2010). As a result, students' skills in using mathematical language, and their mathematical communication skills, increased in the mathematical modeling process. Mathematical modeling assists teachers in recognizing mathematical thinking, skills and the abilities of their students (Fox, 2006). The effect of prospective mathematics teachers' knowledge and skills in mathematical modeling and the mathematical modeling method on prospective mathematics teachers' academic success, self-regulation, learning mathematics, beliefs on mathematics and problem solving skills were examined in many studies conducted with prospective mathematics teachers (Erarslan, 2011; Çiltaş, 2011; Güzel & Uğurel 2010; Kertil, 2008; Lingefjärd, 2002; Özer-Keskin, 2008; Özturan-Sağırlı, 2010; Türker, Sağlam & Umay, 2010).

Many countries tend to feature the mathematical modeling in their teaching programs. However, only several modeling examples are featured in mathematics courses and almost none of the teachers has experience in mathematical modeling (Blum &Ferri, 2009; Frejd, 2012; Kawasaki, Moriya, Okabe & Maesako, 2012). Furthermore, many teachers are not aware of mathematical modeling and its importance in teaching (Çiltaş, Deniz, Akgün, Işık and Bayrakdar, 2011; Siller & Kuntze, 2011). Apart from these studies, Kaiser and Schwarz (2006) offered this opportunity to prospective teachers in their study, in which they examined prospective teachers' ability to implement the mathematical modeling in schools, and they concluded that complicated modeling examples could be implemented in schools. In another study of theirs, Schwarz and Kaiser (2007) focused on the mathematical modeling activities conducted by prospective teachers in schools, and they examined the experiences of prospective teachers regarding modeling in schools in terms of the given examples. In their study, Ferri and Blum (2009) examined prospective teachers' opinions on the modeling process and the difficulties that they experienced throughout this process.

Therefore, it is important to introduce the mathematical modeling method, which has a significant place in mathematics teaching, to prospective teachers in their undergraduate study, and it is also important for them to find the opportunity to implement this method in schools. Otherwise, the integration of mathematical modeling into courses may become difficult. To reveal prospective mathematics teachers' efficacy in application the mathematical modeling method will provide significant information in enabling prospective teachers to realize where they fall short and identifying what they may encounter during application this method in their classrooms in the future.

What makes our study different is the fact that there is no study identifying prospective mathematics teachers' opinions on the applicability of this method and which examines their efficacy in application their acquired mathematical modeling skills in the classroom environment in consideration of the mathematical modeling method and the current high schools in our country following the mathematical modeling teaching that is offered to prospective mathematics teachers.

In this study, it is aimed to identify prospective secondary mathematics teachers' opinions about the mathematical modeling method and the applicability of this method in high schools.

Method

Research Design

In this study, the courses in the teaching practice were observed using the semi-structured observation form in order to identify prospective mathematics teachers' efficacy in application the mathematical modeling method, and interviews were conducted with the prospective teachers about

the applicability of the mathematical modeling method. When looked at from this perspective, case study design, which is among the qualitative research methods, was taken as basis in this research. This is because case study is a method in which one or more situations, environments or other interdependent systems are examined in detail (McMillan, 2000); a current phenomenon is studied in its own real life framework; and the questions "how" and "why" are examined in cases where the researcher has little control over the situations (Yin, 2002).

Participants

The convenience sampling technique, which is among the purposive sampling methods, was used in selecting the participants. The convenience sample is to choose the case that is near and easily accessible (Yıldırım & Şimşek, 2008). The research was conducted with six prospective teachers who were studying on the high school mathematics teaching program and who were taking a teaching practice course. Two of the prospective teachers are females and four are males. The prospective teachers were given the codes A1, A2,..., A6 in order to keep their identities confidential.

