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Predictive Power of 8th Grade Students' Translating Among Multiple Representations Skills on their Algebraic Reasoning

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Abstract

The purpose of the present research is defining the direction and level of the relationship between 8th grade students' translating among multiple representations skills and their algebraic reasoning and revealing the predictive power on algebraic reasoning. The research was conducted in accordance with relational survey model, which is a quantitative research method, and the study group consists of the total of 188 students, who studied at 8th grade in state schools.The data of the research were collected with the Translating Among Multiple Representations Test (TAMRT) and Algebraic Reasoning Evaluation Tool (ARET). Data were analysed using Pearson correlation and multiple linear regression analysis. Findings revealed that there is a significant relationship between students' translating among multiple representations skills and their algebraic reasoning ($r=.59$; $p<.01$). Predictive power of students' translating among multiple representations skill on their algebraic reasoning was found as 40%. According to the analysis on the each translating skill's prediction of the subscales of the algebraic reasoning, only translating to graph and table representation skills predict subscales of algebraic reasoning.

Keywords: Multiple Representations, Algebraic Reasoning, 8th Grade Students.

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INTRODUCTION

One of the most effective and important tools developing individuals' thinking ability is mathematics. Dealing with mathematics refers to a whole of the intellectual efforts including problem solving processes in accordance with a certain order and resulting from the desire to know and understand the truth (Altun, 2011; Yenilmez, & Avcu, 2009). Algebra, which is a special area of mathematics is a language representing quantities and numbers in letter symbols and involving the skills of calculating with these symbols, problem solving, working on models and presenting notations (Dede, & Argün, 2003; Kaf, 2007). Algebraic reasoning is a way of thinking that requires studying or reasoning of the mathematical structures or situations through algebraic symbols by using mathematical models or variables or diagrams, graphs, equations and tables (Herbert, & Brown, 1997; Kaya, 2015; Kriegler, 2004; National Council of Teachers of Mathematics [NCTM], 2000; Trends in International Mathematics and Science Study [TIMSS], 2003). Since algebraic reasoning involves skills requiring mental activity, such as reasoning, problem solving, working on models, understanding variables, expressing ideas using multiple representations and establishing relationships between symbolic notations; algebraic reasoning is considered as a very important concept for all levels of mathematics teaching (Çağdaşer, 2008; Greenes, & Findell, 1998; Warren, & Cooper, 2009) and for deciding on algebraic algorithms (López-Ibáñez, Prasad, & Paechter, 2005). Accordingly, an individual needs to develop their algebraic reasoning skills in order to succeed in understanding and performing in mathematics (Nathan, & Koellner, 2007).

Many previous studies have reported that students have difficulty in understanding algebra related subjects in mathematics teaching (Dede, & Argün, 2003; Ersoy, & Erbaş, 2005). The idea that different representations are required to develop students' comprehension skills in mathematics teaching (especially algebra and geometry) was formed as a result of these studies, which resulted in the emergence of the concept of multiple representations in the mathematics teaching related literature (Adu-Gyamfi, 2007; Akkuş Çıkla, 2004; Schoenfeld, 1992; Selling, 2016). Also, mathematical-algebraic objects cannot be displayed directly; they need to be semantic in multiple representational form (Carraher, Martinez, & Schliemann, 2008). What comes to mind along with the concept of multiple representations is a special language formed of a body of different notions, signs or symbols used to express mathematical concepts, opinions or objects (Durmuş, & Yaman, 2002; Duval, 1999; Kaput, 1998; Özgün Koca, 2004). Many studies have stressed that multiple representation-based approaches and practices create auxiliary environments for mathematics teaching and learning, provide basis for meaningful learning and contribute to students' construction of knowledge on a conceptual level (Ainsworth, 2006; Dreher, & Kuntze, 2015; Goldin, 2004; Ministry of National Education [MoNE], 2009; NCTM, 2008; Sevimli, 2009). In other words, students can re-structure a subject by noticing the necessary-unnecessary details or specific features related to a subject with the help of multiple representations and this way they can make the subject tangible. Therefore, it is obvious that multiple representations are of utmost importance in terms of ensuring the profound learning (Ainsworth, Bibby, & Wood 1997; İzgiol, 2014).

