International Journal of Progressive Education, Volume 19 Number 5, 2023
© 2023 INASED

Mathematical Reasoning Skills as a Predictive of Number Sense

Ahsen Seda Bulut [i]
Kirsehir Ahi Evran University

Okan Kuzu [ii]
Kirsehir Ahi Evran University

Abstract

This study aims to examine the relationship between number sense skills and mathematical reasoning skills of preservice mathematics teachers. The research was conducted with 124 preservice mathematics teachers in the 2018-2019 academic year. Correlational model, one of the quantitative research methods, is used in the research. Data was analysed by the simple linear regression analysis, and the multiple regression analysis. As a result of the analysis, it was determined that the mathematical reasoning skills and the number sense skills of the preservice mathematics teachers were at "High" level. A statistically significant relationshi was observed between preservice mathematics teachers' mathematical reasoning skills and their number sense skills, both in general and in some of its factors. Besides, the main findings of this study was determined that mathematical reasoning skill was a significant predictor of the number sense skill. It was seen that the strongest predictor of number sense was "solving non-routine problems" and the weakest predictor was "recognizing and using mathematical patterns". Thus, it is thought that this study is particularly important in that it emphasizes the strong relationship between number sense and non-routine problems and illustrates that this aspect has a great importance in terms of number sense skills of preservice mathematics teachers.

Keywords: Number Sense, Mathematical Reasoning Skills, Preservice Mathematics Teachers

DOI: 10.29329/ijpe.2023.603.11

Submitted: 07/10/2022 **Accepted:** 20/08/2023 **Published:** 16/10/2023

----

[i] **Ahsen Seda Bulut,** Assist. Prof. Dr., Accounting and Taxation, Kirsehir Ahi Evran University

Correspondence: as_kilic@windowslive.com

[ii] **Okan Kuzu,** Assist. Prof. Dr., Department of Mathematics and Science Education, Kirsehir Ahi Evran Universty

INTRODUCTION

Numbers is an important sub-learning area that forms the basis of mathematics education. Solving problems involving numbers requires processes such as making sense of numbers, recognizing the relationships between numbers and recognizing the connections between numbers and multiplicities. This concept, which is expressed as number sense, has been defined as a good intuition about numbers and the relationships between numbers (Howden, 1989). Number sense is used when presenting practical, flexible and effective strategies (including mental and estimation) in solving numerical problems (Şengül & Gülbağcı-Dede, 2013). There are many definitions and classifications regarding the concept of number sense and its components in the literature. The fact that researchers make different classifications of the components of number sense affects the definition of the concept of number sense, and so different definitions emerge. Reys and Yang (1998) define number sense as a person's ability to understand numbers and operations in a general sense, develop useful strategies for solving complex problems, and use flexible methods to make mathematical judgments. Gersten and Chard (1999) stated that in the definition of number sense, similar to Reys and Yang (1998), the meanings of numbers should be understood, and in addition, the skills of doing mental mathematics and making comparisons are also included in number sense. Hope (1989) defines number sense as the feeling of being able to make logical predictions about the use of numbers, to notice number patterns, to notice arithmetic errors, and to choose the most effective way of calculation. The National Council of Teachers of Mathematics ([NCTM], 1989), which sets principles and standards for mathematics education, states that students with a developed number sense understand what numbers mean and realize the effects of transactions on numbers. The individual who knows the meanings of the transactions can develop flexible methods by going beyond the regular methods. Considering the fact that regular operations cannot be performed continuously in daily life, the individual must be able to produce unique and flexible solutions in order to solve the problems he/she encounters (Şengül & Gülbağcı-Dede, 2013). Students who use number sense adequately do not have problems by making connections between numbers and numerical expressions in mathematics and quantities in the real world (Case, 1998). Therefore, number sense, which has become one of the basic skills to be gained in mathematics education, is considered as a characteristic of students who are good at mathematics and a desired outcome of mathematics teaching (Howell & Kemp, 2005). Hence, there are some studies showing that number sense positively affects mathematics achievement (see e.g., Jordan, Glutting, & Ramineni, 2010; Mohamed & Johny, 2010). Studies on the development of number sense in students indicate that number sense can be developed depending on many different parameters. One of them is undoubtedly the provision of enriched learning experiences to students. (Tsao, 2004; Yang, 2003). This raises the question of the level of number sense in teachers and teacher candidates. It is clear that the level of teachers' number sense will affect the number sense education they will teach students (National Research Council [NRC], 2001). Some studies examining the number sense levels of teachers and pre-service teachers revealed that the number sense of the participants was not at the desired level (Almeida, Bruno, & Perdomo-Díaz, 2016; Yang, 2007; Yang, Reys, & Reys, 2009). This finding reveals the need to identify the components that affect number sense. McIntosh, Reys, and Reys (1992) define number sense as the ability to develop useful strategies in numbers and transactions involving daily life situations and to make mathematical reasoning in flexible ways. Considering that mathematical reasoning is a necessary skill to understand mathematical concepts, generate mathematical ideas, use operations flexibly and reconstruct previously learned mathematical knowledge (Brodie, 2010), it is clear that mathematical reasoning and number sense include common skills. Associating mathematics with its own operation priority with the help of reasoning, questioning its structure and knowing why and what it does provides the formation of mathematics that is both permanent and open to development (Umay & Kaf, 2005). Individuals with mathematical thinking and reasoning skills can develop different solution strategies, make predictions, and prove or generalize their results (Baki, 2008). From the point of view of mathematicians, reasoning is one of the basic tools for the development of mathematical understanding and the construction of new mathematical knowledge (Ball and Bass, 2003).