Instrument and Process

The semi-structured interview and semi-structured observation forms were used as data collection tools in the research. An attempt was made to identify prospective mathematics teachers' opinions about mathematical modeling method and the applicability of this method via conducted interviews. Modeling efficacy requires that the modeling processes are conducted in a suitable manner (Maaß, 2006). That is to say, it is important to follow the steps in the mathematical modeling process. For this reason, an attempt was made to identify the prospective teachers' efficacy in application the mathematical modeling method in the classrooms by considering the mathematical modeling stages (understanding the problem, selecting the variables, forming the model, solving the problem and interpreting the solution in terms of real life) via conducted classroom observations. Furthermore, it is aimed to verify the conducted interviews with observations. The interview questions, which were featured in the studies of Aydın (2008), Çiltaş (2011), Çiltaş et al. (2011), Doruk (2010), Lingefjärd (2007), Özer-Keskin (2008) and Özturan-Sağırlı (2010), were tested in preparing the interview questions whereas the studies of Ciltas et al., (2011), Özer-Keskin (2008) and Özturan-Sağırlı (2010) were used in preparing the observation form. While preparing the observation forms and interview forms, these forms were checked by two experts and revisions were made in order to make them suit our purpose. The study was implemented in the "Teaching Practice" course in the spring semester of the 2011-2012 academic year. In this application, firstly six prospective teachers were selected. The mathematical modeling method was introduced to these selected prospective teachers for five weeks and activity examples that involved mathematical modeling method were presented to them. The prospective teachers were divided into groups of two in the performed activities. Group members firstly tried to solve the activity individually for approximately five minutes and then they studied as a group for approximately 15 minutes. Guidance was avoided as much as possible while giving feedback to the students in order to enable them to reach the results on their own. The prospective teachers shared the results, which they obtained, on the blackboard. While the prospective teachers were presenting the results that they obtained, it was observed that some groups reached the same results through different ways. Following the activities, a discussion was made regarding prospective teachers' preparing mathematical modeling activities and application these activities in schools. In this process, each prospective teacher prepared two activities that involved problems for mathematical modeling method in the teaching practice courses, and they implemented these activities in the schools where they served their internship. Prospective teachers' efficacy in application their activities in accordance with the mathematical modeling method in high school classrooms was observed.

Data Analysis

Content analysis and descriptive analysis methods were used in analyzing the data obtained from the interviews and observations conducted in order to identify prospective secondary mathematics teachers' opinions about mathematical modeling method and their efficacy in application this method in schools. The data obtained from the conducted interviews was transcribed in the same

day. The category and code list was formed for the content analysis of the transcribed data. The frequencies of these codes were identified. While forming the codes, all transcripts were initially read individually, then question by question, and lastly codes, categories and frequencies were identified. Then, these transcripts were read individually again, and an attempt was made to examine prospective teachers' opinions in a holistic manner. Frequencies signify the number of prospective teachers who have each opinion. The formed category-code list was checked by researchers who are experts in their fields, and revisions were made to some codes. In the conducted coding, it was observed that the ratio of harmony among the researchers were considerably close to each other. Moreover, the reliability of the study was maintained by giving excerpts from the data that was obtained from the interviews. Classroom observations were conducted for the purpose of identifying prospective teachers' efficacy in application this method. The data obtained from the observations were analyzed via descriptive analysis by paying attention to the stages within the mathematical modeling process. Data was collected from more than one data sources, and obtained data was checked by two expert researchers in order to maintain the internal validity of the study. An attempt was made to identify the features of the participants and the process in detail in order to maintain the external validity. An attempt was made to give the obtained results in accord with the data in order to maintain the internal reliability of the conducted study. Data collection tools, raw data, the coding performed on the analysis results and reporting were checked by an expert researcher in order to maintain the external reliability of the study.

Results

This section sets forth the findings obtained from the data that was collected via the semi-structured interviews conducted with prospective secondary mathematics teachers and semi-structured observations. The questions within the semi-structured interviews were studied in this section. Each participant was given a number in order to keep the identities of the participants confidential. This numbering was formed in the form of A_x for the prospective teachers.

Categories, codes and frequencies regarding the codes, which were obtained from the analysis of interview data, are given in Table-1.

