International Journal of Progressive Education, Volume 18 Number 1, 2022

Analysis of the Pedagogical Content Knowledge Development of Prospective Teachers in the Lesson Plan Development Process: 4MAT Model*

Feyza Aliustaoğlu[i]
Kastamonu University

Abdulkadir Tuna[ii]
Kastamonu University

Abstract

The aim of this study is to investigate the change of prospective mathematics teachers' pedagogical content knowledge (PCK) on the linear equation and slope subject based on the lesson plan development and implementation process based on the 4MAT model. The concept of PCK was discussed in three components as content knowledge, knowledge of student understanding and knowledge of instructional strategies. The research was carried out based on mixed research methods. The data collection tools consisted of 48 lesson plans developed by prospective teachers in three stages and observation notes taken based on the teachings they did. The lesson plans developed by prospective teachers were analyzed quantitatively using the rubric developed by the researcher. Whether there is a statistically significant difference in PCK components in the lesson plans was analyzed by using appropriate tests. In addition, various qualitative sections from the lesson plans showing the development of prospective teachers were presented. As a result of the research, it was concluded that prospective teachers showed improvement in each of the components of the PCK at the end of the lesson plan development and implementation process based on the 4MAT model.

Key words: Mathematics Education, Pedagogical Content Knowledge, 4MAT Model, Prospective Teachers, Lesson Plans

DOI: 10.29329/ijpe.2022.426.17

---

* This study was produced from a part of the doctoral dissertation prepared by the first author under the supervision of the second author.

[i] **Feyza Aliustaoğlu,** Dr., Mathematics and Science Education, Kastamonu University, ORCID: 0000-0001-9262-5216

Correspondence: fdemirci@kastamonu.edu.tr

[ii] **Abdulkadir Tuna,** Prof. Dr., Mathematics and Science Education, Kastamonu University

INTRODUCTION

It is stated that teachers should have various competencies and these competencies are expressed as "The knowledge, skills, and attitudes that teachers should have in order to fulfill their profession efficiently and effectively" (MoNE, 2017). One of these competencies is planning the education and training process effectively (MoNE, 2017). Lesson planning refers to the technical knowledge necessary to ensure effective classroom performance (Rusznyak & Walton, 2011).

There are various issues to be considered in lesson planning. Baki and Arslan (2015) state that the activities prepared while planning a lesson should be selected according to the level and preliminary information of the students, and the order of the samples and the number of samples provided should be adjusted appropriately. In addition, an effective lesson design to be aware of the learning difficulties and misconceptions that students may have regarding the subject to be taught; it also requires taking precautions for this. From these perspectives, it can be said that lesson planning is also closely related to pedagogical content knowledge concept.

The concept of pedagogical content knowledge (PCK) contains a lot of information such as knowing what will make learning easier and harder, the concepts and prior knowledge of students about the subject, the learning difficulties of students about that subject, what kinds of mistakes they make, which misconceptions they have, and which examples or explanations should be used to eliminate these misconceptions (Shulman, 1987). Again, Ball, Thames, and Phelps (2008) state that this concept includes different skills such as sorting mathematical content, selecting examples that will take students deeper into mathematical content, and guessing what students will find interesting and motivating when choosing an example. Considering the definitions made for the concept of pedagogical content knowledge, it is seen that this concept is important for lesson planning skills.

One of the models, which is based on the constructivism approach and which can be used in lesson plan designs, is the 4MAT (4 Mode Application Techniques) model developed by Bernice McCarthy (McCarthy, 1990). The 4MAT model is an 8-step teaching cycle based on individual learning styles and brain hemispheres (McCarthy, 1990; McCarthy, 2014). The 4MAT model is a model in line with the 21st Century Learning Standards, in which students not only memorize information, but use them in real life environments and explore the creative use of learning (Shaughnessy, 2013).The teaching cycle based on the 4MAT model is presented in Figure 1.

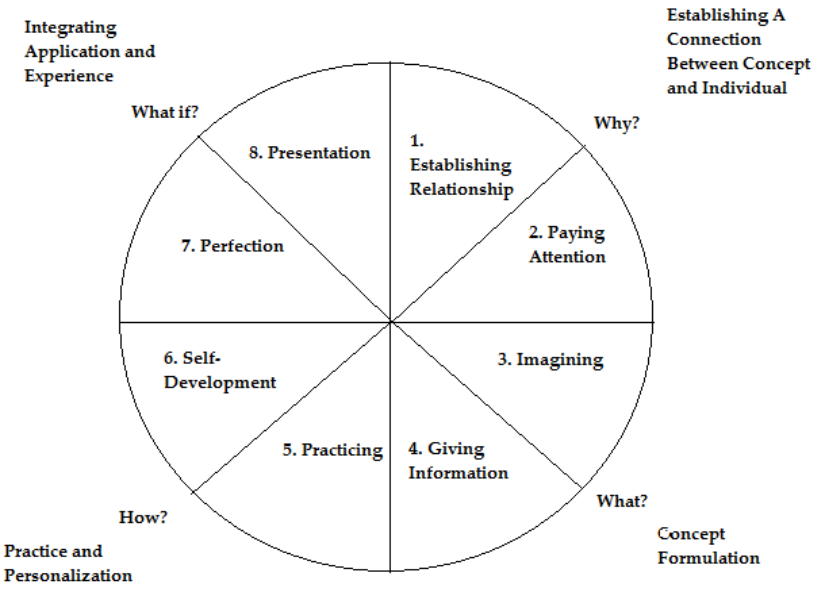


Figure 1. 4MAT Model and Its' Eight Steps (Morris and McCarthy, 1999)

In this model, in terms of learning styles, it is stated that students have individual preferences in the learning process, perceive and process information in their own ways, and there are four learning styles, all of which are of equal importance. It is argued that each individual will participate in the whole learning cycle and learn from each other and develop in different regions (McCarthy, Germain & Lippitt, 2002; Morris & McCarthy, 1999). In terms of brain hemispheres, the 4MAT model is a model that states that the dominance of the right and left hemispheres of the brain varies from person to person, while in some students the left hemisphere is more active, in some students the right hemisphere is more active. Teaching individuals in each of the four learning styles should be done using both the right and left hemisphere techniques of the brain. Thus, those who use the right hemisphere more actively will develop in the left hemisphere and those who use the left hemisphere more actively will develop in the right hemisphere (McCarthy, Germain & Lippitt, 2002; McCarthy, 2014; Morris & McCarthy, 1999).

The first step of the 4MAT model cycle is the step where students are provided with activities to establish a relationship between the concept to be learned and daily life. In the second step, the examples presented in the first step are analyzed. The third step is just before teaching the concepts and various activities are offered for students to visualize the concept in their minds. In the fourth step, the concepts are explained by the teacher. In the fifth step, the aim is to consolidate the learned information. Based on this, various implementations are made. In the sixth step, students expand their learning; they add to what they have learned from themselves. When the seventh step is reached, the evaluation and criticism of the practices and learned are done. In the eighth step, what has been done in the previous steps is presented and shared with the others. In this step, what is learned is integrated (McCarthy, Germain & Lippitt, 2002).

In the literature, there are many studies showing that the 4MAT model has a positive effect on the students' academic achievement (e.g. Aktaş & Bilgin, 2015; Alanazi, 2020; Aydıntan, Şahin & Uysal, 2020; İnel, 2018), the permanence of learning (e.g. Aydıntan, Şahin, & Uysal, 2020; Tsai, 2004) and students' attitudes towards the lesson (Aktaş & Bilgin, 2015; Burkum, 2010; Özgen & Alkan, 2012; Ramirez & Laurinco, 2015; Taylor, 2018). Kaewkiriya (2017) specified that teaching based on the 4MAT model facilitates effective and attractive learning. Additionaly, Omar, Al-Shunnaq & Al-Omari (2018) stated that teaching based on the 4MAT model is effective on metacognitive thinking. On the other hand, In Kelley (1990)'s study, which includes teachers' views based on the 4MAT model, a course was organized to introduce teachers to the 4MAT model, and their views on the model were taken after a while. In addition, administrators participated in this study. Teachers and administrators gave positive opinions about the 4MAT model and stated that they started using this model in their own programs. In the study of Özdoğan (2012), after three middle school mathematics teachers gave teaching based on the 4MAT model in their classes, the teachers' opinions about the 4MAT model were taken. Teachers stated that the 4MAT model is useful in many aspects such as frequency of material use, concretizing the subject, providing permanent learning, the student's production of information himself/herself, better understanding of the subject.