In the standards produced by NCTM (2000), the importance of realizing mathematical thinking and having the necessary mathematical knowledge for daily life mathematics is strongly emphasized. In the Mathematics Curriculum in Turkey, some skills related to every subject of mathematics have been determined. What is expected from the student is to make predictions about the measurement results by using the number sense while facing a problem (Ministry of National Education, 2018). Creating mathematical predictions in accordance with logic is among the characteristics of students with reasoning skills (NCTM, 1989). These reports and the skills to be gained in the curriculum show that individuals need to have a strong number sense and mathematical reasoning to be able to solve daily life problems and produce rational solutions that will reflect mathematics to daily life. The existence of these skills that support each other and the limited number of studies examining the relationship between these skills have been the subject of the problem of this research. Although there are many studies in the literature on mathematical reasoning and mathematical achievement or the number sense and its components, a limited number of studies that deals with these concepts together has been found (see e.g., Almeida, Bruno, and Perdomo-Díaz, 2016; Chrysostomou, Pitta-Pantazi, Tsingi, Cleanthous and Christou, 2013). For this reason, the research problem was determined as examining the relationship between number sense skills and mathematical reasoning skills of PMTs. The sub-problems created to investigate this problem are as follows:

1. How is the relationship between number sense skills and mathematical reasoning skills of PMTs?

2. Do PMTs' mathematical reasoning skills predict their number sense skills?

3. Do PMTs' grade levels predict their mathematical reasoning and number sense skills?

METHOD

Research Design

In this study, in which the quantitative research approach was adopted, the correlational model was used. The correlational model involves collecting data in order to determine the degree to which a relationship exists between two or more variables (Fraenkel & Wallen, 2006). In this context, in this study, the relationship between mathematical reasoning and number sense skills of primary school preservice mathematics teachers (PMTs) and the relationship between their grade level and mathematical reasoning and number sense skills were examined using a correlational model.

Sample

The sample of the study was composed of 124 PMTs studying in a state university in the Central Anatolia Region of Turkey in the spring term of the 2018-2019 academic year. While the convenience sampling method was used in the selection of the relevant universities, the purposive sampling method (Cohen and Manion, 1994) was used in the selection of the PMTs studying at this university. Percentage and frequency distribution of the sample are presented in Table 1 in detail.

Table 1. Percentage and frequency distribution of the sample

Variable		n	%
Grade	1	44	35,5
	2	28	22,6
	3	34	27,4
	4	18	14,5

Data Collection

In this study, two data collection tools were used. First, the two-stage "Mathematical Reasoning Assessment Scale" (MRAS) consisting of 20 multiple-choice and six open-ended items developed by Çoban (2010) was used to measure preservice teachers' mathematical reasoning skills.

The Kuder Richardson-20 (KR-20) reliability coefficient of the multiple-choice items of the scale was calculated as .74. The reliability of the part consisting of open-ended items was calculated by the correlation between the two raters and was found to be .85. A calculated reliability coefficient of .70 and higher is generally considered sufficient for the reliability of test scores (Yockey, 2016). In this context, it was seen that a measurement tool with high reliability was obtained. The measurement tool developed by Çoban (2010) consists of seven factors: "MRF1: Estimating", "MRF2: Recognizing different representations of the same data", "MRF3: Recognizing and using mathematical patterns", "MMF4: Deciding the correctness of the process and result", "MRF5: Ability to make mathematical generalizations", "MRF6: Developing logical arguments for solutions" and "MRF7: Solving non-routine problems". The relevance, content validity and scientific accuracy of this scale were obtained by taking the opinions of four experts in the field of mathematics education and two experts in the field of educational sciences. Each multiple-choice item of the scale was scored as “3.8: true, 0=false”, corresponding to a total of 76 scores. In the evaluation of open-ended items, a scoring key between 0 and 4 scores was used, corresponding to a total of 24 scores. In this context, the MRAS was evaluated over a total of 100. (For detailed information, see Çoban, 2010). In this study, in order for the total scores of the scales and factors to be similar calculation was made out of 100 scores (total score from factors) x 100 / (the highest total score that can be obtained from factors). For example, for the MRF6, which consists of six multiple-choice and two open-ended items, four correct (4x3.8) in multiple-choice items; In the open-ended item, a sample calculation of the total score of a PMT who got one full correct (1x4) and two points partially correct (1x2) is presented in Table 2.

Table 2. Total score calculation for the mathematical reasoning assessment scale

Factor	Total score received	Total score possible	Total score out of 100
MRF6	4x3,8+1x2+1x4=21,2	6x3,8+2x4=30,8	21,2x100/30,8=68,1

Another measurement tool used in this study was measured with the "Number Sense Test (NST)" prepared by Yang (2007). This measurement tool consists four factors: “NSF1:Understanding the meaning of numbers, operations and their relationships”; “NSF2: Recognizing relative number size”, “NSF3: Developing and using benchmarks appropriately”; "NSF4: Judging the reasonableness of a computational result by using the strategies of estimation” (For detailed information, see Yang, 2007). In this study, if the process and response to the use of number sense in the evaluation of each item of the test is correct, the researchers defined “2 (Correct)”; “1 (Partially correct)” if the process for using number sense is correct and the response is incorrect; in other cases, it is coded as “0 (false)”. Afterwards, the number sense test was also evaluated over 100 scores in terms of being similar to the mathematical reasoning assessment scale over the total score. For example, for the NSF3, which consists of three items, a sample calculation of the total score of a fully correct (1x2), a partially correct (1x1) and a wrong (1x0) is presented in Table 3.