Table 1. Categories, codes and frequencies regarding prospective mathematics teachers' opinions

Category	Code	Frequency
mathematical model	equation	2
	function	1
	relationship between	2
	mathematics and daily life	
	mathematical solution to daily	3
	life problems	
mathematical modeling	process	5
	mathematical solution to daily	1
	life problems	
method in application mathematical modeling process	application the mathematical	4
	modeling steps	
mathematical modeling	mathematical solution to daily	2
problems and other mathematics problems	life problems	
	comprehensive	3
	interesting	2
	difficult	3
	not prototype	2
	the same with other activities	1
its applicability in high school	applicable	3

	not applicable	3
contributions of application	relationship between	2
process	mathematics and daily life	
	preparing for the course	2
	application the mathematical modeling	3
	communication	1
	classroom management	1
	permanent learning	1
	different perspective	2
difficulties experienced in school applications	time	3
	classroom management	2
	number of students per	2
	classroom	
	student level	1
	structure of the groups	1
	no difficult	1
Mathematics teaching in	mathematical modeling course	4
universities	applicability of mathematics	2
use in the future	plan to use it	3
	plan to use it rarely	3
positive aspects of	educational	2
mathematical modeling	applicability of mathematics	3
	interesting	2
	active participation	2

The answers given by the interviewed prospective teachers regarding the meaning of the mathematical model concept were coded under the "Mathematical Model" category as follows: equation, function, relationship between mathematics and daily life, mathematical solution to daily life problems.

The opinion of A6, who thinks that the mathematical model is an equation or a function, is as follows:

"The mathematical model can be regarded as – how can I say – an equation that has been actually mentioned under the given formula till now. As students, we learned it not with a mathematical model but with questions in the form of writing a function related to it or writing a general statement related to it or finding its formula."

The opinion of A1 for the code "relationship between mathematics and daily life", which was studied to form a relationship between mathematics and daily life, of the mathematical model is as follows:

"In my opinion, in order to associate the mathematical model with daily life, we must not only ask questions and find solutions but also contemplate on how to add something from daily life and how to model daily life mathematically. For instance, we can liken a function to a factory."

Here, A1 regarded the mathematical model as associating mathematics with daily life. However, he was not able to give the definition of the mathematical model in his statement. That is

because he thought of the mathematical model as "likening a function to a factory". It is clear that this example does not fit into mathematical model examples.

While defining the mathematical model, many prospective teachers without knowing described mathematical modeling. For instance, the opinion of A3, who thinks that the mathematical model is the mathematical solution to daily life problems, is as follows:

"I think that the mathematical model is a way of generating solutions for the real life problems by translating them into mathematical language."

When the applications of the prospective teachers were observed, it was seen that they used equations, formulas, tables and figures as mathematical models. For instance, in his first application, A1 used a graph, which presented year-based population growth, as a mathematical model.

The answers given by the interviewed prospective teachers regarding the meaning of mathematical modeling concept were coded under the "Mathematical Modeling" category as follows: process and mathematical solution to daily life problems.

The opinion of A3, who thinks that the mathematical modeling is a process, is as follows:

"In my opinion, we can define mathematical modeling as the process of preparing the mathematical model, that is to say, the process of modeling."

The opinion of A2, who thinks that the mathematical modeling is the mathematical solution to daily life problems, is as follows:

"The mathematical model is the equation that we draw, but modeling is to express and solve a daily life problem with mathematical expressions."

In view of these statements, it was observed that the prospective teachers define mathematical modeling as the process of solving real life problems.

Since it was considered in the conducted interviews that the answers given by the prospective teachers about the meaning of the mathematical model actually defined mathematical modeling, they were asked whether the mathematical model and mathematical modeling are the same thing. The prospective teachers stated that these concepts are indeed different from each other. The opinion of A3, who defined the mathematical model and mathematical modeling in a similar way on this subject, is as follows:

"The mathematical model and mathematical modeling are not the same thing. The mathematical model can be a function or a rule whereas mathematical modeling is a level and a process."

When we look at this statement, it can be stated that he could distinguish between the concepts of the mathematical model and mathematical modeling, but he was confused since these concepts are very similar to each other.

When the above given statements were examined, it was observed that the prospective teachers were generally able to define mathematical modeling. However, it was witnessed in the conducted observations that they were not able to fully use this in classroom applications. In the classroom observations and in two applications, which were performed in order to reveal prospective teachers' efficacy in application the mathematical modeling activities, we can say the following regarding the use of mathematical modeling by the prospective teachers:

For the first application, it was observed that only two (A1 and A2) of the six prospective teachers were able to fully implement mathematical modeling. Furthermore, it was observed that one

prospective teacher (A5) did not fully implement mathematical modeling whereas three prospective teachers (A3, A4 and A6) had inadequacies. It was observed that those inadequacies resulted from the fact that they were not able to interpret the formed model in terms of real life. For the second application, it was also observed that only two (A1 and A3) of the six prospective teachers were able to fully implement mathematical modeling. It was observed that three prospective teachers (A1, A4 and A5) were not able to implement mathematical modeling whereas one prospective teacher (A6) implemented it with deficiencies. When we look at both applications, we can state that only A2 was able to fully implement mathematical modeling in both of them.