Some of the previous studies on the concept of representation have suggested that students' skill of choosing and forming representations among the relationships they are provided with will be more important than their calculating skills and that students with representation awareness will also develop in terms of metacognition skills (Ainsworth, 2006; Goldin, 1998; Kaput, 1998). Considering that it can be observed that the subjects are based on its different representations in multiple representations-based approaches and practices in the curriculum of primary school mathematics curriculum (MoNE, 2017). Additionally, it has been reported that teaching by enabling students expressing mathematical concepts or opinions through symbols, tables or any concrete model or any event they encounter in their daily lives increases meaningful and quality learning (MoNEf, 2009), stimulates ideas (Abdullah, Zakaria, & Halim, 2012; Parkinson, & Redmand, 2002; Stylianou, 2002), provides profound comprehension (Abdullah et al., 2012; Adu Gyamfi, 2003; Hoyles, Noss, & Kaput, 2002; Parkinson & Redmand, 2002; Roschelle et al., 2000; Stylianou, 2002) and enables algebraic thinking and reasoning (Akkan, 2009). It was suggested that extra attention should be paid to creating learning environments that enable students to establish relationships between mathematical information (Kilpatrick et al., 2001). Adu-Gyamfi (2007) stated that individual differences among students, such as learning styles or intelligence types can provide a more effective learning process in learning environments enriched with multiple representations. While explaining the importance of the multiple representations in education NCTM (2000) data emphasizes the importance of encouraging students to use multiple representations continuously in order to systematize the mathematical ideas, choose among representations in problem solving, and model and interpret the different situations in daily life.

Many studies have been conducted on the concept of multiple representations in mathematics teaching. Generally, these studies have investigated the role of representations in understanding in mathematics (Çetin & Aydin, 2020; Dreher, & Kuntze, 2015; Duncan, 2010; Gilbert, 2010; Goldin, 1998), representation transformation processes (Adu-Gyamfi, 2007), representations awareness, representations use and representation preference among students (Ahmad, Tarmizi, & Nawawi, 2010; Akkuş Çıkla, 2004; Akkuş Çıkla, & Çakıroğlu, 2006; Bal, 2014; Dündar, 2015; Gagatsis, & Elia, 2004; Herman, 2007; İpek, & Okumuş, 2012; Kılıç, & Özdaş, 2010; Sevimli, 2009), and the concept of representations in technology assisted learning with the inclusion of technology in the learning environments (Durmuş, & Yaman, 2002; Erbaş, 2005; İzgiol, 2014; Kendal, 2002; Mallet, 2007). Similarly, studies on the algebraic reasoning have focused on defining the students' levels of algebraic thinking in algebraic and conceptual terms (Çağdaşer, 2008; Dede, & Argün, 2003; Kaf, 2007; Kaya, 2015; Yenilmez, & Teke, 2008); meta-synthesis of Algebra I interventions (Dibbs, Hott, Martin, Raymond, & Kline, 2020). It is known that evaluation is not possible through knowledge-level questions in mathematics. It was reported that skills, such as reasoning, deduction, synthesis, inference, and interpreting are more valuable instead of these in the area of mathematics teaching (NCTM, 2000; Kabael, & Tanışlı, 2010). Developing 8th grade students' algebraic reasoning, which is known as the basis of mathematics, is necessary. Accordingly, investigating the relationship between algebraic reasoning and translating among multiple representations skills in the area of algebra learning is important.

The purpose of the present research is defining the direction and level of the relationship between 8th grade students' translating among multiple representations skills and their algebraic reasoning and revealing the predictive power on algebraic reasoning. Accordingly, the answers to the following research questions are sought:

1) Is there a relationship between 8th grade students' translating among multiple representations skills and their algebraic reasoning?