Table 3. Calculation of the total score for the number sense test

Factor	Total score received	Total score possible	Total score out of 100
NSF3	1x2+1x1+1x0=3	3x2=6	3x100/6=50

During the evaluation process of the responses given to the test, scores were made independently by two academicians who are experts in the field of mathematics education, and the agreement between the two raters was calculated with the "weighted kappa", a type of kappa statistics. It is recommended that the values obtained should be at least .60. On the other hand, values between .60-.80 indicate good agreement between raters; Values above .80 correspond to a very good fit among raters (Wood, 2007). In this context, when Table 4 is examined, it is seen that the agreement between raters is very good (Table 4).

Table 4. Concordance values between raters

NST	1	2	3	4	5	6	7	8	9	10	11	12
κ	.88	.89	.86	.91	.90	.91	.96	.95	.92	.90	.93	.89

Data Analysis

During the data analysis process, firstly, the data obtained from 141 PMTs were transferred to the SPSS 23 program and the z score of each item was calculated for data cleaning. Then, normality tests were performed on the total score of the MRAS and NST. Histogram, boxplot and normal probability graphs were examined and the resulting extreme values were deleted. As a result, it was seen that boxplot and Q-Q graphs showed normal distribution over 124 data and the histogram curve met the assumption of normality (Figure 1).

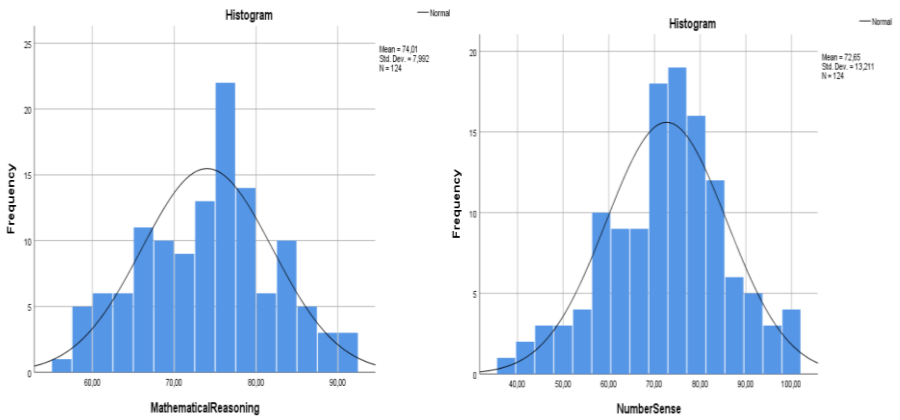

Figure 1. Histogram of mathematical reasoning and number sense total scores

On the other hand, when the descriptive statistics of the distribution are examined, the values such as arithmetic mean, mode and median are close to each other; it was observed that the skewness and kurtosis coefficients did not differ significantly between the values of -3 and +3 (Kim, 2013). In addition, although the Kolmogorov-Smirnov test results were found to be p>.05 for the total score of the MRAS, this value was found to be p<.05 for factors of the MRAS and also both the number sense test and its factors. Therefore, it was determined that it did not meet the normality condition for Kolmogorov-Simirnov. However, when Kolmogorov-Simirnov results were evaluated together with the descriptive and graphical results, the normality of the distribution was seen in this study (Table 5).

Table 5. Descriptive statistics results on the distribution of data

	Mode	Median	$\bar{X}$	SD	Skewness	Kurtosis	Min	Max	Kolmogorov Simirnov
MRAS	75.00	75.00	74.01	7.99	-.08	-.64	56.40	90.20	.20
MRF1	100.00	50.00	70.04	32.38	-.63	-.58	.00	100.00	.00
MRF2	100.00	100.00	85.89	22.60	-.98	-1.06	50.00	100.00	.00
MRF3	60.00	72.89	71.80	16.30	.14	-.64	40.00	100.00	.00
MRF4	75.00	75.00	77.62	20.86	-.63	.16	.00	100.00	.00
MRF5	91.67	83.33	78.63	19.80	-1.26	1.19	8.33	100.00	.00
MRF6	65.58	68.83	70.29	13.31	-.06	-.38	34.42	100.00	.00
MRF7	74.36	74.36	72.29	8.24	-1.34	2.65	48.72	87.18	.00
NST	75.00	75.00	72.65	13.21	-.25	-.01	37.50	100.00	.00
NSF1	100.00	83.33	87.10	15.88	-1.27	1.27	33.33	100.00	.00
NSF2	83.33	83.33	74.33	18.94	-.71	.95	.00	100.00	.00
NSF3	83.33	83.33	71.77	23.10	-.96	.48	.00	100.00	.00
NSF4	50.00	50.00	52.02	22.95	-.28	-.27	.00	100.00	.00

After providing the normality, the mathematical reasoning and number sense skills of the PMTs were examined. In addition, simple linear regression analysis to determine whether the mathematical reasoning skills of the PMTs significantly predicted their number sense skills; multiple regression analysis was performed regarding the extent to which the factors of the scale predicted the number sense skill. In addition, multiple regression analysis was used to determine whether it predicted mathematical reasoning and number sense skills by considering the grade levels of PMTs as a dummy variable.