In addition, in the literature, there are studies in which tools such as concept caricatures and concept maps are used to reveal and eliminate misconceptions while preparing a lesson plan suitable for the 4MAT model (Ergin, 2011). In addition, classroom discussions, analyses, and brainstormings are conducted in teaching based on the 4MAT model, and all these processes provide an examination of student thinking (McCarthy, 1990). In addition, in the 4MAT model, the emphasis is placed on activating students in the learning process and using different strategies/methods and techniques to better understand the subject to be taught (Morris & McCarthy, 1999). Considering the role of examining student thinking and designing the teaching process in pedagogical content knowledge, the importance of using 4MAT model in researches related to pedagogical content knowledge is more clearly seen.

In the studies of Wright (2009) and Shuilleabhain (2016), the PCK development of teachers and in the study of Baki (2012), the PCK development of prospective teachers were examined within the scope of lesson study method. As the data collection tool, focus group interviews, observation

notes, lesson plans developed by teachers/prospective teachers, etc. used. In the studies of Cavin (2007), Ferna'ndez (2010), Yeşildere-İmre and Akkoç (2012), the development of prospective teachers' pedagogical content knowledge was examined within the scope of micro-teaching/micro-teaching lesson study methods. Similarly, one of the data collection tools used in these studies is the lesson plans developed by prospective teachers. On the other hand, in the studies of Eroğlu (2016) and Tataroğlu Taşdan and Çelik (2017), the PCK developments of the teachers were examined within the scope of the action research, and interpretations were made based on the lesson plans developed by the teachers.

When the studies are examined, it is seen that lesson plans are used as an important data collection tool in studying the development of PCK. From this point of view, in this research, the PCK development of prospective teachers was examined within the scope of the lesson plans they developed and the trainings they did. It is thought that the experiences that prospective teachers will gain based on developing a lesson plan have an impact on their PCK. Indeed, Lim, Son and Kim (2018), in the studies where prospective teachers' skills of developing lesson plans are examined, they stated that preparing lesson plans and teaching is the skills that are constantly developing during the professional lives of teachers. Although it is stated that the 4MAT model is a model used in lesson planning (McCarthy, 1990), there is no study in the literature that examines the PCK developments of teachers or prospective teachers based on the lesson plan development and teaching process. In this sense, the study is thought to contribute to the gap in the literature.

In this research; the concept of PCK is discussed within the scope of components: Content knowledge, knowledge of student understanding, and knowledge of instructional strategies. The aim of this study is to investigate the change of prospective mathematics teachers' pedagogical content knowledge (PCK) on the linear equation and slope subject based on the lesson plan development and implementation process based on the 4MAT model. In line with this aim, the question of the research is, "How does the pedagogical content knowledge of prospective mathematis teachers change in the process of developing and implementing a lesson plan based on the 4MAT model? In line with this problem, answers to the following sub-problems were sought:

- How does the content knowledge of prospective mathematis teachers change in the process of developing and implementing a lesson plan based on the 4MAT model?
- How does the knowledge of student understanding of prospective mathematis teachers change in the process of developing and implementing a lesson plan based on the 4MAT model?
- How does the knowledge of instructional strategies of prospective mathematis teachers change in the process of developing and implementing a lesson plan based on the 4MAT model?

METHOD

Research Model

This research was carried out based on mixed research methods. The mixed research method focuses on collecting, analyzing and collating both quantitative and qualitative data. Its basic premise is the combined use of qualitative and quantitative data, providing a much better understanding of the research problem than any method used alone. (Creswell & Plano Clark, 2007). The quantitative part of the study consisted of the analysis of 48 lesson plans developed by 16 prospective teachers in three stages. In the analysis of the lesson plans, the lesson plan evaluation rubric developed by the researcher was used, and the qualitative data were quantified. In the qualitative part of the research, various qualitative sections from the lesson plans showing the development of prospective teachers were presented. In addition, observation notes were kept for the teachings made by prospective

teachers based on the second lesson plans. An example of the observation notes held by prospective teachers is given in Appendix 1.

Sample

The study group of the research consisted of 16 prospective teachers who were at the third grade level of the Elementary Mathematıcs Education Department. The prospective teachers who participated in the study were determined by using a random sampling method among 30 prospective teachers who are taking Special Teaching Methods-I and II courses. The random sampling method is a method in which the sample has the power to represent the universe, and the probability of sampling units to be sampled in this method is equal and independent (Büyüköztürk et al., 2014). During the separation of the prospective teachers into two groups, the prospective teachers were not told about the 4MAT model and the process.

Until prospective teachers reached third-grade level, they took mathematics-related courses such as General Mathematics, Analysis-I, Analysis-II, and education courses such as Introduction to Educational Science, Instructional Technologies and Material Design, Teaching Principles and Methods. Taking into account the ethics of the research, the real names of prospective teachers who participated in the study were not used, and prospective teachers were given codes from PT1 to PT16.

Implementation

The implementation process was presented in detail in Table 1.

Table 1. Implementation Process

Stage	What is held at this stage
Before the prospective teachers are given any information about the development of lesson plan	Acquisitions for linear equation and slope subjects at the 8th grade level of secondary school mathematics curriculum are "*Creates and interprets the tables, graphics, and equations of real-life situations involving linear relationship*" and "*Explains the slope of the line with models; relates linear equations, graphics and related tables to slope*". First of all, these acquisitions were randomly divided among prospective teachers and they developed a lesson plan for these acqusitions.
Giving information about 4MAT model and PCK to prospective teachers	Prospective teachers were given training based on the 4MAT model. This training includes introducing the 4MAT model and presenting examples from the developed lesson plans based on the 4MAT model. Some of these lesson plans are the translation of the lesson plans on the websites www.aboutlearning.com , www.4mationweb.com to Turkish, while others are lesson plans previously developed for the 4MAT model directly by the researcher. These lesson plans were examined with prospective teachers and discussions were made on the lesson plans. In addition, a presentation was made to prospective teachers about the meaning of the concept of PCK and the meaning of the PCK components' (content knowledge, knowledge of students understanding, and knowledge of instructional strategies) The prospective teachers re-developed the lesson plans they developed after the training given the 4MAT model and the PCK components.
After the prospective teachers has been informed about 4MAT model and PCK	Each prospective teacher made teachings based on these lesson plans, and after these teachings, discussions were held on the teachings. In addition, observation notes were kept by the researcher and other prospective teachers during the teaching. At the end of the teaching, the prospective teachers who taught the subject were evaluated by the prospective teachers themselves, the other prospective teachers, and the researcher. In this evaluation process, self/peer/ expert evaluation forms developed by the researcher were used. These assessments addressed both the steps of the 4MAT model and the PCK components. The prospective teachers reorganized the lesson plans they prepared considering these evaluations and discussions. Thus, each prospective teacher developed a total of 3 lesson plans, while the first lesson plan not adhering to any model and the second and third lesson plans are based on the 4MAT model.

Data Analysis

The lesson plans developed by prospective teachers were analyzed according to the lesson plan evaluation rubric developed by the first researcher. This rubric consists of two parts. The first part is to the compliance of the lesson plans with the 4MAT model; and the second part is to examine the compatibility with PCK components. While developing items in the rubric, the steps of the 4MAT model, and the definitions for the components included in the PCK were taken into consideration.

After the rubric was developed, the opinions of three mathematics educators and one assessment and evaluation specialist were taken. In addition, the rubric has been piloted. During the pilot implementation phase, the lesson plans that the prospective teachers developed based on the 4MAT model for different acquisitions in the previous periods were analyzed through the rubric. Some changes were made in the rubric according to the experts' opinions and pilot application results. For example, while the first version of the rubric included items between 1-5, it was decided that the evaluation would include 1-3 coding in order to make a more reliable analysis, and some items in the rubric were removed. In addition, the fact that the expected behavior was not reflected in the lesson plan was evaluated as "*not observed*" (0) by examining the studies in the field (Gökkurt, 2014). Some items belonging to the rubric were given in the findings section and the entire rubric was given in Appendix 2. The rubric consisted of 15 items belonging to the 4MAT model; 6 items belonging to the content knowledge component, 5 items belonging to the knowledge of student understanding component, and 9 items belonging to the knowledge of instructional strategies component.

For the reliability of the research, the analysis of 6 lesson plans which were randomly selected from the data set and consisting of 3 stages of 2 prospective teachers was made separately. While one of the analysts is the first researcher, the other is a mathematics teacher who has experience in developing a lesson plan based on the 4MAT model and is related to the master's thesis 4MAT model. Inter-rater compliance rates were calculated using the inter-rater reliability coefficient formula, % of consensus = consensus / (consensus + divergence) (Miles & Huberman, 1994), and percentages were 0.84, 0.78, 0.80, and 0.80 for 4MAT model, content knowledge, knowledge of student understanding, and knowledge of instructional strategies, respectively. A reliability calculation above 70% is considered reliable for researches (Miles & Huberman, 1994). Based on this, it can be said that the analysis made were reliable. Regarding the non-compliance items, two evaluators came together and the reason for the difference in the evaluation was discussed. After this stage, the researcher continued the lesson plan analysis individually according to the rubric she developed.