2) Is 8th grade students' translating among multiple representations skill (graph, table, equation, verbal) a significant predictor of their algebraic reasoning?

METHOD

Research Model

The present research was conducted in accordance with relational survey model, which is used for revealing the relationships between two or more variables, or in other words, whether variables affect each other. It enables explaining of the relationships between variables and prediction of the results (Plano Clark & Creswell, 2011; Mills & Gay, 2016). In relational research, the exploratory and prediction models were used together. In exploratory model, the relationship between variables was defined with correlation analysis, and continuous variable type scores obtained from the participants were analysed as a single group (Creswell, & Creswell, 2017). In this research, multiple linear regression analysis was conducted by assigning the independent variable as the predicting variable and the dependent variable as the predicted variable.

Study Group

The study group consisted of a total of 188 students, 57 male (30.3%) and 131 female (69.7%), who studied at 8^{th} grade in Turkey. In socio-culturally low – middle - high profile state public schools are included in the study. Informed consent form was presented to the participants before the administration. It was explained to the participants that the study will be carried out on a voluntary basis and will not be used except the purpose of the study. Non-volunteer participants were not included in the study.

Data Collection Tools

To measure students' translating among multiple representation skills the Translating Among Multiple Representations Test (TAMRT) developed by Gürbüz and Şahin (2015) was used. The sub-scales of the test are presented in Table 1 below.

Table 1 TAMRT skill and subscales of the skill (Gürbüz & Şahin, 2015)

Skill	Subscales of the skill
Translating Among Multiple Representations Skill	TAMRT1**:** Translating from verbal expression, table and equation to graph representation
	TAMRT2**:** Translating from verbal expression, table and graph to equation representation
	TAMRT3: Translating from verbal expression, equation and graph to table representation
	TAMRT4: Translating from table, equation and graph to verbal expression representation

The Cronbach α reliability coefficient was calculated as .848 at this step. In order to enable students to detect the relationship between different representations of the same data in the TAMRT consisting of twelve open-ended questions, the present research is based on four basic problems (verbal, table, equation, graph) named as flower, pool, book and quadrangle problems and participants were asked to relate the each with other three representations (Gürbüz, & Şahin, 2015). The administration of the test took 40 minutes. An example question of TAMRT is like below.

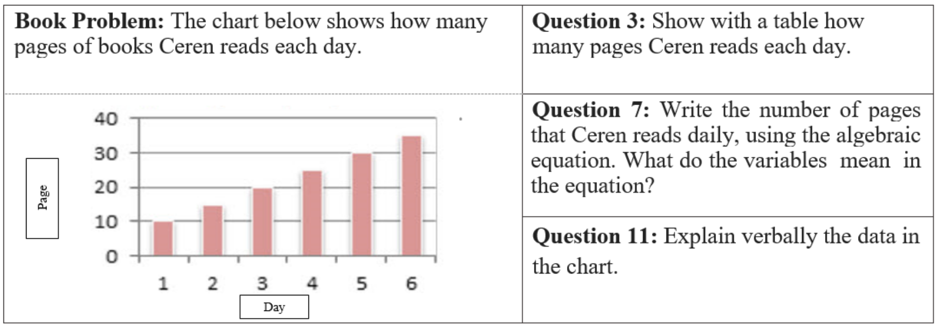

Figure 1. Example question of TAMRT

Data collected with TAMRT were scored according to an analytic scoring scale (Cetin & Ertekin, 2011) (Table 2).

Table 2 TAMRT analytic scoring scale

Test dimension	Score	Student behaviour	Min-Max score
Open-ended question	3	Solution steps are correct, reached a correct result.	0-36
	2	Solution steps are correct, result is wrong.	
	1	Solution steps are partially correct, result is wrong.	
	0	Solution steps are wrong or there is none, result is wrong or there is none.	