FINDINGS

In this section, mathematical reasoning and number sense skills of PMTs are examined and the findings obtained as a result of the analyzes are presented. In this study, considering the limit values specified by Kuzu (2021), the levels are determined according to the total score that can be obtained from both measurement tools. Such that 0 ≤ score ≤ 20: Very low, 20< score ≤ 40: Low, 40< score ≤ 60: Medium, 60< score ≤ 80: High, 80 < score ≤ 100: Very high. In this context, when Table 5 is examined, it is seen that the mathematical reasoning and number sense skills of the PMTs are high ($\bar{X}_{\mathrm{MRAS}} = 74.01$; $\bar{X}_{\mathrm{NST}} = 72.65$). On the other hand, when examined in terms of factors, the skill levels of the "MRF2: Recognizing different representations of the same data" factor of the MRAS are very high; other skill levels were determined to be high. In terms of the NST, the skill levels of the factor "NSF1: Understanding the meaning of numbers, operations and their relationships" are very high; The skill levels of the factor "NSF4: Judging the reasonableness of a computational result by using the strategies of estimation" were medium; other skill levels were found to be high (Table 5). On the other hand, in this study, the relationship between PMTs' number sense skills and mathematical reasoning skills was examined with the Pearson correlation test and the findings are presented in Table 6.

When Table 6 is examined, it is seen that there is a statistically significant correlation between the mathematical reasoning skills of the PMTs and their number sense skills both in general and in some of its factors (p<.05). The obtained correlation coefficient (r) is very weak if r<.20; If .20<r<.40, weak; If .40<r<.60, medium; If .60<r<.80, it is high; r>.80 indicates that there is a very high level (Evans, 1996).

Table 6. The relationship between PMTs' mathematical reasoning skills and number sense skills

r		Number Sense				
		NST	NSF1	NSF2	NSF3	NSF4
Mathematical Reasoning	MRAS	.242*	.140	.180*	.193*	.280*
	MRF1	.207*	.182*	.189*	.059	.100
	MRF2	.234*	-.077	.160	.100	.212*
	MRF3	.109	.146	.133	.036	-.004
	MRF4	.145	.011	.026	.077	-.025
	MRF5	.239*	-.008	.109	.147	.123
	MRF6	.280*	.098	.223*	.165	.250*

*: p<.05

Accordingly, it was observed that there was a weak positive correlation between the mathematical reasoning skills of the PMTs and their number sense skills (r=.242). When the total of the mathematical reasoning scale is examined in terms of the factors of the number sense test, there is a difference between the mathematical reasoning skills of the PMTs and the skills of "SHF2: Recognizing the magnitudes of relative numbers" (r=.180) and "NSF3: Developing and using benchmarks appropriately" (r=.193). It was determined that there was a very weak correlation. In addition, it was observed that there was a positive but weak (r=.280) significant correlation between the mathematical reasoning skills of the PMTs and the skill of “NSF4: Judging the reasonableness of a computational result by using the strategies of estimation”.

On the other hand, when examined in terms of the factors of the mathematical reasoning scale and the number sense test, it has been determined that there is a very weak and significant positive correlation between "MRF1: Estimating" and "NSF1:Understanding the meaning of numbers, operations and their relationships” (r=.182); and between MRF1 and “NSF2: Recognizing relative number size” (r=.189). There was a weak and positive correlation between the "MRF4: Deciding the correctness of the process and result" and "NSF4: Judging the reasonableness of a computational result by using the strategies of estimation" (r=.212). There was a weak and positive correlation between the "MRF4: Deciding the correctness of the process and result" and the "NSF4: Judging the reasonableness of a computational result by using the strategies of estimation" (r=.212). There was a weak positive correlation between the skills of "MRF6: Developing logical arguments for solutions" and "NSF2: recognizing relative number size" (r=.223) and "NSF4: Judging the reasonableness of a computational result by using the strategies of estimation" (r=.250). When the sum of the NST is analyzed in terms of the factors of the MRAS, a significant relationship the positive and weak level (r=.207; r=.234; r=.239; respectively; r=,280) was found between the number sense skills and MRF1, MRF2, MRF5, MRF6 of the mathematical reasoning skills of the PMTs. Simple linear regression analysis was used to determine whether the mathematical reasoning skills of PMTs significantly predicted their number sense skills, and the findings are presented in Table 7.

Table 7. Simple linear regression analysis results on the prediction of mathematical reasoning skill on number sense skill

	B	Standart Error	r	r^2	Standardized β	t	F	p
Mathematical Reasoning	.741	.134	.448	.20	.448	5.542	30.711	.000*

*: p<.05

When Table 7 is examined, it is seen that the result of analysis of variance (F=30.711; p<.05) is significant. According to the results of analysis of variance, it was determined that the relationship between number sense and mathematical reasoning skills was linear, and that the mathematical reasoning skills of the PMTs predicted their number sense skills significantly positively. It was seen that 20% of the PMTs' number sense skills were explained by their mathematical reasoning skills. Multiple regression analysis was performed regarding the degree of predicting the number sense skill of the factors of the mathematical reasoning scale, and the findings are presented in Table 8.