An example based on the evaluation of the lesson plans of PT1 (prospective teacher-1) regarding the item 7 of the instructional strategies knowledge components, "*Being able to use different teaching strategies/methods and techniques suitable for the teaching of the subject*" was presented in Table 2.

Table 2. An Example of How to Evaluate the Lesson Plans for the Instructional Strategies Knowledge Component

Lesson plan	Scoring	Description for scoring
First lesson plan	1 (insufficient)	A completely teacher-centered education was carried out.
Second lesson plan	2 (partly sufficient)	The teacher is in the center, but sometimes teaching techniques such as question-answer, discussion, and analogy were used.
Third lesson plan	3 (sufficient)	Teaching techniques such as question-answer, discussion, and analogy were used as in the second lesson plan. In addition, the student was able to access the information himself/herself in many parts of the lesson plan.

Whether there is a statistically significant difference in PCK components in the lesson plans developed by prospective teachers was analyzed by using appropriate tests (Friedman test, Wilcoxon test, one-way ANOVA for repeated measurements test). One-way ANOVA for repeated measurements test is used to test whether there is a statistically significant difference between the mean of the data

obtained as a result of successive measurements from the same data source. The non-parametric form of the one-way Anova for repeated measurements test is the Friedman test (Can, 2017). The Friedman test gives whether there is a significant difference between the measurements, but does not give between which measurements this difference is. It is recommended to use the Wilcoxon test (non-parametric test in which two measurement results are compared for related measurements) to reveal between which measurements there are significant differences (Can, 2017). The tests used and why these tests are used were explained in Table 3.

Table 3. The tests used to examine the changes in the pedagogical content knowledge in the lesson plans of prospective teachers

Component	The tests used	Why this test was used
Content knowledge	Friedman Test and then Wilcoxon test	According to the normality results obtained by making different examinations, it was concluded that the first lesson plan content knowledge scores did not show normal distribution, while the second and third lesson plan content knowledge scores showed normal distribution. Since two of the measurements showed a normal distribution and one did not, whether there was a statistically significant difference between the three measurements was analyzed using the Friedman Test. And then, Wilcoxon test was used to reveal between which measurements were significant differences.
Knowledge of student understanding	One-way ANOVA for repeated measurements test	According to the normality results obtained by making different examinations, it was concluded that the comprehension knowledge scores of all three lesson plan students showed a normal distribution. Since all three of the measurements showed normal distribution, whether there was a statistically significant difference between the three measurements was analyzed using the one-way Anova test for repeated measurements.
Knowledge of instructional strategies	Friedman Test and then Wilcoxon test	According to the normality results obtained by different examinations, it was concluded that the first and third lesson plan knowledge of instructional strategies scores were normally distributed, but the second lesson plan instructional strategies knowledge scores did not show normal distribution. Since two of the measurements showed a normal distribution and one did not, whether there was a statistically significant difference between the three measurements was analyzed using the Friedman Test. And then, Wilcoxon test was used to reveal between which measurements were significant differences.

After analyzing quantitatively whether the prospective teacher showed a statistically significant improvement in the PCK components, various qualitative sections from the lesson plans showing the development of the prospective teachers were presented. These sections were presented within the framework of the items in the lesson plan evaluation rubric for each pedagogical content knowledge component. Thus, quantitative results were supported by qualitative sections.

RESULTS

Findings for the First Sub-problem

The first sub-problem of the research was consisted of "How does the content knowledge of prospective mathematis teachers change in the process of developing and implementing a lesson plan based on the 4MAT model?. First of all, quantitative analyzes were presented to seek an answer to this sub-problem. The mean scores of prospective teachers in the content knowledge component were given in Figure 2.

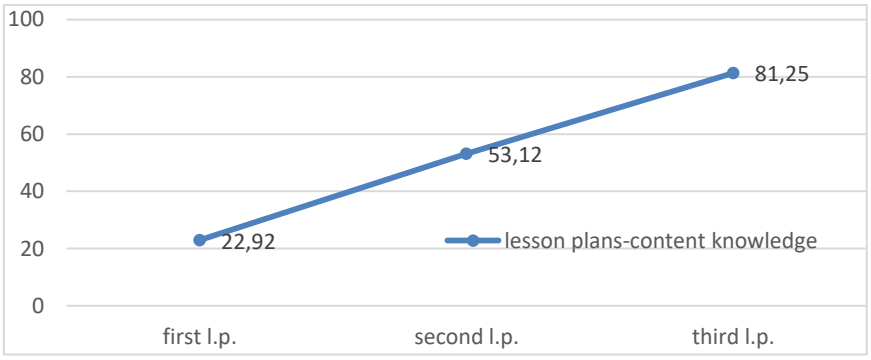


Figure 2. The Mean Scores of Content Knowledge Obtained From the Lesson Plans

The Friedman test results were presented in Table 4.

Table 4. "Friedman Test" Results Regarding the Lesson Plans-Content Knowledge Mean Scores

Lesson plans	N	Mean	Sd	Mean rank	χ^2	df	p
First lesson plan	16	22,92	9,70	1,00	31,524	2	,000
Second lesson plan	16	53,12	9,73	2,03			
Third lesson plan	16	81,25	10,90	2,97			

When Table 4 was examined according to the results of the Friedman test, it was seen that the prospective teachers' content knowledge differed significantly $[\chi^2_{(2,N=16)} = 31,524, p < 0,05]$. The Wilcoxon test results were presented in Table 5.

Table 5. "Wilcoxon Test" Results Regarding the Lesson Plans-Content Knowledge Mean Scores

Lesson plans	Ranks	N	Mean rank	Sum of ranks	z	p
First l.p-Second l.p.	Negative rank	0	,00	,00	-3,523	,000
	Positive rank	16	8,50	136,00		
	Equal	0				
Second l.p-Third l.p.	Negative rank	0	,00	,00	-3,418	,001
	Positive rank	15	8,00	120,00		
	Equal	1				
First l.p-Third l.p.	Negative rank	0	0,00	,00	-3,529	,000
	Positive rank	16	8,50	136,00		
	Equal	0				

As seen in Table 5, a statistically significant difference was observed between the content knowledge scores obtained from all lesson plans. As a result, it can be said that the implementation process based on the 4MAT model contributed to the development of the content knowledge of the prospective teachers at each stage of the lesson plan development. In order to better see the progress of prospective teachers in the content knowledge component, various examples from the content knowledge developments in the lesson plans were presented.

The presented example is based on the examination of PT5's lesson plans in terms of "*Giving sufficient and appropriate examples to create mathematical content*". The examples that PT5 included in the lesson plans respectively were shown in Table 6.

Table 6. Examples of PT5 in the Lesson Plans

Lesson plan	Examples
First lesson plan	$y = 2x$
Second lesson plan	$y = -x + 2$, $y = 2x - 1$, $y = 4x - 16$, $y = 3x$, $y = 2x + 4$
Third lesson plan	$y = x$, $y = -x$, $y = \frac{x}{2}$, $y = x + 2$, $y = -2x - 6$, $y = \frac{3}{2}x + 1$, $3x - 2y + 2 = 0$

When examining the given examples, it was seen that the first lesson plan of PT5 included only one example that will represent "$y = 2x$" equation. The expression in the lesson plan for this example is as in Figure 3.

If we write an example on the blackboard;

x	1	2	3	4	5	6
y	2	4	6	?	?	?

According to the data in the table;

- Fill in the sections "?"
- Create an equation using x and y.
- Draw the graph of the equation in the coordinate system.

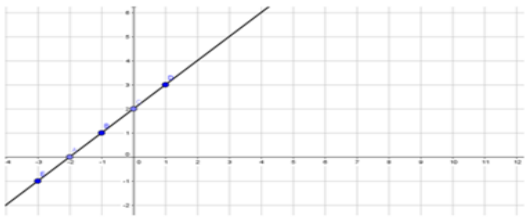
Figure 3. A Section from the First Lesson Plan of PT5

In the second lesson plan, it can be said that although there were different examples about the lines passing through and not passing through the origin, PT5 did not give any examples for the lines in type "$ax + by + c = 0$". In addition, it was seen that PT5 did not include negative slope line graphs passing through the origin, and examples of lines that do not pass through the origin were given before then the lines passing through the origin. In the third lesson plan, PT5 first included the line graphs passing through the origin and then the line graphs that did not pass through the origin. In addition, PT5 gave appropriate examples of different line equations with positive/negative slope, passing/not passing through the origin and which can be expressed in different ways. From this point of view, it was concluded that the examples included in the third lesson plan of PT5 were sufficient and appropriate. A section from the third lesson plan was presented in Figure 4.

Let's plot the line indicated by the equation y = x + 2.

x	-2	-1	0	1
y	0	1	2	3
Ordered pair	(-2,0)	(-1,1)	(0,2)	(1,3)

Let's mark the points we have determined in the coordinate system and draw the graph.