The answers given by the interviewed prospective teachers regarding the method of application mathematical modeling process were coded under the "method of application Mathematical Modeling Process" category as follows: application the mathematical modeling steps.

The prospective teachers were not able to fully express the mathematical modeling steps. The opinion of A5, who thinks that these steps are composed of understanding the problem, selecting the variable and forming the mathematical model, is as follows:

"First of all, it is absolutely important to understand the question. We evaluate the data. Then, we try to understand in what kind of an order it functions. Then, as I have just stated, we try to transform this order into a form from which we can move towards a generalization by creating a formula or a function."

The opinion of A2, who thinks that the steps of mathematical modeling are to select a variable and then to make a certain generalization, is as follows:

"Firstly, variable selection is important. After selecting the appropriate variable, I try to substitute it into the expression several times, that is to say, for several values. I try it for real values. Then, I check whether I can reach a generalization from the results that I tried."

In the conducted classroom observations, it was observed that the majority of the prospective teachers did not fully use the steps followed in the mathematical modeling process. Among the prospective teachers, A1, A3 (first application), A4 (second application), A5 and A6 (first application) followed the steps of understanding the problem, selecting the variables, understanding the mathematical model and solving the mathematical model from among the mathematical modeling steps, but they did not perform the step of interpreting the mathematical model in terms of real life. A2, A3 (second application), A4 (first application) and A6 (second application) followed these steps. In view of these findings, we can state that the prospective teachers generally had deficiencies in the step of interpreting in terms of real life from among the steps of mathematical modeling. Furthermore, it was observed that the prospective teachers, who were not able to fully define the steps of mathematical modeling, followed these steps in the application process. Thus, it can be stated that the prospective teachers indeed knew these steps, but they were not able to fully define them.

The answers given by the interviewed prospective teachers regarding the comparison between mathematical modeling activities and other activities were coded under the "Problems in Mathematical Modeling Activities and Other Mathematics Problems" category as follows: *mathematical solution to daily life problems, comprehensive, interesting, difficult, not cliché and same with other activities*.

The opinion of A4, who thinks that the problems in mathematical modeling activities are more difficult when compared to other mathematics problems, is as follows:

"In the problems of mathematical modeling it is a little more difficult to identify the similar variables. We may find some clues from there according to the problem."

The opinion of A1, who thinks that the problems in mathematical modeling activities are not cliché, is as follows:

"There are certain cliché in problem solving. This is not the case with mathematical modeling. We form the model on our own. However, when you come across a problem, the solving process of this problem is cliché. It is done directly by us. It can be performed even if we really do not understand the logic behind it."

The opinion of A2, who thinks that the problems in mathematical modeling activities are more comprehensive when compared to other mathematics problems, is as follows:

"I mean they are similar in some subjects. For instance, they are similar in the calculation of turn of work days of nurses and doctors in the subject of modes. However, the mathematical model can generate problems from much larger, much more comprehensive and much more different fields. Problems in some subjects in schools are similar to this, but we cannot state that they are exactly the same."

The opinion of A3, who thinks that the problems in mathematical modeling activities are the mathematical solution to real life problems and they are interesting when compared to other mathematics problems, is as follows:

"Of course, mathematical modeling activities are more effective compared to other activities and other problem solving activities. That is because the things which are solved with models are the problems that we come across in real life. They can attract more attention from students and appeal to their needs more."

According to the above-mentioned findings, the prospective teachers stated that the problems in mathematical modeling activities are more interesting, comprehensive and related to daily life when compared to other mathematics problems. However, A6 stated, as written below, that the problems in mathematical modeling activities are the same thing as other mathematics problems:

"No, I think there is no difference among them. The other ones are also activities. That is to say, we were doing the same thing in other problems. However, we did not know that its name was mathematical modeling until now. We were finding a generalization or a function."