Table 8. Multiple regression analysis results on the prediction of the factors of mathematical reasoning skills on the number sense skill

	Unstandardized		Standardized						
Mathematical Reasoning	B	Standart Error	β	t	p	F	r	Adjusted r^2	p
Constant	-7.59	13.405		-.566	.572	6.180	.521	.23	.000
MRF1	.076	.033	.186	2.273	.025				
MRF2	.110	.049	.188	2.241	.027				
MRF3	.086	.065	.106	1.325	.188				
MRF4	.089	.051	.141	1.758	.081				
MRF5	.121	.054	.181	2.219	.028				
MRF6	.178	.082	.179	2.163	.033				
MRF7	.420	.129	.262	3.256	.001				

When Table 8 is examined, it is seen that the result of analysis of variance (F=6.180; p<.05) is significant. According to the analysis of variance results, it was determined that the relationship between the MRF3 and MRF4 factors and number sens was not statistically significant, and that these two variables did not significantly predict the number sense skill. On the other hand, it was seen that the relationship between the other factors of the MRAS and the number sense skill was linear and predicted the number sense skill significantly positively. When the factors of the MRAS were taken together, it was seen that they explained 23% of the total variance regarding the number sense skill. On the other hand, when the standardized regression coefficients (β) are examined, the order of importance of the factors of the MRAS on the number sense skill was determined as MRF7 ($\beta = .262$; t=3.256; p<.05), MRF2 ($\beta = .188$; t=2.241; p<.05), MRF1 ($\beta = .186$; t=2.273; p<,05), MRF5 ($\beta = .181$; t=2.219; p<.05), MRF6 ($\beta = .179$; t=2.163; p<.05), MRF4 ($\beta = .141$; t=1.758; p>.05), MMF3 ($\beta = .106$; t=1.325; p>.05). The results of multiple regression analysis regarding the prediction of mathematical reasoning and number sense skills of PMTs grade levels are presented in Table 9.

Table 9. Multiple regression analysis results on the prediction of the grade levels on mathematical reasoning and number sense skills

		Unstandardized		Standardized						
	Grade	B	Standart Error	β	t	p	F	r	Adjusted r^2	p
MRAS	Constant	71.836	1.070		67.142	.000	11.986	.480	.21	.00
	2	.076	.858	.008	.089	.929				
	3	.619	.540	.104	1.146	.254				
	4	2.815	.496	.498	5.670	.000				
NST	Constant	68.750	1.751		39.268	.000	13.055	.496	.23	.00
	2	-1.11	1.404	-.071	-.795	.428				
	3	2.288	.884	.233	2.588	.011				
	4	4.340	.812	.465	5.343	.000				

When Table 9 is examined, it is seen that the result of analysis of variance is significant for both mathematical reasoning (F=11.986; p<.05) and number sense (F=13.055; p<.05). The average mathematical reasoning skill score of the 1st grade students was high (B=71,836); It was observed that this score was relatively higher and did not differ significantly (p>.05) among the PMTs studying in the 2nd grade (B=71,912) and the 3rd grade (B=72,455). It was determined that the mathematical reasoning skills of the 4th grade students differed significantly (B=74,651). Accordingly, it was determined that 21% of the PMTs' mathematical reasoning skills were explained by their 4th grade education. On the other hand, the number sense skill score of the PMTs studying in the 1st grade high (B=68,750); It was observed that this score was relatively lower in the PMTs studying in the 2nd grade (B=67.64) and did not differ significantly (p>.05). It was determined that the number sense skills of the PMTs studying in the 3rd grade (B=71,038) and the 4th grade (B=73,09) were significantly higher. Accordingly, 19% of the PMTs' number sense skills are in the 4th grade; 5% is explained by their education in the 3rd grade. When the PMT studying in the 3rd and 4th grades are considered together, it was seen that it explained 21% of the total variance. When the order of importance of the grade level on the mathematical reasoning skill was examined, it was seen that the first three classes did not have a significant difference between them and did not have a significant effect on mathematical reasoning. It was observed that studying in the 4th grade had a significant effect on mathematical reasoning skills (β=.498; t=5.670; p<.05). In terms of number sense skills, it was determined that the most important effect was to be educated in the 4th grade (β=.465; t=5.343; p<.05) and 3rd grade (β=.233; t=2.588; p<.05) respectively. It was observed that studying in the 1st and 2nd grades did not have a significant effect on the number sense skill.

CONCLUSION DISCUSSION AND SUGGESTIONS

Number sense requires reaching the result by using short, effective, and practical solution methods instead of calculation methods that require long time (Kayhan-Altay, & Umay, 2013). The fact that we need practical solution methods in order to encounter mathematics in almost every field of our daily life and to solve the problems we encounter reveals the importance of the sense of number we have. Individuals need skills related to number sense not only in the short term, but at every stage of their lives. Therefore, it is thought that considering every factor that shapes number sense of individuals will shed light on the trainers in order to develop these skills.