Let's draw our graph in a different way. Let's find the points where the line crosses the x and y axes.

For x = 0, y = 0 + 2, y = 2. Point (0,2) is the point where the graph crosses the y-axis.

For y = 0, 0 = x + 2, x = -2. Point (-2.0) is the point where the graph crosses the x axis.

Let's mark these points in the coordinate system. Let's draw a graph of the line passing through these points.

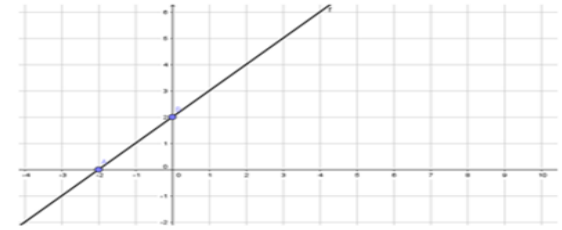


Figure 4. A Section from the Third Lesson Plan of PT5

From the point of view of the content knowledge component, in the process of using the 4MAT model, prospective teachers noticed the mistakes and deficiencies in the content knowledge while organizing the content information in the fourth step. It was observed that they overcomed these shortcomings during the next lesson plan development phase. This situation led to an improvement in the items related to the content knowledge in the lesson plan evaluations. For example, when it was evaluated in terms of "*Making the necessary mathematical explanations appropriately*" and "*Giving the logical grounds underlying the concepts*", it was concluded that the prospective teachers had deficiencies in the first lesson plans. In the last lesson plans, the prospective teachers showed an improvement, for example, they explained the situations where the slope was zero and undefined in detail by using the change rate meaning of the slope.

Findings for the Second Sub-problem

The second sub-problem of the research was consisted of "How does the knowledge of student understanding of prospective mathematis teachers change in the process of developing and implementing a lesson plan based on the 4MAT model?" The mean scores of prospective teachers in the knowledge of student understanding component were given in Figure 5.

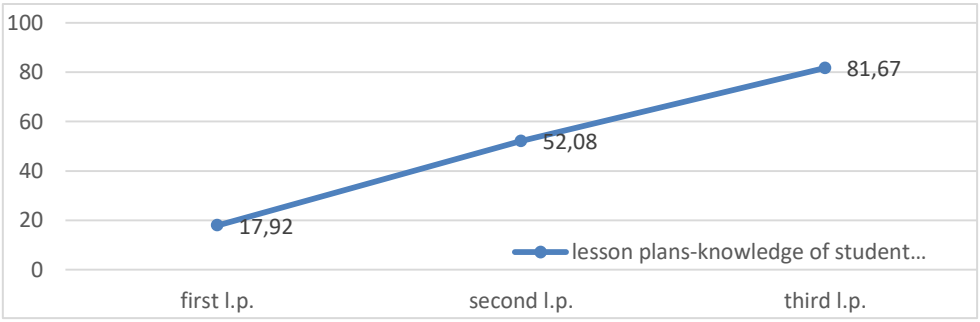


Figure 5. The Mean Scores of Knowledge of Student Understanding Obtained From the Lesson Plans

The One-way Anova for repeated measurements test results were presented in Table 7.

Table 7. "One-Way Anova for Repeated Measurements Test Results" Regarding the Lesson Plans-Knowledge of Student Understanding Mean Scores

Source of variance	Sum of squares	df	Average of squares	F	p
Between subjects	4074,504	15	271,634		
Measurement	32569,120	2	16284,560	168,601	,000
Error	2897,591	30	96,586		
Total	39541,22	47			

As seen in Table 7, a statistically significant difference was found between the knowledge of student understanding mean scores obtained from all lesson plans[$F_{(2,30)} = 168,601, p <, 05$]. The results

of the analysis based on determining which measurements differ significantly were presented in Table 8.

Table 7. Comparison of the Differences in the Lesson Plans

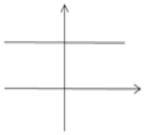
Compared lesson plans	Average difference	Standart error	p
First l.p-Second l.p.	-34,166	3,985	,000
Second l.p-Third l.p.	-29,584	2,787	,000
First l.p.-Second l.p.	-63,751	3,546	,000

When Table 8 was examined, it was seen that there is a statistically significant difference between the averages of all measurements $(p < 0,05)$. Based on these results, it can be said that the application process based on the 4MAT model contributed to the development of knowledge of student understanding of the prospective teachers in each lesson plan development stage.

In order to better see the progress of the prospective teachers in the knowledge of student understanding component, various examples from the knowledge of student understanding developments in the lesson plans were presented. The first example presented is for examining the lesson plans of PT13 based on the criteria of "*Selecting examples that students will find interesting and motivating*". When the lesson plans of PT13 were examined, it was seen that the first lesson plan included slope calculation only through the algebraic and graphic representation of the equations, and did not include any examples that could attract the students' interest. A section from the first lesson plan of PT13 was presented in Figure 6.

Lines with a slope of 0:

The slope of the lines parallel to the x axis is 0.



Let's assume that the line is the line y = 3.

Since y = 3+0.x, the slope is 0.

Looking at the graph, there is no change in y values. So I teach that the slope of these lines is 0.

Lines with slope not calculated (without slope):

The slope of the lines parallel to the y axis cannot be calculated, so there is no slope.

Looking at the graph, it is seen that y values change but x values do not. So I teach that the slope of these lines cannot be calculated.

Figure 6. A Section from the First Lesson Plan of PT13

Various sections from the examples that PT13 includes in the second and third lesson plans were presented in Figure 7.

"What can you say about the slope of the figures below?"

"Betül walks for 10 minutes at 1 mile per hour. She stops for 3 minutes and watches the nest with baby birds. Then he walks for another 5 minutes at a speed of 2 miles per hour. What will the graph look like during the 3 minutes that Betül stopped?"

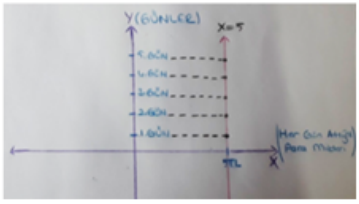
(Second lesson plan)

Ayşe has a favorite flower and this flower needs to be watered every day. Ayşe gives her flower 1 glass of water a day and her flower consumes all the water she gives in a day. According to this;

- Plot the graph representing the relationship between the number of days (x) and the amount of water (y) given to the flower each day.
- Find the equation that expresses this relationship.
- Find the slope of this line.

Ece throws 5 TL into her piggy bank every day. What is the graph showing the relationship between the amount of money (x) and day (y) that Ece puts in his piggy bank every day?

The amount of money she throws in her piggy bank every day (x)	Days (y)
5 TL	First day
5 TL	Second day
5 TL	Third day
5 TL	Fourth day
5 TL	Fifth day



(Third lesson plan)

Figure 7. Sections from the Second and Third Lesson Plans of PT13

When examining these lesson plans, it was concluded that the examples given by PT13 included daily life situations and are more suitable for attracting students' attention.

Another item that is addressed in the knowledge of student understanding component is "*Being aware of students' mistakes and misconceptions about the mathematical content taught and including it in the lesson plans*". An example based on the development of prospective teachers in this item was given from the lesson plans of PT14. When the first lesson plan of PT14 was examined, no statement was found to show that PT14 was aware of students' mistakes and misconceptions. In the second lesson plan, PT14 included the activity in Figure 8.

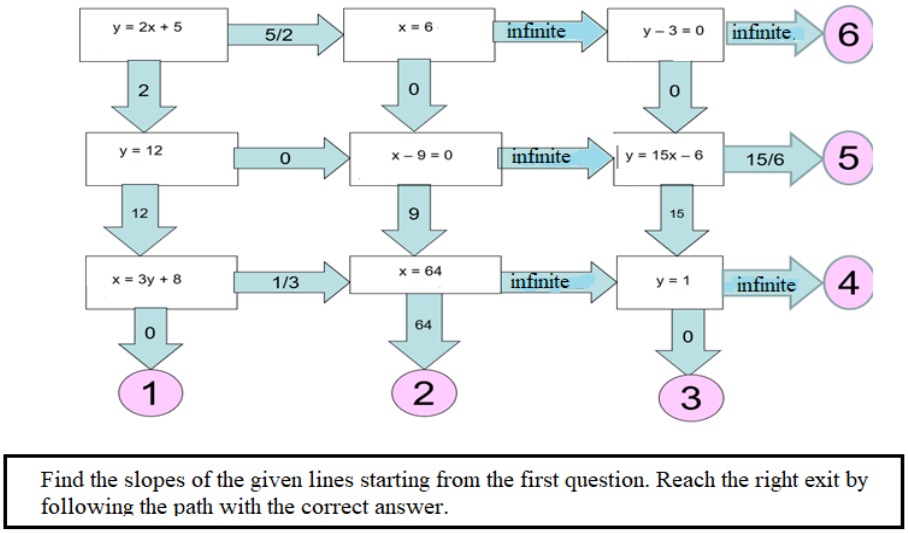


Figure 8. A Section from the Second Lesson Plan of PT14

When the second lesson plan of PT14 was analyzed, it was concluded that PT14 had an idea partially about the students' mistakes/misconceptions and tried to design an activity based on this. However, PT14 also had learning deficiencies such as "*Expressing the undefined state of the slope as being infinite*". In the third lesson plan, PT14 included concept caricatures on the subject. One of the concept caricatures was presented in Figure 9.