The second data collection tool used is Algebraic Reasoning Evaluation Tool (ARET) developed by Kaya (2015) and consisting of seven subscales in order to define school students' algebraic reasoning. The tool consists of 16 multiple-choice and 22 open-ended, the total of 38 items including test dimension and skills obtained by grouping the similar of the reasoning related skills defined by research and curriculum documents (Kaya, Keşan, İzgiol, & Erkuş, 2016). An example question concerned "defining suitable algebraic reasoning" subscale of ARET is like below.

Serkan, Canan and Ozkan will share 45 ₺ in accordance with the following conditions:

• Serkan will take 20 ₺ more than Canan.
• Canan will take twice as much money as Ozkan.

Accordingly, how much ₺ will Ozkan get ?

....

Figure 2. Example question of ARET

The subscales of ARET and the distribution of items by each subscale are presented in Table 3 (Kaya, 2015).

Table 3Subscales of ARET and the distribution of items

Title	Subscales
Algebraic Reasoning	ARET1: Defining and using algebraic constructs and relations
	ARET2: Using different algebraic expressions of the same data
	ARET3: Defining suitable algebraic reasoning
	ARET4: Making deductions of algebraic expressions
	ARET5: Establishing algebraic relations related to the deduction
	ARET6: Deciding on the correctness of the result and the correct solution steps
	ARET7: Solving non-routine problems

Cronbach α reliability coefficient of ARET was calculated as .93 by Kaya (2015). This test was administered to students in two sessions, each lasting 40 minutes. Analytic scoring scale used in the evaluation of ARET is presented in Table 4.

Table 4 Analytic scoring scale for ARET

Test dimension	Score	Student behaviour	Min-max score
Multiple choice	1	Correct	0-16
	0	Wrong	
Open-ended	4	Reasoning is clear and full and student used the reasoning correctly and answered	0-88
	3	Answer is correct, reasoning is not full and clear	
	2	Answer is wrong, but the students defined the correct reasoning, tried to use it but couldn't complete it	
	1	Answer is wrong, developed reasoning is partially correct and used only partially in the solution steps	
	0	Student made no reasoning	

Both instruments were administered in the second semester of the academic year, after the teaching of the related learning domains was completed. This test was administered to students in two sessions, each lasting 40 minutes. Initially TAMRT was administered then ARET was completed. The data were collected in mathematics courses by the researchers. The data collection process took three weeks.

Data Analysis

Pearson correlation analysis was conducted in the present research to study the relationship between 8^{th} grade students' translating among multiple representations skills and algebraic reasoning. Additionally, linear regression analysis was conducted to find out whether students' translating among multiple representations skills (verbal, graph, table, equation) are a significant predictor of their algebraic reasoning. Quantitative data analysis was conducted on SPSS 21.0 packaged software.

To conduct multiple linear regression analysis, whether the conditions of sufficient work group, multiple linear relations, extreme values and normality were met was tested. Taking the condition that there should be at least 40 participants for each predicting variable (Pallant, 2001) into consideration, it can be stated that 188 participants were sufficient for the analysis. Besides, that the values of correlation between predicting variable (translating among multiple representation skills; translating into graph, table, equation, verbal representations) is much lower than .90 (see Table 5) and that the studied Durbin Watson values are lower than 10 (1.84) indicate that there is no linear relation problem. Moreover, because 4 predicting variables were included in the present research by studying the mahalanobis values, extreme values were cleared by excluding the ones having mahalanobis values higher than 18.47 (Pearson, & Hartley, 1958; cited in Seçer, 2015). Skewness and kurtosis values indicated a normal distribution, and Pearson correlation and multiple linear regression analyses (enter method) were conducted. According to enter method, all predicting variables are processes at the same time, and so the common predictive power of all predicting variables on predicted variables is found (Seçer, 2015).

FINDINGS

In order to find out whether there is a relationship between students' translating among multiple representations skills and algebraic reasoning, Pearson correlation analysis was conducted between TAMRT subscales and whole ARET and ARET subscales, and the results are presented in Table 5.