In this study, it was determined that the number sense level of PMTs was at "High" level. While a group of studies in the literature state that preservice teachers' number sense skills are at a moderate level (Gülbağcı-Dede, 2015), a group of studies indicate that they are low (Almeida, Bruno, and Perdomo-Díaz, 2016; Can, 2020; Courtney-Clarke & Wessels, 2014; Yang, 2007; Yang, Reys, & Reys, 2009). This difference may be related to the different measurement tools used, as well as the fact that the participants are teacher candidates from different branches. Can (2020) stated that primary school teacher candidates have a low level of number sense and they prefer to use computational solution methods further. In the study, which focused on the relationship between logical thinking skill and number sense, she stated that candidates with low logical thinking skills also had low number sense. In this study, the remarkable finding regarding the number sense of PMTs is that the scored

"Very High" in the factors of "Understanding the meaning of numbers, operations, and relations between them", "Recognizing the magnitudes of relative numbers" and "Using and developing appropriate criteria", they received a “Medium” score in the factor of “Evaluation of the result of the transaction using estimation strategies”.

Gülbağcı-Dede (2015), who reached a similar conclusion, stated that PMTs' number sense showed the lowest performance in the "Guess" component. Similarly, in the study by Şenol, Dündar and Gündüz (2015) focused on preservice classroom teachers’ number sense and detected that they had difficulty in deciding the rationality of the results by using estimation strategies which are the most employed. Boz-Yaman and Bulut (2017) stated that mathematics teachers have limited definitions of estimation skills and revealed that mathematics teachers have limited knowledge of estimating strategies. Sayın, Özdemir and Öner (2022) stressed that teachers use estimation skills in questions that clearly require estimation, but they do not use estimation skills other than that. The fact that preservice teachers cannot make inferences about making predictions in daily life problems shows that they cannot use number sense sufficiently. The estimation skill which is common in the lives of children and adults and is also an important part of mathematical understanding gives information about how mathematical concepts and relationships are generally understood in terms of the strategies used (Siegler & Booth, 2005; Dowker, 1992). Therefore, the fact that preservice teachers' estimation performances are not at the desired level may be an indication that they cannot adequately perceive and make sense of mathematical concepts and relationships. In this context, as stated by Kuzu, Kuzu and Sıvacı (2018), it is beneficial to design a learning environment and teaching process suitable for pre-service teachers' understanding in order to make sense of mathematical concepts.

In this study, it was found that PMTs' mathematical reasoning skills were at a "High" level. The striking situation here is that while PMTs perform at high and very high levels in all other factors of the scale, they score "Intermediate" in the factors of "Recognizing and using mathematical patterns" and "Developing logical arguments for solutions". Yeşildere and Akkoç (2011) revealed that the majority of pre-service mathematics teachers had various difficulties in the process of generalizing and using shape patterns.

In the study, it was determined that there was a significant and positive relationship between number sense and mathematical reasoning ability. The highest correlation is between number sense and the factor of "Developing logical arguments for solutions" of mathematical reasoning skill. Considering that number sense is making the fastest and most logical decision for solving problems, the fact that mathematical reasoning includes similar thinking processes in this respect is a finding of this research. As a matter of fact, Greeno (1991) describes number sense as making judgments and inferences about quantities. Similarly, Toulmin, Rieke, and Janik (1984) stated that reasoning is making the best decision about a particular situation, issue or event rather than generating new ideas. Almeida, Bruno, and Perdomo-Díaz (2016) stated that pre-service teachers who do not use number sense know how to solve activities in different ways, but they use algorithms rigidly in their solutions while they can reach a solution with good mathematical reasoning. Since PMTs can develop logical arguments for solutions and include high-level thinking skills, it can be ensured that they practice with activities that include different thinking skills, which are thought to increase this skill to develop number sense. Solving the problem by considering multiple and different strategies can help them develop their number sense in parallel with developing logical arguments.

It has been determined that PMTs' mathematical reasoning skills are a significant predictor of number sense skills. It was seen that the strongest predictor of number sense was "Solving non-routine problems", and the weakest predictor was "Recognizing and using mathematical patterns". Işık and Kar (2011) stated that there is a positive relationship between number perception and non-routine problem-solving skills. Non-routine problems, by their nature, require skills beyond just processing skills, such as organizing data, classifying, seeing relationships, and performing certain actions one after the other (Altun, 2005). In this respect, non-routine problems require some skills, such as reasoning along with mathematical thinking, in order for students to find an algorithm that is different from the one they learn in the class (Işık & Kar, 2011). The result of this research shows that number sense is used to a great extent in problem solving skill, which is considered important in terms of helping students establish the connection between mathematics and real-life situations, especially in the solution process of non-routine problems that require reasoning. In this manner, it is thought that this study is particularly important in that it emphasizes the strong relationship between number sense and non-routine problems and illustrates that this aspect has a great importance in terms of number sense skills of PMTs. In this direction, in the process of developing number sense skills, it is recommended to include activities aiming at developing this process, especially with non-routine problems, in parallel with targeting individuals' mathematical reasoning skills.

In the analyses made to see how the grade levels affect the mathematical reasoning of the PMTs, it was seen that studying in the 4th grade had a significant effect on the mathematical reasoning skills. It is a finding in the literature that mathematical reasoning skills increase with age (Dündar & Yaman, 2015). It is thought that such a result emerged due to the fact that the teachers took courses related to mathematical reasoning during their undergraduate education with their increasing mathematics experience.