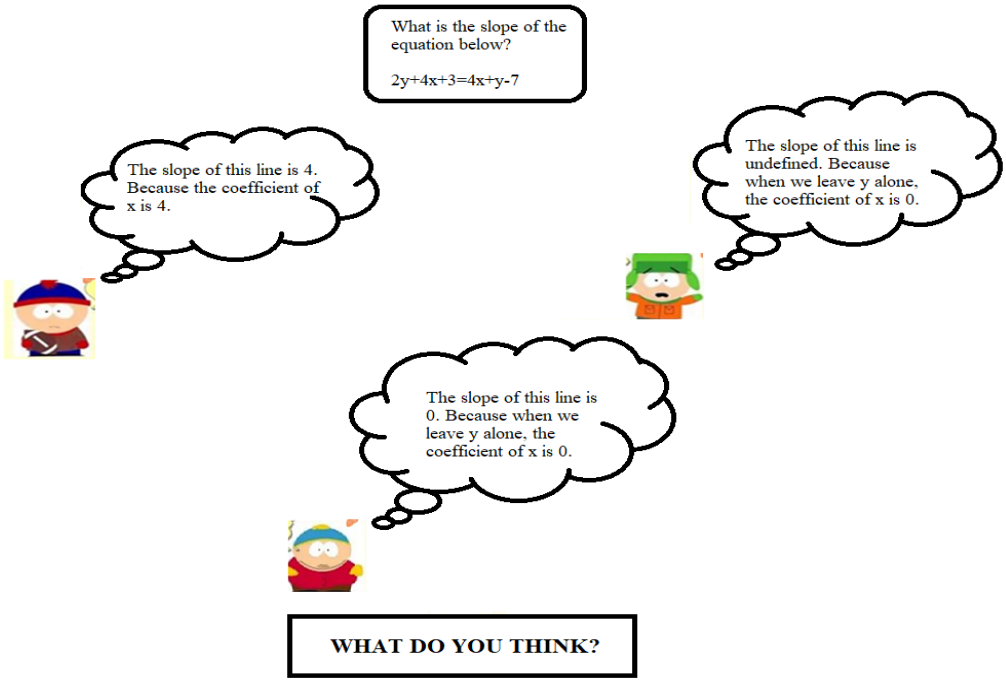


Figure 9. A Section from the Third Lesson Plan of PT14

When examining the concept caricature included in the third lesson plan of PT14, it was seen that PT14 mentioned the mistakes that the students actually made. These are mistakes such as students thinking of the slope as the coefficient of x in any case without going to edit the given equation, and mixing the situations where the slope is undefined and 0. In addition, while PT14 expressed the undefined state of the slope as infinity in the second lesson plan, she corrected her own wrong learning about the subject in the third lesson plan. When the lesson plans were analyzed as a whole, it was

concluded that PT14 showed improvement regarding the awareness of the mistakes/misconceptions in the students.

In terms of the knowledge of student understanding component, it was seen that the prospective teachers did not mention student mistakes in their first lesson plans. Considering the second and third lesson plans, it was concluded that the concept caricatures, crossword activities, and discussion environments based on these activities contributed to addressing students' different thoughts and interpreting students' thoughts.

Findings for the Third Sub-problem

The third sub-problem of the research was consisted of "How does the knowledge of instructional strategies of prospective mathematis teachers change in the process of developing and implementing a lesson plan based on the 4MAT model?" The mean scores of prospective teachers in the knowledge of instructional strategies component were given in Figure 10.

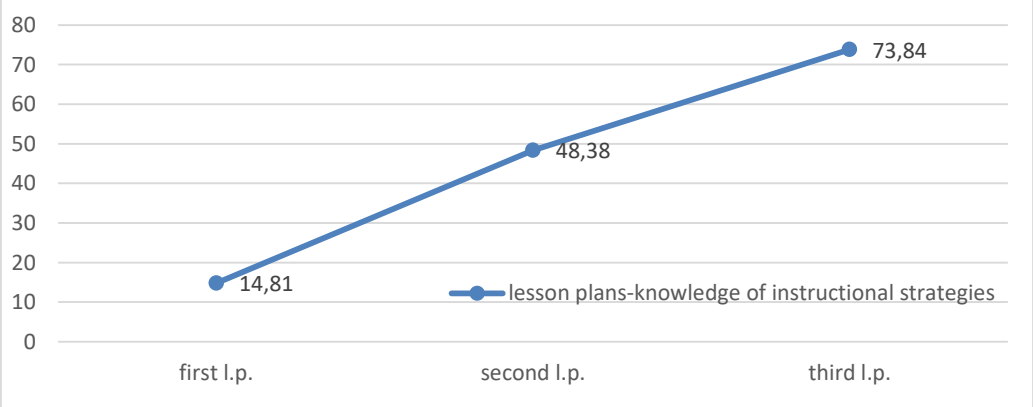


Figure 10. The Mean Scores of Knowledge of Instructional Strategies Obtained From the Lesson Plans

The Friedman test results were presented in Table 9.

Table 9. "Friedman Test" Results Regarding the Lesson Plans-Knowledge of Instructional Strategies Scores

Lesson plans	N	Mean	Sd	Mean Rank	χ^2	df	p
First lesson plan	16	14,81	5,89	1,00	32,00	2	,000
Second lesson plan	16	48,38	11,75	2,00			
Third lesson plan	16	73,84	14,09	3,00			

When Table 9 was examined according to the results of Friedman test, it was seen that the prospective teachers' knowledge of instructional strategies differed significantly $[\chi^2_{(2,N=16)} = 32,00, p < 0,05]$. The Wilcoxon test results were presented in Table 10.

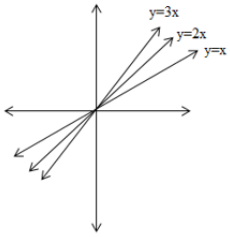
Table 10. "Wilcoxon Test" Results Regarding the Lesson Plans-Knowledge of Instructional Strategies Mean Scores

Lesson plans	Ranks	N	Mean rank	Sum of ranks	z	p
First l.p-Second l.p.	Negative rank	0	,00	,00	-3,540	,000
	Positive rank	16	8,50	136,00		
	Equal	0				
Second l.p-Third l.p.	Negative rank	0	,00	,00	-3,529	,000
	Positive rank	15	8,50	136,00		
	Equal	1				
First l.p-Third l.p.	Negative rank	0	0,00	,00	-3,519	,000
	Positive rank	16	8,50	136,00		
	Equal	0				

As seen in Table 10, a statistically significant difference was observed between the knowledge of instructional strategies mean scores obtained from all lesson plans ($p < 0,05$). As a result, it can be said that the application process based on the 4MAT model contributed to the development of the knowledge of instructional strategies of the prospective teachers at each stage of the lesson plan development.

In order to better see the progress of prospective teachers in the knowledge of instructional strategies component, various examples from the instructional strategies developments in the lesson plans were presented. The presented example is based on the examination of PT9's lesson plans in terms of "*Using different teaching methods and techniques appropriate for the teaching of the subject*", "*Engaging activitely students in the lesson" and "Designing learning environments for the students to solve learning difficulties, mistakes, and misconceptions and to teach concepts effectively*". In Figure 11, a section from the first lesson plan of PT9 was given.

I want them to graph the lines y = x, y = 2x, y = 3x. I ask how there are changes in these lines. Here, the student is provided to show that the change in the coefficient of x is the slope.



After these comments, it is stated that the ratio of the change in y to the change in x is the slope.

After interpreting from the table, it is stated that the slope is the ratio of the change in y to the change in x.

Figure 11. A Section from the First Lesson Plan of PT9

When examining the first lesson plan of PT9, it was seen that a lesson in which the teacher was more active than the students was planned and the information was generally presented by the teacher. From time to time, students were tried to access some information themselves (Like the expression "*Here the student is provided to show that the change in the coefficient of x is the slope*"). In addition, different teaching methods/techniques were not included in the first lesson plan too much. In the second lesson plan, class discussions and group studies were conducted based on the 4MAT model, and students were asked to question the slope concept through the activity presented in Figure 12 before informing by the prospective teacher about the slope concept.