The answers given by the interviewed prospective teachers regarding the contributions the mathematical modeling method process had made to them were coded under the "Contributions of the Application Process" category as follows: relationship between mathematics and daily life, preparing for the course, application the mathematical modeling, communication, classroom management, permanent learning and different perspective.

The opinion of A1, who states that the application process guides them into thinking differently and contributes to forming a real life relationship, is as follows:

"First of all, the mathematical modeling method enabled me to think differently. We have a habit that dates back to high school. We follow certain steps and suggest a solution when we come across a cliché problem. However, when we come across a question or an activity in mathematical modeling, we look for what it wants from us and what we must do. It associates mathematics with real life through real life problems."

The opinion of A3, who states that the application process contributes to classroom management and mathematical modeling application, is as follows:

"What is a model? What is modeling? What must be taken into account while performing modeling or while application modeling or the model in the classroom? How can I maintain classroom order, classroom discipline and classroom management while performing modeling? I have learned these kinds of things. It benefitted me in such things. In other words, I have learned how to give a course with a different method. It benefitted me this way."

The opinion of A6, who states that the application process facilitates coming to the course prepared and teacher-student communication as well as student-student communication, is as follows:

"For instance, it enables us to maintain a healthier communication with the students during that process. The communication between me and the students as well as the communication among them are strengthened."

"It motivated me to come to the course prepared. That is what all teachers do in our time. It is something beyond the sense of giving the course with what you know already instead of preparing for the course at all. You try to accomplish something and you make an effort in doing so. In its preparation stage, you undoubtedly contemplate whether or not the student can reach a solution about it."

The opinion of A2, who states that the application process contributes to permanent learning and forming the relationship between mathematics and daily life, is as follows:

"I mean mathematics is an abstract thing in terms of what you know. You know many things, but you do not use these things in real life. However, I realized with the model that we could indeed solve all our daily life problems by knowing mathematics. It maintains permanence in what we learn. We can transfer what we know to daily life. In other words, we learn for living. I have found it more applicability and useful."

In this statement, A2 expressed that the application of the mathematical modeling method in schools contributes to forming the relationship between the mathematics course and daily life, thereby maintaining a permanent learning.

A2's opinion shows parallelism with the opinion of the high school students in the group who stated that they learned more permanently by seeing the applicability of the mathematics course in daily life right after the applications when we asked them what they thought of these applications.

When the answers given by the interviewed prospective teachers on whether or not they will use the mathematical modeling method in their courses were examined, it was observed that all of them stated that they would use this method in their courses. However, when the given statements were examined, it was observed that the frequency of the use of this method could change. The data obtained from the interviews were coded under the "future use" category as follows: *I plan to use it and I plan to use it rarely*. All prospective teachers stated that they would use this method in their courses in the future.

A3, who plans to use the mathematical modeling method in his courses in the future, stated the following:

"Yes, I plan to use it. That is because I do not want the students to perceive mathematics as something that is disconnected with daily life, that has no relation with daily life, and as if someone formed a range of rules for them to experience difficulty. I want them to realize that mathematics is a branch of science that can be utilized within daily life and in every field of life. I plan to use the modeling method for them to become more effective and more active in the courses."

Here, A3 stated that the reason for using the mathematical modeling is that the students realized that mathematics has a relationship with daily life and they became more active in the course during this process.

A4, who plans to use the mathematical modeling method rarely in his courses in the future, stated the following:

"I think I will use it very rarely. That is because it would slow the curriculum. I mean we were able to solve only two questions. Besides, it is a little difficult to prepare. If there are suitable subjects

in the future, I can perform it since I think that it functions very well. Moreover, we were able to discuss only two questions due to its difficult application while we normally solve ten questions in a normal course hour. Although it has options, I think I will not use it very much since it progresses slowly.

A4 stated that he would use the mathematical modeling very rarely in the future since its process is time consuming and its application is difficult.

Since the prospective teachers stated the positive aspects of the mathematical modeling in the conducted interviews, a category named "positive aspects of the mathematical modeling" was formed. *Educative, applicability of mathematics, interesting* and *active participation* codes are featured in this category.

The opinion of A4, who thinks that the mathematical modeling is interesting and it ensures active student participation, is as follows:

"It attracts more attention than a teacher solving problems on the blackboard. It makes the student participate in the course more."