Table 5 Analysis of correlation between TAMRT subscales and ARET subscales

	ARET	TAMRT	ARET 1	ARET 2	ARET 3	ARET 4	ARET 5	ARET 6	ARET 7	TAMRT 1	TAMRT 2	TAMRT 3	TAMRT 4
ARET	1												
TAMRT	.595**	1											
ARET1	.431**	.236**	1										
ARET2	.265**	.159*	.310**	1									
ARET3	.916**	.547**	.391**	.231**	1								
ARET4	.708**	.398**	.380**	.286**	.584**	1							
ARET5	.873**	.537**	.280**	.215**	.732**	.556**	1						
ARET6	.874**	.524**	.328**	.206**	.739**	.631**	.726**	1					
ARET7	.910**	.533**	.367**	.167*	.754**	.632**	.764**	.725**	1				
TAMRT 1	.599**	.866**	.269**	.122	.557**	.381**	.510**	.545**	.540**	1			
TAMRT 2	.581**	.851**	.282**	.142	.539**	.388**	.497**	.550**	.501**	.751**	1		
TAMRT 3	.400**	.801**	.104	.117	.357**	.277**	.402**	.304**	.382**	.530**	.507**	1	
TAMRT 4	.416**	.844**	.137	.158*	.382**	.292**	.392**	.360**	.361**	.611**	.620**	.635**	1

According to the results of Pearson correlation analysis presented in Table 5, there is a positive and significant correlation (r=.595, p<.01) between translating among multiple representations skills and algebraic reasoning, and there are positive and significant correlations between translating among multiple representations test 1^{st} dimension (translating to graph representation) and algebraic reasoning (r=.599, p<.01); between translating among multiple representations test 2nd dimension (translating to table representation) and algebraic reasoning (r=.581, p<.01); between translating among multiple representations test 3rd dimension (translating to equation representation) and algebraic reasoning (r=.400, p<.01); and between translating among multiple representations test 4th dimension (translating to verbal representation) and algebraic reasoning (r=.416, p<.01). Accordingly, it can be claimed that students' algebraic reasoning scores increase as their translating among multiple representations skills develop.

Multiple linear regression analysis was conducted to investigate whether students' translating among multiple representations skills subscales predict their algebraic reasoning total scores significantly, and the results are presented in Table 6.

Table 6 Results of the multiple linear regression analysis for the translating among multiple representations test subscales' prediction of algebraic reasoning total score

	B	Std Error	β	t	p
Constant	19.831	3.502		5.662	.000
TAMRT1	3.562	.911	.357	3.911	.000
TAMRT2	3.336	1.033	.294	3.229	.001
TAMRT3	.893	.772	.088	1.157	.249
TAMRT4	-.480	.987	-.041	-.487	.627

Dependent variable: ARET
R=.635, R^2=.403, p<.01

As presented in Table 6, according to the results of the multiple linear regression analysis there are positive and significant relationships between algebraic reasoning total score and between translating among multiple representations test 1st dimension (translating to graph representation), 2nd dimension (translating to table representation), 3rd dimension (translating to equation representation) and 4th dimension (translating to verbal representation) (*R=.635, R^2=.403, p<.01*). Students' translating among multiple representations skill (1st and 2nd dimensions; translating to graph and table representations) explains the 40% of the total variance in their algebraic reasoning. According to standardized (β) coefficient and t values, in the order of importance, translating to graph representation and table representation skills are significant predicators of algebraic reasoning. The 3rd and 4th dimensions of translating among multiple representations test (translating to equation and verbal representations) do not predict algebraic reasoning at a significant level.

Since the correlations between subscales of translating among multiple representations test and the 1st dimension (defining and using algebraic constructs/relations) and 2nd dimension (using different algebraic expression of the same data) of the algebraic reasoning evaluation tool is low (see Table 3), the prediction is not significant. Therefore, the analysis for the prediction of TAMRT subscales on other subscales of algebraic reasoning evaluation tool (3rd, 4th, 5th, 6th, and 7th) are presented.

Results of the multiple regression analysis for TAMRT subscales' prediction of ARET 3rd dimension (defining suitable algebraic reasoning) are presented in Table 7.