Activity2: Each group is asked to examine the situations below and fill in the table. They are said to note the answers to the questions and then the answers will be discussed.

Imagine you have a piggy bank. There is no money in your piggy bank.
1 a. You throw 1 TL into the piggy bank every day.
1 b. You throw 2 TL into the piggy bank every day.

According to these cases, fill in the table containing the relationship between the number of days (x) and the amount of money in the piggy bank (y). How does y values change as x values increase? Consider what the increase or decrease in y values depend on.

For the 1a;

The amount of money in the piggy bank (y)						
Days past (x)	0	1	2	3	4	5

For the 1b;

The amount of money in the piggy bank (y)						
Days past (x)	0	1	2	3	4	5

They are asked to show the values in the table on the graph and combine the dots to create lines and write the equations of these lines. Then they are asked to examine the lines. They are expected to make an inference.

- What are the differences in the graphics of the lines? What would be the reason?
- What can be said about the steepness of the graphics?
- How do the values of x and y variables change in lines? Then consider the relationship between the two (x, y) ordered pairs you choose. Try to relate it to the differences in the graphics.

Figure 12. A Section of the Second Lesson Plan of PT9

As seen from the activity, in the second lesson plan, PT9 designed a teaching process to make students think about the concept of slope. She did not give the definition of the slope directly and provided the students to reach the results themselves. From this point of view, it can be said that a better teaching process was designed to teach concepts effectively. However, at this stage, there were also cases where PT9 presented the information directly in some cases. The statement given reflects this situation: "*For example* $y = x$, $y = 2x$. *They were always leaning to the right. But in* $y = -3x - 5$, *the situation is different. We see that the x values increase while the y values decrease. Then we can say that if we are generalizing, the slope in such lines is equal to the coefficient of x. At the same time, we can say that the slope of the lines leaning to the left will be negative*". When examining this expression, it was seen that a generalization was made by PT9 as "*The slope of the lines leaning to the left is negative*". A section from the third lesson plan of PT9 was presented in Figure 13.

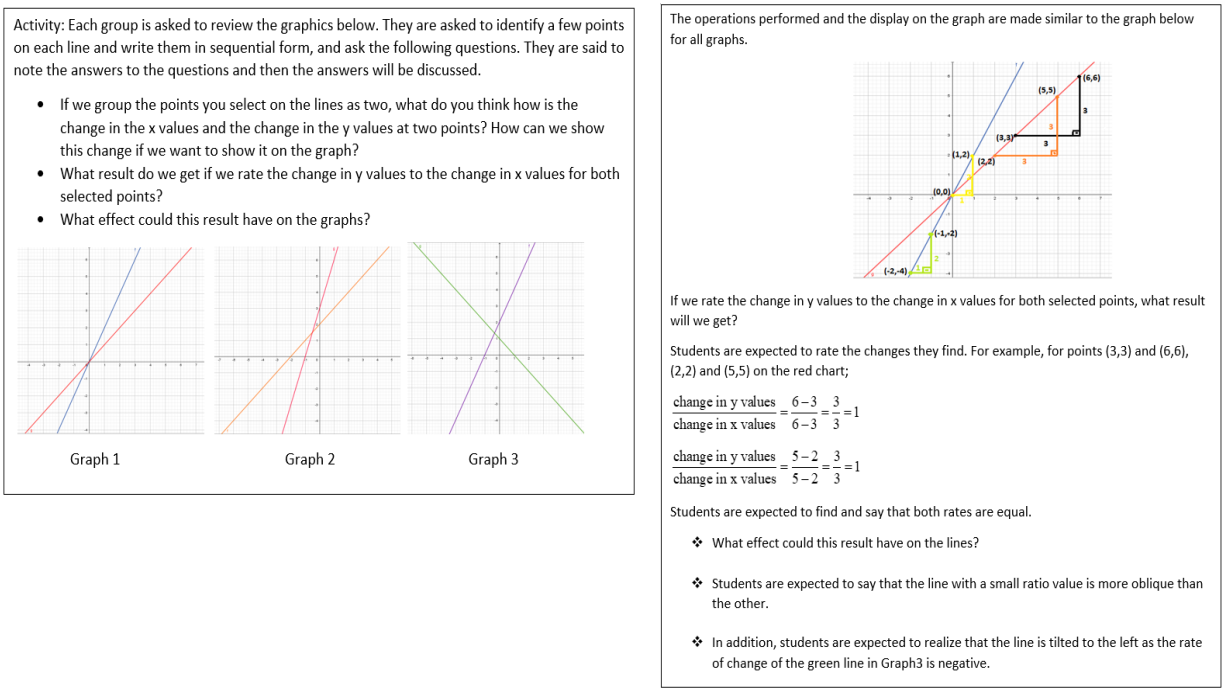


Figure 13. A Section of the Third Lesson Plan of PT9

When the activity in Figure 13 was analyzed, it was seen that PT9 wanted students to question the slopes of all lines that pass/do not pass through the origin and have a negative/positive slope. With this activity, it was aimed that the students see that the slope is constant even if different points are taken on the line and in which cases the slope of the line is negative. When the lesson plan was analyzed as a whole, it was concluded that PT9 developed in terms of active participation of the students in the lesson, providing the student to reach the information herself/himself, and designing a more effective process for teaching the concepts.

In terms of the knowledge of instructional strategies component, in the first lesson plans, it was seen that prospective teachers generally designed the lesson as teacher-centered, used different teaching strategies/methods/techniques very little or not at all, and did not consider the student's active participation in the lesson. In the second lesson plans, based on the 4MAT model, question-answer, discussion, etc. methods were used. Based on the discussions they have made over the concept caricatures they have prepared, they have tried to design teaching aimed at eliminating students' mistakes/misconceptions. In the third lesson plans, generally, a transition was made from the teacher-centered teaching process to the student-centered teaching process. Emphasis was placed on the active participation of students in the lesson, and students were provided with access to information through various questions, activities, and directions. In this development, it is considered that the process of developing a lesson plan based on the 4MAT model, conducting, and evaluating these trainings with self/peer and expert evaluations is effective.

CONCLUSION AND DISCUSSION

As a result of the research, it was seen that the prospective teachers' content knowledge showed improvement during the lesson plan development and implementation process based on the 4MAT model. This result is similar to some studies in the literature. As a result of lesson study applications in Wright (2009)'s work and as a result of professional training given in the studies of Nielsen (2009) and Naseer (2016); it was concluded that the content knowledge of the teachers improved. After the training given in the study of Nielsen (2009), it was stated that the lecture processes of the teachers were recorded by video and that the content knowledge improved as a result of the feedback given in this process. In the study of Naseer (2016), it was stated that teachers' content knowledge was improved in terms of making algebraic explanations, solving problems, noticing

wrong expressions in textbooks. In this study, similar to the studies mentioned, it was concluded that prospective teachers' content knowledge improved as a result of the teachings and discussions on these teachings. Another result related to the content knowledge component is the development of the prospective teachers' definition skills. This result is similar to the findings of Seviş (2008). Similarly, in this research, it has been concluded that prospective teachers' ability to make definitions improved after the implementations made within the scope of the mathematics teaching methods course.

In the process of developing and implementing a lesson plan based on the 4MAT model, prospective teachers also improved in the knowledge of student understanding component. Based on professional training provided in the study of Naseer (2016), lesson study practices in the study of Baki (2012), and action research in the studies of Eroğlu (2016) and Tataroğlu Taşdan and Çelik (2017); it was concluded that teachers and prospective teachers improved in terms of determining in which points students may have learning difficulties and misconceptions. On the other hand, Yeşildere İmre and Akkoç (2012) stated that, as a result of micro-teaching practices, prospective teachers improved in terms of considering students' understandings and difficulties about patterns. Although the methods used are different, the results obtained from this research are similar to the results obtained from the mentioned studies. Similarly, in these studies, it was concluded that prospective teachers showed improvement in the knowledge of student understanding component. It is thought that the teachings and discussions on these teachings were effective in the development of the prospective teachers' knowledge of student understanding component. Another result obtained in terms of knowledge of student understanding component is the development of prospective teachers' giving examples that will appeal to students. It can be said that this development is compatible with the study of Shuilleabhain (2016), which shows that prospective teachers' interesting examples giving skills have improved based on the use of the lesson study model.

In the process of developing and implementing a lesson plan based on the 4MAT model, prospective teachers showed improvement in many aspects of the knowledge of instructional strategies component. When the lesson plans were analyzed, it was seen that prospective teachers switched from teacher-centered to student-centered teaching approach and gained the ability to keep the student active during the lesson. Similarly, Tataroğlu Taşdan and Çelik (2017) stated as a result of the action research, Baki (2012) and Shuilleabhain (2016) stated as a result of the lesson study practices that teachers/prospective teachers showed improvement in the specified directions. Another result is that prospective teachers showed improvement in the use of multiple representations. Similarly, based on the action research conducted by Eroğlu (2016) and Tataroğlu Taşdan and Çelik (2017); and as a result of the practices carried out based on the lesson study model in Shuilleabhain (2016)'s study, it was reached the conclusion that teachers developed in terms of using different forms of representations.