The opinion of A1, who thinks that the mathematical modeling is easily educative, is as follows:

"There are some concepts that the students experience difficulty in learning. However, I think we can convey these concepts more comfortably to the student by performing mathematical modeling..."

The opinion of A3, who thinks that the mathematical modeling is practicable in daily life, is as follows:

"I do not want the students to perceive mathematics as something that is disconnected to daily life, that has no relation to daily life, and as if someone formed a range of rules for them to experience difficulty. I want them to realize that mathematics is a branch of science that can be utilized within daily life and in every field of life."

Apart from the opinions of the prospective teachers, when the students in the study group were asked what they thought of this application right after the conducted classroom observations, they stated that they found the applications generally entertaining, enjoyable, motivating, exciting, useful and educative. Furthermore, they stated that such activities must be performed from time to time; they can visualize some subjects in their minds more clearly and they understand them better when they see that they are used in real life. Lastly, they stated that working in groups is very useful.

In view of the feedbacks from students and the conducted observations, we can state that the applications of the prospective teachers had positive aspects such as revealing the applicability of mathematics and attracting the attention of students.

In accordance with the analysis of the prospective teachers' opinions, the difficults experienced in school applications of mathematical modeling method were coded under the "Difficults Experienced in School Applications" category as follows: classroom management, student level, number of students per classroom, time, structure of the groups and no difficult.

A2 stated the difficults that he experienced in school applications regarding the number of students per classroom, student levels and structure of the groups as follows:

"For instance, the number of students in the classroom, for which we were responsible, was high. We were able to check their answers one by one. However, I do not know what will happen when the number exceeds 20. That is to say, it is difficult to find the mistakes of each of five groups. There is another issue regarding the groups. Let's say that we formed the groups randomly. These groups must

be heterogeneous, and students must be accustomed to group work. For instance, only one student performed all activities in a group of ours while the other two students stated that they were looking at what he wrote. They talked, but they did not write anything. Thus, those two students remain completely passive in the course. We perform this application in order for the students to become active. However, since the students are not accustomed to this application, they say that they have an answer as a group, but they do not make an effort to write; they just talk. Moreover, the course becomes considerably unproductive for passive students. That is because the application depends on the solution of the teacher. All students talk concurrently. The student profile is important. The efficacy and classroom management skills of the teacher is very important. A teacher's good command of the application beyond his field knowledge is also important. I was not able to do it, for instance. I think that I experienced many problems because of that.

In the conducted observations, it was witnessed that A2, A4 and A5 experienced difficulty in classroom management, particularly in maintaining silence in the classroom. Moreover, it was observed that the applications were generally time consuming. The opinion of A2 on this issue is as follows:

"Yes, I experienced great difficulty, particularly in planning the duration of the course. It takes a lot of time to present the subject, to feature simple problems that a normal student can understand and to move on to the modeling. The preparation stage of the modeling also takes a very long time. The physical conditions of the school are also not suitable for group work. It takes 5-6 minutes for the students to form the groups. That's it. I think it is difficult to implement in crowded classrooms considering students' discussions, reaching a verdict and explaining that verdict."

The opinion of A1, who stated that he did not experience difficulty in the classroom applications, is as follows:

"I did not experience much difficulty in the classroom while application the activities."

However, when the application of A1 was observed, it was seen that he experienced a great difficulty in switching between subjects.

When the above given statements were examined, it was observed that only one prospective teacher (A1) stated that he did not experience difficulty in mathematical modeling applications. The prospective teachers stated the difficults that they experienced as follows: high number of students per classroom, difficulty of classroom management and the fact that not all students participated in the activities due to the structure of the groups. Apart from these, they stated that the difficult that they most frequently experienced was the fact that it was time consuming to implement this method.

The answers given by the interviewed prospective teachers regarding the applicability of mathematical modeling method in high schools were coded under the "Its Applicability in High schools" category as follows: *applicable* and *not applicable*.

The opinion of A3, who thinks that the mathematical modeling can be implemented in high schools, is as follows:

"I think it can be implemented. Why? Because it does not require an extra cost. It is just that the classrooms are rendered into group order if the collaborative teaching technique is to be used. However, there is a problem here: teacher efficacy. As long as teacher efficacy is at a level to implement the modeling, curriculums and physical structure of the classrooms are convenient to do this. Everything is convenient. This model can be implemented."