In terms of number sense, it was determined that the number sense skills of the PMTs studying in the 3rd and 4th grades were significantly higher. There are many studies in the literature that support this finding (Ak & Ertekin, 2020; Aunio, Niemirta, Hautamaki, Van Luit, Shi, & Zhang, 2006; Sturdevant, 1991; Singh, 2009; Takir, 2016; Tunalı, 2018). This can be explained as number sense which is a developmental process and is generally expected to increase with age (Kayhan-Altay & Umay, 2013). The increase in the mathematical experience of the individual and the development of abstract thinking as a result of increasing age may lead to an increase in the individual's performance of number sense.

As a result, in this study, it was determined that PMTs had mathematical reasoning skills in explaining their number sense skills. Determining that the strongest predictor of number sense is "solving non-routine problems", it can be said that non-routine problems should be included in teaching to increase number sense skills. Tsao (2004) stated that pre-service teachers' number sense performances increased when problem solving approaches were used in lessons. If it is desired to develop students' number sense for high-quality mathematics education, first of all, teachers' number sense should be developed (Yang et al., 2009). Therefore, the first thing to do is to carry out studies to increase the number sense performances of teachers. It is obvious that the quality of teaching in terms of number sense will increase if teachers have high number sense and the components related to number sense are revealed.

Conflicts of Interest: The authors declare that there is no conflict of interest.

Funding Details: The authors declare that this study has not received any financial support.

CRediT Author Statement: Ahsen Seda Bulut: Writing-Original Draft Preparation, Conceptualization, Methodology, Investigation, Formal Analysis, Writing-Reviewing and Editing, Supervision. Okan Kuzu: Writing-Original Draft, Methodology, Validation, Visualization.

Ethical Statement: This study does not require an ethics committee approval.

REFERENCES

Altun, M. (2005). *İlköğretim ikinci kademe (6-7 ve 8.sınıflarda) matematik öğretimi [Teaching mathematics in primary education second level (6-7 and 8th grades)]*. Bursa: Aktüel Yayınları [Bursa: Aktüel Publications].

Almeida, R., Bruno, A., & Perdomo-Díaz, J. (2016). Strategies of number sense in pre-service secondary mathematics teachers. *International Journal of Science and Math Education*, 14, 959–978.

Ak, Y. & Ertekin, E. (2020). A study on the relationship between number sense and math anxiety of 7th and 8th grade students. *OPUS International Journal of Society Researches, 16*(31) , 4047–4076.

Aunio, P., Niemivirta, M., Hautamäki, J., Van Luit, J. E., Shi, J., & Zhang, M. (2006). Young children's number sense in China and Finland. *Scandinavian Journal of Educational Research, 50*(5), 483–502.

Baki, A. (2008). *Kuramdan uygulamaya matematik eğitimi [Mathematics education from theory to practice]*. Ankara: Harf Eğitim Yayıncılığı [Ankara: Harf Eğitim Publishing].

Ball, D. L., & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin, & D. Schifter (Ed.) A Research Companion to principles and standards for school mathematics. 27-44. Reston, VA: National Council of Teachers of Mathematics.

Boz-Yaman, B., & Bulut, S. (2017). Middle school mathematics teachers’ opinions on estimation. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 11*(1), 48–80.

Brodie, K. (2010). Teaching mathematical reasoning in secondary school classrooms. London: Springer.

Can, D. (2020). An analysis on the number sense performances of preservice primary school teachers based on their logical thinking ability. *Cumhuriyet International Journal of Education, 9*(2), 367-389.

Case, R. (1998). A psychological model of number sense and its development. *Paper presented at the annual meeting of the American Educational Research Association,* San Diego, CA.

Chrysostomou, M., Pitta-Pantazi, D., Tsingi, C., Cleanthous, E., & Christou, C. (2013). Examining number sense and algebraic reasoning through cognitive styles. *Educational Studies in Mathematics, 83*(2), 205–223.

Cohen, L., & Manion, L. (1994). *Research methods in education (Fourth Edition).* London: Routledge.

Courtney-Clarke, M., & Wessels, H. (2014). Number sense of final year pre-service primary school teachers: Original research. *Pythagoras, 35*(1), 9 pages.

Çoban, H. (2010). *Investigating the relationship between the level of students? using mathematical reasoning skills and using metacognitive learning strategies.* Master's thesis, Gaziosmanpaşa University Institute of Social Sciences, Tokat.

Dowker, A. (1992). Computational estimation strategies of professional mathematicians. *Journal for Research in Mathematics Education, 23*(1), 45–55.

Dündar, S. & Yaman, H. (2015). To examine how the skills of class teacher candidates in terms of interpreting tables and graphics hange according to mathematical reasoning skills. *Kastamonu Education Journal, 23*(4), 1695–1710.

Evans, J. D. (1996). *Straightforward statistics for the behavioral sciences.* Pacific Grove, CA: Brooks/Cole Publishing.

Fraenkel, J. R., & Wallen, N. E. (2006). *How to design and evaluate research in education,* Sixth Edition, New York: McGraw-Hill.

Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *The Journal of Special Education, 33*(1), 18–28.

Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education, 22*(3), 170–218.

Gülbağcı-Dede, H. (2015). *Investigation of number sense of primary and secondary school mathematics teacher candidates.* Doctoral dissertation, Marmama University Institute of Educational Sciences, Istanbul.

Hope, J. (1989). Promoting number sense in school. *The Arithmetic Teacher, 36*(6), 12.

Howden, H. (1989). Teaching number sense. *Arithmetic Teacher,* 36, 6–11.

Howell, S., & Kemp, C. (2005). Defining early number sense: A participatory Australian study. *Educational Psychology, 25*(5), 555–571.