On the other hand, prospective teachers' development in terms of the transition from rule-based and memorized explanations to conceptual explanations is similar to the findings of Bütün (2012), and the development in terms of instructional explanation qualities is similar to Baki (2012). Besides, the development of prospective teachers' skills to include class discussions is similar to the findings of Eroğlu (2016) and Shuilleabhain (2016). In addition, prospective teachers also improved in terms of using different teaching strategies. Similarly, Naseer (2016), as a result of professional training and Yeşildere İmre and Akkoç (2012), as a result of micro-teaching practices, reached the conclusion that prospective teachers showed improvement in terms of suggesting teaching strategies aimed at eliminating students' mistakes and using subject-specific strategies.

In the research of Ferna'ndez (2010), which uses lesson plans as a data collection tool, the micro-teaching lesson study model was used. Similar to the results of this research, when the lesson plans developed by prospective teachers at the first and last stages were examined, it was concluded that the knowledge of teaching mathematics has improved. When the mentioned studies are examined, although the methods used in the process are different methods such as action research, lesson study, micro-teaching, the findings are similar to the findings obtained from the research in terms of improving the PCK of prospective teachers. In this research, prospective teachers developed lesson

plans; as a result of the teachings and discussions based on the lesson plans, it was concluded that the prospective teachers' PCK improved.

When the literature was examined it was observed that the opinions of both the students and the teachers who used this model in their lessons based on the 4MAT model were positive and these results can also be associated with the pedagogical content knowledge. For instance, in the study of Özgen and Alkan (2012), the derivative subject was taught based on the 4MAT model, and the students stated that this way of learning is beneficial in terms of teaching the subject in a student-oriented lesson, working with groups, practicing, associating, etc. Similarly, Özdoğan (2012) stated that the 4MAT model is useful in many aspects such as frequency of material use, the student's production of information himself/herself. In addition, Kaewkiriya (2017) specified that teaching based on the 4MAT model facilitates effective and attractive learning. On the other hand, Ramirez and Laurinco (2015) stated that while maintaining the learning style of each student in their learning process according to the 4MAT model, teachers should systematically expose all students to multiple learning techniques. When the mentioned research results are examined, it is seen that the positive effects of the 4MAT model are also related to the pedagogical content knowledge components. In the light of these results and based on the positive results obtained from the study, it is recommended to use the 4MAT model in lesson plan designs and studies to be conducted for the development of prospective teachers' PCK.

REFERENCES

Aktas, İ., & Bilgin, İ. (2015). The effect of the 4MAT learning model on the achievement and motivation of 7th grade students on the subject of particulate nature of matter and an examination of student opinions on the model. *Research in Science & Technological Education, 33*(1), 1-21.

Alanazi, F. H. (2020). The effectiveness of the 4MAT teaching approach in enhancing conceptions of electricity in physics for female students in the kingdom of Saudi Arabia. *Journal of Turkish Science Education, 17*(2), 271-288.

Aydıntan, S., Şahin, H., & Uysal, F. (2020). The effect of 4MAT learning style model on academic achievement and permanence while teaching "Fractions". *Mehmet Akif Ersoy University Journal of Education Faculty, 23*(1), 408-427.

Baki, M. (2012). *Sınıf öğretmeni adaylarının matematiği öğretme bilgilerinin gelişiminin incelenmesi: bir ders imecesi (lesson study) çalışması* [Investigating development of prospective primary teachers' mathematical pedagogical content knowledge: lesson study] [Unpublished doctoral dissertation]. Karadeniz Technical University.

Baki, M., & Arslan, S. (2015). Ders imecesinin (lesson study) sınıf öğretmeni adaylarının matematik dersini planlama bilgilerine etkisinin incelenmesi [Examining the effect of lesson study on prospective primary teachers' knowledge of lesson planning]. *Turkish Journal of Computer and Mathematics Education, 6*(2), 209-229.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407.

Burkum, K. (2010). *Retention: Diverse institutions= diverse retention practices*. Chicago, IL: ACT.

Bütün, M. (2012). *İlköğretim matematik öğretmeni adaylarının uygulanan zenginleştirilmiş program sürecinde matematiği öğretme bilgilerinin gelişimi* [The development of pedagogical content knowledge of preservice mathematics teachers in the process of applied enriched program] [Unpublished doctoral dissertation]. Karadeniz Technical University.

Büyüköztürk, Ş., Çakmak, E. K., Akgün, Ö. E., Karadeniz, Ş., & Demirel, F. (2014). *Bilimsel araştırma yöntemleri [Scientific research methods].* Ankara: Pegem Academy.

Can, A. (2017). *SPSS ile bilimsel araştırma sürecinde nicel veri analizi [Quantitative data analysis in the scientific research process with SPSS]*. Ankara: Pegem Academy.

Creswell, J. W., Plano Clark, V. L. (2007). *Designing and conducting mixed methods research*. Thousand Oaks, CA: Sage.

Ergin, S. (2011). *Fizik eğitiminde 4MAT öğretim yönteminin farklı öğrenme stillerine sahip lise öğrencilerinin iş, güç ve enerji konusundaki başarısına etkisi* [Effects of the 4MAT instruction method on the achievement of high school students with different learning styles on the subjects of work, power and energy in physics education] [Unpublished doctoral dissertation]. Gazi University.

Eroğlu, D. (2016). *Ortaokul matematik öğretmenlerinin tahmini öğrenme yollarına dayalı öğretimlerindeki pedagojik yollarının desteklenmesi* [Supporting middle school mathematics teachers' pedagogical ways in their teachings based on hypothetical learning trajectories] [Unpublished doctoral dissertation]. *Anadolu University.*

Ferna´ndez, M. L. (2010). Investigating how and what prospective teachers learn through microteaching lesson study. *Teaching and Teacher Education, 26*(2), 351– 362.

Gökkurt, B. (2014). *Ortaokul matematik öğretmenlerinin geometrik cisimler konusuna ilişkin pedagojik alan bilgilerinin incelenmesi* [An examination of secondary school mathematics teachers' pedagogical content knowledge on geometric shapes] [Unpublished doctoral dissertation]. Atatürk University.

İnel, Y. (2018). The effect of 4MAT method on the academic achievement of students in social studies education. *Review of International Geographical Education Online (RIGEO), 8*(3), 440-458.

Kaewkiriya, T. (2017). Design of e-learning content for student guidance based on ILS and 4MAT theory. in Communication Systems, *Computing and IT Applications (CSCITA), 2nd International Conference on*. 2017. IEEE.

Kelley, L. S. (1990). Using 4MAT to improve staff development, curriculum assesment and planning. *Educational Leadership, 48*(2), 38-39.

Lim, W., Son, J. W. and Kim, D. J. (2018). Understanding preservice teacher skills to construct lesson plans. *International Journal of Science and Mathematics Education, 16*(3), 519-538.

McCarthy, B. (1990). Using the 4MAT System to bring learning styles to schools. *Educational Leadership, 48*(2), 31–37.

McCarthy, B., Germain, C., & Lippitt, L. (2002). *The 4MAT research guide*. Wauconda, IL: About Learning Inc.

McCarthy, B. (2014). *4MAT pure and simple: Learning about learning*. Wauconda, IL: About Learning Inc.

Miles, M, B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. Thousand Oaks, CA: Sage.

Morris, S., & B. McCarthy. (1999). *4MAT in action*. Barrington, IL: Excel, Inc.

Naseer, M. S. (2016). *Algebraic content and pedagogical knowledge of sixth grade mathematics teachers* [Unpublished doctoral dissertation]. University of Walden.

Nielsen, L. S. (2009). *The relationship between pedagogical content knowledge and mathematics teacher questioning strategies* [Unpublished doctoral dissertation]. Louisiana Tech University.

Omar, M. S., Al-Shunnaq, M. M., & Al-Omari, W. H. (2018). The effectiveness of using 4MAT model in the development of metacognitive thinking in mathematics among 7th grade students in palestine. *Journal of Al-Quds Open University for Educational & Psychological Research & Studies, 9*(26), 184-197.

Özdoğan, Z. B. (2012). *4MAT öğretim modelinin ilköğretim matematik öğretmenleri tarafından uygulanması sürecinden yansımalar* [Reflections of practice of 4MAT education model by elementary school mathematic teachers] [Unpublished master's thesis]. Karadeniz Technical University.

Özgen, K., & Alkan, H. (2012). Yapılandırmacı öğrenme ortamında öğrenme stillerine uygun geliştirilen etkinliklere yönelik öğrenci görüşlerinin incelenmesi [An analysis of student views on activities developed according to learning styles within a constructivist learning environment] *Dicle University Ziya Gökalp Education Faculty Journal, 18,* 239-258.