Table 7 Results of the multiple regression analysis for translating among multiple representations test subscales' prediction of defining suitable algebraic reasoning dimension

	B	Std Error	β	t	p
Constant	3.796	1.228		3.091	.002
TAMRT1	1.139	.319	.341	3.566	.000
TAMRT2	1.043	.362	.275	2.879	.004
TAMRT3	.197	.271	.058	.728	.467
TAMRT4	-.131	.346	-.033	-.380	.705

Dependent variable: ARET3
R= .588, R^2=.346, p<.01

As presented in Table 7, according to the results of the multiple linear regression analysis, there are positive and significant correlations between 1st dimension (translating to graph representation), 2nd dimension (translating to table representation), 3rd dimension (translating to equation representation) and 4th dimension (translating to verbal representation) of translating among multiple representations skill and 3rd dimension (defining suitable algebraic reasoning) of algebraic reasoning (*$R=.588$, $R^2=.346$, $p<.01$*). Students' translating among multiple representations skill (1st and 2nd dimensions; translating to graph and table representations) explains the 34% of the total variance in defining suitable algebraic reasoning subscale. According to standardized (β) coefficient and t values, in the order of importance, translating to graph representation and table representation skills are significant predicators of defining suitable algebraic reasoning. The 3rd and 4th dimensions of translating among multiple representations test (translating to equation and verbal representations) do not predict defining suitable algebraic reasoning skill at a significant level.

Results of the multiple regression analysis for TAMRT subscales' prediction of ARET 4th dimension (making deductions of algebraic expressions) are presented in Table 8.

Table 8 Results of the multiple regression analysis for translating among multiple representations test subscales' prediction of making deductions of algebraic expressions dimension

	B	Std Error	β	t	p
Constant	2.646	.249		10.647	.000
TAMRT1	.109	.065	.181	1.685	.094
TAMRT2	.146	.073	.214	1.995	.048
TAMRT3	.043	.055	.070	.783	.435
TAMRT4	.003	.070	.004	.041	.967

Dependent variable: ARET4
$R= .415$, $R^2=.173$, $p<.01$

As presented in Table 8, according to the results of the multiple linear regression analysis, there are positive and significant correlations between 1st dimension (translating to graph representation), 2nd dimension (translating to table representation), 3rd dimension (translating to equation representation) and 4th dimension (translating to verbal representation) of translating among multiple representations skill and 4th dimension (making deductions of algebraic expressions) of algebraic reasoning ($R= .415$, $R^2=.173$, $p<.01$). Students' translating among multiple representations skill 2nd dimension (translating to table representation) explains the 17% of the total variance in defining suitable algebraic reasoning subscale. According to standardized (β) coefficient and t values, translating to table representation skill is a significant predicator of making deductions of algebraic expressions. The 1st, 3rd and 4th dimensions of translating among multiple representations test (translating to graph, equation and verbal representations) do not predict making deductions of algebraic expressions skill at a significant level.

Results of the multiple regression analysis for TAMRT subscales' prediction of ARET 5th dimension (establishing algebraic relations related to the deduction) are presented in Table 9.

Table 9 Results of the multiple regression analysis for translating among multiple representations test subscales' prediction of establishing algebraic relations related to the deduction dimension

	B	Std Error	β	t	p
Constant	1.165	.752		1.549	.123
TAMRT1	.525	.196	.264	2.686	.008
TAMRT2	.509	.222	.226	2.295	.023
TAMRT3	.304	.166	.151	1.834	.068
TAMRT4	-.012	.212	-.005	-.058	.954

Dependent variable: ARET5
$R= .553$, $R^2=.305$, $p<.01$

According to the results of the multiple linear regression analysis, there are positive and significant correlations between 1st dimension (translating to graph representation), 2nd dimension (translating to table representation), 3rd dimension (translating to equation representation) and 4th dimension (translating to verbal representation) of translating among multiple representations skill and 5th dimension (establishing algebraic relations related to the deduction) of algebraic reasoning (R= .553, R^2=.305, p<.01). Students' translating among multiple representations skill 1st dimension (translating to graph representation) and 2nd dimension (translating to table representation) explain the 30% of the total variance in establishing algebraic relations related to the deduction subscale. According to standardized (β) coefficient and t values, translating to graph representation and table representation skills are significant predicators of establishing algebraic relations related to the deduction skill. The 3rd and 4th dimensions of translating among multiple representations test (translating to equation and verbal representations) do not predict establishing algebraic relations related to the deduction skill at a significant level.