Işık, C., & Kar, T. (2011). Investigating of the number perception and nonroutine problem solving skills of 6th, 7th and 8th grade students. *Ahi Evran University Journal of Kırşehir Education Faculty, 12*(1), 57–72.

Jordan, N.C., Glutting, J., & Ramineni, C. (2010), The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences, 20* (2), 82–88.

Kayhan-Altay, M., & Umay, A. (2013). The development of number sense scale towards middle grade students. *Education and Science, 38*(167), 241–255.

Kim, H. Y. (2013). Statistical notes for clinical researchers: Assessing normal distribution using skewness and kurtosis. *Restorative Dentistry & Endodontics, 38*(1), 52–54.

Kuzu, O. (2021). Diagnostic assessment of preservice mathematics and science teachers' attributes on integral. *YYU Journal of Education Faculty*, *16*(1), 1402–1418.

Kuzu, O., Kuzu, Y., & Sıvacı, S. Y. (2018). Preservice teachers' attitudes and metaphor perceptions towards Mathematics. *Cukurova University Faculty of Education Journal, 47*(2), 897–931.

McIntosh, A., Reys, B. J., & Reys, R. E. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics, 12*(3), 2–9.

Ministry of National Education, (2018). Mathematics (1-2-3-4-5-6-7-8th grade) curriculum. Board of Education and Discipline.

Mohamed, M., & Johnny, J. (2010). Investigating number sense among students. *Procedia-Social and Behavioral Sciences, 8*, 317–324.

National Counsil Of Teachers of Mathematics (1989). Curriculum and evalution standards for school mathematics. Reston, VA: NCTM.

NCTM, (2000). Principles and standards for school mathematics. https://www.nctm.org/uploadedFiles/Standards_and_Positions/PSSM_ExecutiveSummary.pdf. Accessed June 10, 2022.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* In J. Kilpatrick, J. Swafford, B. Findel (Eds.), Mathematics learning study committee, center for education, division of andbehavioral and social sciences and education. Washington, DC: National Academy Press

© 2023 INASED

Reys, R. E., & Yang, D. C. (1998) Relationship between computational performance and number sense among sixth- and eighth-grade students in Taiwan, *Journal for Research in Mathematics*, 29, 225–237.

Sayın, S., Özdemir, Ü., & Oner, A. T. (2022). Mathematics teachers' ınstructional explanations and estimation skills about area measurement. *Ahi Evran University Journal of Kırşehir Education Faculty,* 23, 84–127. Special Issue.

Siegler, R. S.& Booth, J. L. (2005). Development of numerical estimation a review. Edited Jamie I.D. Campbell. Handbook of Mathematical Cognition.

Singh, P. (2009). An assessment of number sense among secondary school students. *International Journal for Mathematics Teaching and Learning,* 155, 1–29.

Sturdevant, R.J. (1991). *Investigating the use of number sense by elementary students in grades 4, 6, and 8.* PhD Thesis, University of Missouri, Colombia.

Şengül, S., & Gülbağcı-Dede, H. (2013). An investigation of classification of number sense components. *International Journal of Social Science, 6*(8), 645–664.

Şenol, A., Dündar, S., & Gündüz, N. (2015). Analysis of the relationship between estimation skills based on calculation and number sense of prospective classroom teachers. *International Journal of Progressive Education, 11*(3), 90–105.

Takır, A. (2016). Investigating the relations number sense ability of 6th, 7th and 8th grade students between their grade level, gender and mathematics self-efficiency level. *Dicle University, Journal of Ziya Gökalp Faculty of Education,* 29, 309–323.

Toulmin, S., Rieke, R., & Janik, A. (1984). An introduction to reasoning (Second Edition). New York: Macmillan Publishing Co.

Tsao, Y. L. (2004). Exploring the connections among number sense, mental computation performance, and the written computation performance of elementary preservice school teachers. *Journal of College Teaching & Learning, 1*(12), 71–90.

Tunalı, C. (2018). *The determination of gifted students' level of number sense.* Master's thesis, Dokuz Eylul University Institute of Educational Sciences, Izmir.

Umay, A., & Kaf, Y. (2005). A study on flawed reasoning in mathematics. *Hacettepe University Journal of Education*, 28, 188–195.

Wood, J. M. (2007). Understanding and computing Cohen's kappa: A tutorial. Web Psych Empiricist. Retrieved from http://wpe.info/ vault/wood07/wood07ab.html.

Yang, D. C. (2003). Teaching and learning number sense–an intervention study of fifth grade students in Taiwan. *International Journal of Science and Mathematics Education, 1*(1), 115–134.

Yang, D. C. (2007). Investigating the strategies used by pre-service teachers in Taiwan when responding to number sense questions. *School Science and Mathematics, 107*(7), 293–301.

Yang, D. C., Reys, R. E., & Reys, B. J. (2009). Number sense strategies used by pre-service teachers in Taiwan. *International Journal of Science and Mathematics Education*, *7*(2), 383–403.

Yeşildere, S., & Akkoç, H. (2011). Pre-Service mathematics teachers' generalization processes of visual patterns. *Pamukkale University Journal of Education, 30*(30), 141–153.

© 2023 INASED

Yockey R.D., (2016). *SPSS Demystified: A simple guide and reference* (Second edition). United Kingdom: Taylor & Francis.