The opinion of A2, who thinks that the mathematical modeling cannot be implemented in high schools, is as follows:

"The mathematics course hour is limited. I served my internship in the school for two years. During these two years, I observed that teachers were in a rush to complete the subjects. Besides, our

system is based on examinations and since this modeling type does not come up in examinations, this method is not used by our teachers at all."

A2 stated that this method could not be implemented in high schools since the problems appropriate to this method are time consuming and they do not come up in the examinations.

Prospective teachers' opinions on the nature of the education that must be given in universities in order for prospective teachers who will be mathematics teachers in the future to use the mathematical modeling method at some point were collected under the "Mathematics Education That Must Be Given in Universities" category. The codes *mathematical modeling course* and *applicability of mathematics* are featured in this category.

Here, the code *mathematical modeling course* is discussed with the idea that the mathematical modeling course and its applications must be featured in the education given in universities whereas the code *applicability of mathematics* is discussed with the idea that the applicability of mathematics in daily life must be emphasized.

A3 stated the following about featuring mathematical modeling course and its applications in universities:

"We learn all subjects in theory in universities, but an education must be given towards further practice. I do not know whether or not the modeling can be a separate course. However, I believe that the prospective teachers themselves must implement this method in the classrooms so that they can see the advantages and disadvantages of this method, whether or not it can be implemented, and how it can be implemented. After all, theoretical information does not serve any purpose. It must be put into practice. Thus, education must be oriented towards practice.

Here, A3 stated that the information about the mathematical modeling in universities must be oriented towards practice rather than being theoretical information.

A1 stated the following about emphasizing the applicability of mathematics in daily life in universities.

"As I said, if mathematics is given in universities not only as the meanings of countless subjects but also to reveal what it tries to tell us, we can further facilitate the mathematical modeling process. For instance, what is function? We must know that it does not only consist of a definition. We must know where it can be used in daily life..."

As can be observed from the above given statements, A1 stated that not only mathematical concepts but also how they are used in daily life must be explained in universities.

Conclusion and Discussion

It is important to earn the prospective teachers this efficacy in order for the mathematical modeling method, which has an important place in mathematics teaching, to be implemented in high school education at the desired levels and to serve its purpose. Therefore, this study examined prospective teachers' opinions about mathematical modeling and their efficacy in application this method.

According to the findings obtained from the interviews that were conducted with the prospective teachers after classroom applications, it was revealed that prospective teachers regard the mathematical models as equations, functions, the relationship between mathematics and daily life and mathematical solution to daily life problems. As stated by Meyer (1984), we can argue that many of the prospective teachers regard the mathematical models as mathematical concept segments such as variables, functions, equalities and inequalities. However, when A1's example, in which he explained

the mathematical model as associating mathematics with daily life, was examined, it was observed that he likened a mathematical concept to a real life condition, and that was not an appropriate example to mathematical model concept. When the school applications of the prospective teachers were observed, it was seen that they used equations, formulas, graphs, tables and figures as mathematical models. When the statements given by the prospective teachers regarding the mathematical modeling were examined, it was found that they regarded the mathematical modeling as a process and mathematical solution to daily life problems. When the statements given by the prospective teachers were examined, it was observed that they agreed with the opinion of Özturan-Sağırlı (2010) which stated that mathematical modeling represented solving real life problems via mathematical terms. When the statements given by the prospective teachers regarding the mathematical model and the mathematical modeling were examined, it was observed thatthe mathematical modeling while giving the definition of the mathematical model.

Although the prospective teachers were able to define the mathematical modeling, they were not able to fully implement it in classroom applications. According to the findings obtained from the observations, only one out of six prospective teachers exhibited a more suitable last application than first application in terms of mathematical modeling. It was observed that the deficiencies of the prospective teachers in presenting the activities in the classroom were generally experienced in interpreting the mathematical model into real life.

The prospective teachers stated that the problems in the mathematical modeling activities were more comprehensive, interesting, difficult, not clichéd and related with daily life when compared to other mathematics problems. Only one prospective teacher stated that these activities are the same as ordinary problems. In his study, Bonotto (2010) concluded that mathematical modeling problems increase students' interest in the course and make it easy for them to form a relationship with life outside the school. On the other hand, in his study, English (2006a) concluded that modeling activities are real life problems that attract the interest of students, and these problems are more comprehensible for students. Similarly, in his study, Fox (2006) concluded that students take an interest in complicated and difficult modeling activities.