Results of the multiple regression analysis for TAMRT subscales' prediction of ARET 6th dimension (deciding on the correctness of the result and the correct solution steps) are presented in Table 10.

Table 10 Results of the multiple regression analysis for translating among multiple representations test subscales' prediction of deciding on the correctness of the result and the correct solution steps dimension

	B	Std Error	β	t	p
Constant	.914	.778		1.174	.242
TAMRT1	.681	.202	.322	3.366	.001
TAMRT2	.813	.230	.338	3.543	.001
TAMRT3	-.031	.171	-.015	-.183	.855
TAMRT4	-.093	.219	-.037	-.425	.671

Dependent variable: ARET6
R= .586, R^2=.344, p<.01

As shown in Table 10, according to the results of the multiple linear regression analysis, there are positive and significant correlations between 1st dimension (translating to graph representation), 2nd dimension (translating to table representation), 3rd dimension (translating to equation representation) and 4th dimension (translating to verbal representation) of translating among multiple representations skill and 6th dimension (deciding on the correctness of the result and the correct solution steps) of algebraic reasoning (*R=.586, R^2=.344, p<.01*). Students' translating among multiple representations skill 1st dimension (translating to graph representation) and 2nd dimension (translating to table representation) explain the 34% of the total variance in deciding on the correctness of the result and the correct solution steps subscale. According to standardized (β) coefficient and t values, translating to graph representation and table representation skills are significant predicators of deciding on the correctness of the result and the correct solution steps skill. The 3rd and 4th dimensions of translating among multiple representations test (translating to equation and verbal representations) do not predict deciding on the correctness of the result and the correct solution steps skill at a significant level.

Results of the multiple regression analysis for TAMRT subscales' prediction of ARET 7th dimension (solving non-routine problems) are presented in Table 11.

Table 11 Results of the multiple regression analysis for translating among multiple representations test subscales' prediction of solving non-routine problems dimension

	B	Std Error	β	t	p
Constant	3.105	1.108		2.802	.006
TAMRT1	1.048	.288	.353	3.637	.000
TAMRT2	.715	.327	.212	2.185	.030
TAMRT3	.396	.244	.132	1.624	.106
TAMRT4	-.244	.312	-.070	-.781	.436

Dependent variable: ARET7
R= .568, R^2=.322, p<.01

According to the results of the multiple linear regression analysis, there are positive and significant correlations between 1st dimension (translating to graph representation), 2nd dimension (translating to table representation), 3rd dimension (translating to equation representation) and 4th dimension (translating to verbal representation) of translating among multiple representations skill and 7th dimension (solving non-routine problems) of algebraic reasoning (*$R= .568$, $R^2=.322$, $p<.01$*). Students' translating among multiple representations skill 1st dimension (translating to graph representation) and 2nd dimension (translating to table representation) explain the 32% of the total variance in solving non-routine problems subscale. According to standardized (β) coefficient and t values, translating to graph representation and table representation skills are significant predicators of solving non-routine problems skill. The 3rd and 4th dimensions of translating among multiple representations test (translating to equation and verbal representations) do not predict solving non-routine problems skill at a significant level.

The present research revealed a general significant relationship between students' translating among multiple representations skills and their algebraic reasoning. According to the findings related to students' skills of translating to each representation predicting subscales of algebraic reasoning (defining suitable algebraic reasoning, making deductions of algebraic expressions, establishing algebraic relations related to the deduction, deciding on the correctness of the result and the correct solution steps, and solving non-routine problems skills), it was found that only translating to graph and table representations skills predict subscales of algebraic reasoning at a significant level.