Shaughnessy, M.F. (2013). An Interview with Bernice McCarthy: Creator of The 4MAT® System. *Journal of Social Sciences (COES&RJ-JSS), 2*(4), 196-198.

Seviş, Ş. (2008). *The effects of a mathematics teaching methods course on pre-service elementary mathematics teachers' content knowledge for teaching mathematics* [Unpublished master's thesis]. Middle East Technical University.

Shuilleabhain, A. N. (2016). Developing mathematics teachers' pedagogical content knowledge in lesson study. *International Journal for Lesson and Learning Studies*, *5*(3), 212-226.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1-23.

Ramirez, I., & Laurencio,O. (2015). English oral communicative competence of future teachers: A second work-integrated experience at Bindura University of Science Education. *Journal of Teaching and Education (JTE), 4*(2), 91-94.

Rusznyak, L., & Walton, E. (2011). Lesson planning guidelines for student teachers: A scaffold for the development of pedagogical content knowledge. *Education as Change*, *15*(2), 271-285.

Tataroğlu Taşdan, B., & Çelik, A. (2017). Matematik öğretmenlerinin matematiksel düşünmeyi destekleme bağlamındaki pedagojik alan bilgileri nasıl geliştirilebilir? [How could the mathematics teachers' pedagogical content knowledge in the context of supporting mathematical thinking be developed?]. *International Journal of New Trends in Arts, Sports & Science Education (IJTASE)*, *6*(2), 40-55.

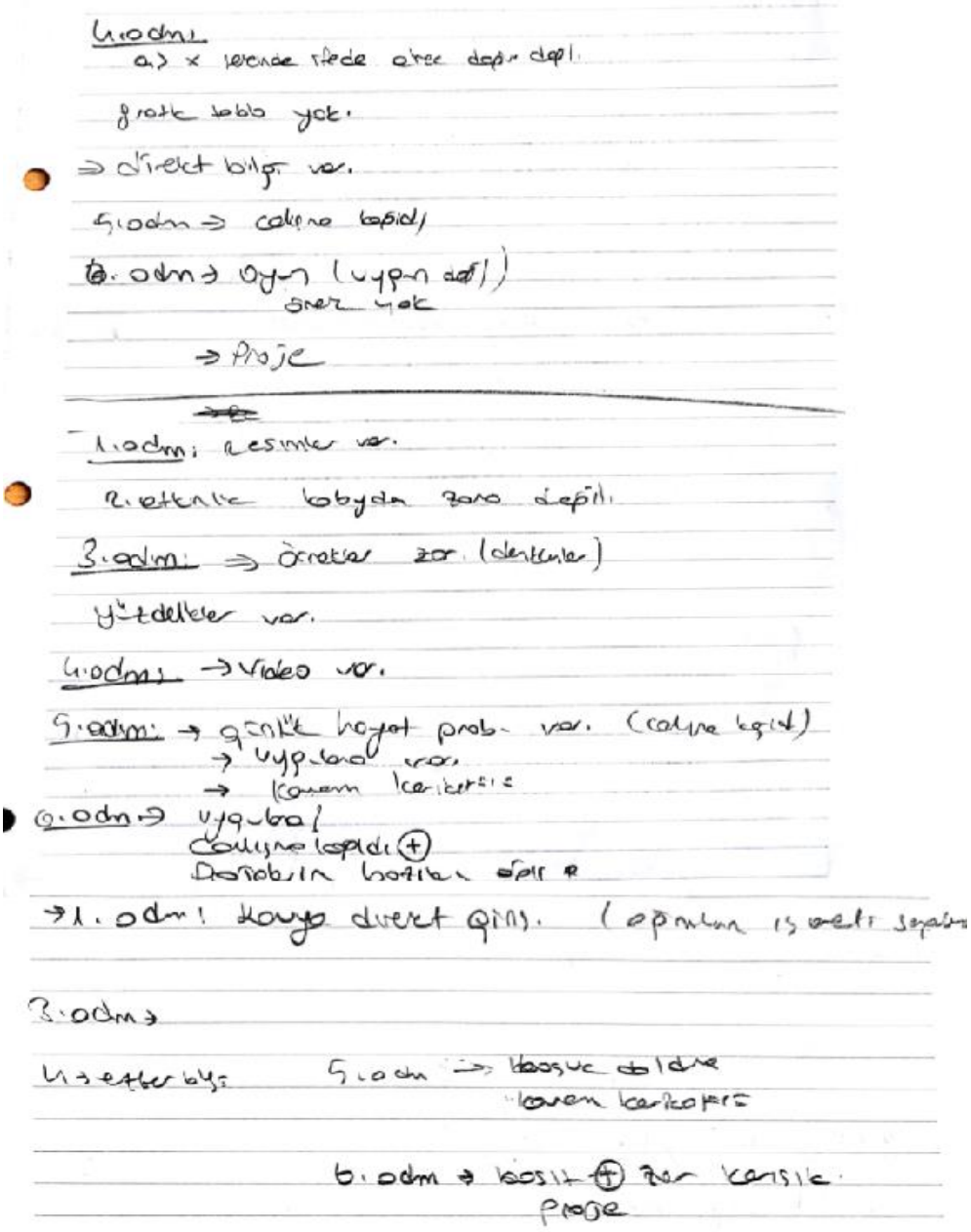
Taylor, B. (2018). Activating creativity in elementary EFL writing. In E. Polyudova (Ed.), *Acquiring lingua franca of the modern time current issues and strategies in ESL studies* (pp. 119-150). UK: Cambridge Scholars Publishing.

Tsai, H. S. (2004). *Learning achievement satisfaction and retention with wholebrain instruction among nursing students at a technology college in taiwann* [Unpublished doctoral dissertation]. Idaho State University.

Wright, T. D. (2009). *Investigating teachers' perspectives on the impact of the lesson study process on their mathematical content knowledge, pedagogical knowledge, and the potential for student achievement* [Unpublished doctoral dissertation]. University of New Orleans.

Yeşildere İmre, S., & Akkoç, H. (2012). Investigating the development of prospective mathematics teachers' pedagogical content knowledge of generalising number patterns through school practicum. *Journal of Mathematics Teacher Education, 15*(3), 207-226.

Appendix 1. Example of Observation Notes Held by Prospective Teachers



Appendix 2. Lesson Plan Evaluation Rubric

	Not obsorved (0)	Insufficient (1)	Partiallly Sufficient (2)	Sufficient (3)
Compliance with the 4MAT Model				
1. Starting the lesson appropriately based on what the students knew before				
2. Making the students feel why they should learn the subject				
3. Presenting a suitable experience that will enable students to establish a relationship between the subject and daily life				
4. Enabling students to analyze/discuss the experience presented				
5. Presenting visuals that will enable students to visualize the subject in their minds				
6. Using techniques such as brainstorming and analogy				
7. Presenting mathematical content appropriately to the students				
8. While presenting the content, made use of visual or audio tools such as web resources, CDs, and movies				
9. Enabling students to acquire the necessary skills by using worksheets and exercise books.				
10.Enabling students to make applications other than worksheets related to the defined concepts				
11. Enabling students to make innovations / inventions / projects by making additions on the applications made				
12. Enabling students to improve their knowledge by using worksheets that require high-level thinking				
13. Enabling students to analyze their own work.				
14. Enabling students to analyze the work of their friends				
15. Giving an opportunity to students to present/exhibit what they did.				
Compliance with the Pedagogical Content Knowledge Components				
Content Knowledge				
1. Knowing the basic definitions of the subject and reflecting them on the lesson plan				
2. Giving the logical grounds underlying the concepts				
3. Giving sufficient and appropriate examples to create mathematical content				
4. Using terms and notations correctly				
5. Making the necessary mathematical explanations appropriately				
6. Asking productive mathematical questions				
Knowledge of Student Understanding				
1. Being aware of students' mistakes and misconceptions about the mathematical content taught and including it in the lesson plans				
2. Predicting what students will find easy and difficult and to act in accordance with this order in the lesson plan				
3. Selecting examples that students will find interesting and motivating				
4. Being aware of the underlying causes of mistakes and misconceptions				
5. Acting in accordance with the level of students				
Knowledge of Instructional Strategies				
1. Reminding students' prior knowledge				
2. Talking about the importance and justification of the subject to be told				
3. Relating the subject to daily life				
4. Engaging actively students in the lesson				
5. Including different representations of concepts				
6. Designing learning environments for the students to solve learning difficulties, mistakes, and misconceptions and to teach concepts effectively				
7. Using different teaching methods and techniques appropriate for the teaching of the subject				
8. Benefiting from various course tools/equipment while explaining the subject				
9. Utilizing technology where necessary				