In the conducted interviews, it was observed that the application contributed to the prospective teachers as follows: forming a relationship between mathematics and daily life, coming to course prepared, increasing communication in classroom environments, maintaining permanent learning, application the mathematical model in classroom and thinking differently. In his study conducted in order to determine whether or not mathematical modeling increased communication, English (2006b) observed that students freely expressed their opinions in written and verbal forms, and he stated that this condition provided the students with communication skills. Regarding the positive aspects of the mathematical modeling, the prospective teachers stated that it was interesting, it maintained active student participation, it was easily educative and it ensured the applicability of mathematics in daily life. When the student groups were asked what they thought of these activities after the completion of the activities, they stated that the activities were enjoyable, entertaining, assisting in permanent learning and motivating. In his study, Frejd (2012) concluded that the mathematical modeling holds an important place for students to learn the applicability of mathematics in daily life. Furthermore, it was concluded in some studies that prospective teachers (Zbiek & Conner, 2006) and students were motivated in the mathematical modeling process (Ferrucci & Carter, 1999).

In the interviews conducted regarding the difficults that were experienced in application the mathematical model, it was observed that these difficults were mostly due to inadequate time whereas other difficults were as follows: high number of students per classroom, difficult classroom management, low student level in the group and homogeneous structure of the groups. It was observed in the conducted observations that the prospective teachers experienced difficults in classroom management and transition between subjects, and their application, was time consuming. In their studies, Schwarz and Kaiser (2007) and Blum (1991) stated that it was time consuming to implement the mathematical modeling activities. In their study, Ikeda and Kaiser (2005) stated that teachers had trouble since they had little previous experience in modeling. In their study, Blum and Ferri (2009) the

application of mathematical modeling was difficult for both teachers and students due to disputes on education and the space among daily school applications.

The prospective teachers, who participated in the interviews, thought that the mathematical modeling method could be or could not be implemented in high schools. In his study, Kaiser (2005) concluded that it was possible to implement complicated modeling examples in schools. In their study conducted with prospective teachers, Kaiser and Schwarz (2006) concluded that the mathematical modeling could be implemented in schools. The prospective teachers who thought that it could not be implemented stated that this method could not be implemented in high schools due to the fact that it was time consuming and the problems appropriate to this method were not asked in the examinations. In their study conducted to determine whether or not the examples appropriate to mathematical modeling were asked in the examinations, Kawasaki et al. (2012) stated that the mathematical modeling were not appropriate to the examinations in their countries, and the mathematical modeling must be effectively implemented by paying attention to the educational traditions.

When they were asked whether or not they would use this method in their courses in the future, the prospective teachers stated that they would want to use this method due to its applicability in daily life. However, some prospective teachers stated that they would use this method very rarely since the method was time consuming and classroom management was difficult. In the conducted interviews, the prospective teachers stated that courses on the application of mathematical modeling must be given in universities for the mathematics teachers of the future and the place of mathematics course in daily life must be emphasized. In the research conducted to determine the role of mathematical modeling in Japan and Germany, Ikeda and Kaiser (2005) found that teachers in Japan did not plan to use such applications in the future whereas teachers in Germany planned to use such applications more as such examples are given in the examinations more.

Although the prospective teachers found the opportunity to learn the mathematical modeling method in this process, it was observed that some difficults were experienced and the prospective teachers had some deficiencies. The reasons for these difficults can be listed as follows: the prospective teachers and students encountered such an application for the first time, the number of students per classroom was very high in high schools and there were many subjects in a special mathematics course. A content in which the prospective teachers can learn and implement the mathematical modeling must be featured in the faculties of education for the prospective teachers in order for mathematical modeling, which helps our students associate mathematics with real life, to be implemented better in schools and for prospective teachers to correct their deficiencies. In order to bring these accomplishments to high schools, the intensity of curriculums and the number of students per classroom must be reduced. As also stated by Eric (2010), more studies are required in this field in order to maintain a proper balance among modern and traditional approaches; to establish a connection between theory and practice; and to raise teachers.

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