DISCUSSION, CONCLUSION AND RECOMMENDATIONS

According to the findings of the present research, there is positive correlation 8th grade students' translating to multiple representations skills and their algebraic reasoning. This finding is in agreement with the findings of the study conducted by Akkuş Çıkla (2004) on 7th grade students that multiple representations have a significant effect on algebra performance. Similarly, it is in agreement with the findings of the study conducted by Kaya (2015) on algebraic reasoning that multiple representations are effective in developing 7th grade students' using different algebraic expressions of the same data, defining suitable algebraic reasoning, establishing algebraic relations related to the deduction, deciding on the correctness of the result, deciding on the solution steps and solving non-routine problems skills. It is also in agreement with the finding of the study conducted by İzgiol (2014) that multiple representations-based teaching has a significant effect on pre-service teachers' linear algebra achievement.

The present research also found that 8th grade students' translating to graph and table representations skills among the multiple representation skills predict defining and using algebraic constructs and relations, using different algebraic expressions of the same data, making deductions of the algebraic expressions, establishing algebraic relations related to the deduction, deciding on the correctness of the result and the correct solution steps, and solving non-routine problems skills subscales of algebraic reasoning at a significant level. Accordingly, translating among multiple representations skill (1st and 2nd dimensions; translating to graph and table representations) explain the 40% of the total variance in algebraic reasoning. What is different than expected here is that inference representations (graph, table) rather than equation representation predict subscales of algebraic reasoning. The related literature suggests that alternative representations other than equation representation are very important in deciding suitable algebraic algorithms (López-Ibáñez, Prasad, & Paechter, 2005). Additionally, the definition that "Algebraic reasoning is a way of thinking that requires studying mathematical constructs and situations with the help of algebraic symbols, using mathematical models and variables or with diagrams, graphs, equations and tables" (Herbert, & Brown, 1997; Kaya, 2015; Kriegler, 2004; NCTM, 2000; TIMSS, 2003) verifies the findings of the present research theoretically.

While the development of algebraic reasoning is considered as a basic requirement for individuals' understanding mathematics and achievement in performing mathematics (Nathan, & Koellner, 2007) it comes along with the need of students for methods to follow in acquiring skills, such as interpreting, transferring information to representations and reasoning. Taking the relationship between students' algebraic reasoning and translating among multiple representations skills into consideration, development of translating among multiple representations skills is only possible with representations assisted teaching. According to the findings of the studies conducted in this context (Adu-Gyamfi, 2007; Akkuş Çıkla, 2004; Bal, 2014; Çetin & Aydin, 2020; Dreher, & Kuntze, 2015; Duncan, 2010; Durmuş, & Yaman, 2002; Gilbert, 2010; Goldin, 1998; Kendal, 2002; Mallet, 2007; Sevimli, 2009) translating among multiple representations skills must be developed, and for this multiple representations-assisted teaching can be utilized in mathematics teaching. In a study by (Carraher, Martinez, & Schliemann, 2008), multiple representations used even in early age (3rd grade students) contributed to their algebraic expressions. In general, the use of multiple representations had benefit for students in algebra topic (Dibbs, Hott, Martin, Raymond, & Kline, 2020; Selling, 2016).

Based on the findings of the present research, attaching extra importance to expressing graph and table representations among multiple representations can be suggested in algebra teaching. In accordance with the finding that algebraic reasoning is not predicted by procedural skills, verbal representations in mathematics education, more attention can be paid on graph and table interpretations.

It can be suggested that instead of giving procedural learnings prominence, studies on students' acquisition of translating between representations using different representations together can be conducted in algebra teaching. Teachers can contribute to the development of students' algebraic reasoning by including multiple representations-assisted practices in the teaching of the subject of algebra. Based on the present research, the relationship between translating among multiple representations skills and algebraic reasoning can be studied in the context of different variables on wider samples with structural equation modelling.

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