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Pre-Service Elementary Mathematics Teachers' Specialized Content Knowledge: The Case of Integer Addition and Subtraction

Ali Sabri İpek
Recep Tayyip Erdoğan University

Abstract

Pre-service mathematics teachers' content knowledge is an important issue. Therefore, detailed studies are needed to be conducted on mathematical topics. The study examines preservice elementary mathematics teachers' (PEMTs) special content knowledge (SCK) of integer addition and subtraction in the context of using multiple representations, explaining mathematical reasons lying behind the concepts and justifying them. The findings obtained from the written responses of 42 PEMTs reveal that preservice teachers do not have sufficient and balanced special content knowledge. This is especially more so in the case of addition and subtraction of numbers with opposite signs. The preservice teachers were observed to have more difficulty in using the number line model compared to the use of other representations. The findings offer some indicators about how PEMTs understand integer addition and subtraction through multiple representations and why more emphasis on the SCK components

Keywords: Specialized content knowledge, Integer addition and subtraction, Pre-service elementary mathematics teachers.

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[i] **Ali Sabri İpek**, Assoc. Prof. Dr., Department of Education, Recep Tayyip Erdogan University, Rize, Turkey.

Correspondence: ali.ipek@erdogan.edu.tr

Introduction

Although there are various studies on the qualifications of mathematics teachers and the quality of mathematics teaching (Silverman & Thompson, 2008; Fennema & Franke, 1992; Hill, Rowan & Ball, 2005; Ma, 1999), there has been a growing interest in studying mathematics teachers' special content knowledge. Greenberg & Walsh (2008) report that such interest in the improvement of teachers' mathematical knowledge is observed both in the educational policies and the studies conducted. There are alternative beliefs regarding the structure and characteristics of content knowledge in-service and pre-service mathematics teachers have. We can show the studies conducted by Shulman (1986) and Ball (2000) as the leading studies in this regard. Emphasizing that mathematics teaching knowledge requires more than content knowledge, Shulman (1986) developed a model of teacher knowledge which has been a pioneering model in mathematics education just like in other fields. Ball, Thames and Phelps (2008) took Shulman's model as a basis and developed the Mathematical Knowledge for Teaching (MKT) model, the scope of which covers mathematics education. In this model, SCK is the most important component for teachers to increase success in their classes.

Specialized Content Knowledge

The model called MKT developed by Ball et al. (2008) regarding the knowledge that teachers must have consists of two dimensions: content knowledge and pedagogical content knowledge. SCK is one of the three categories of content knowledge. SCK is defined as the mathematical knowledge specific to teachers. Delaney, Ball, Hill, Schilling and Zopf (2008) defines specialized content knowledge as the mathematical knowledge and skills used by teachers. Studies on mathematical success of students have been focusing more and more on the content knowledge of mathematics teachers. In this sense, pre-service and in-service processes of mathematics teachers have been addressed in a more detailed way. Pointing out the importance of this issue, Morris, Heibert and Spitzer (2009) indicate that SCK serves as a good candidate for topics to be covered in preservice teacher education.

Mathematical knowledge of teachers and pre-service teachers in a way characterizes their ability to understand and use the subject knowledge during mathematics teaching. Hill, Rowan, and Ball (2005) reports that these tasks covers having knowledge of mathematical teaching programs, explaining clearly the mathematical concepts and concepts to students and assessing the thoughts of students as well as choosing and using the best representations and examples in teaching mathematical concepts. Given the expression "Knowledge for teaching" which can be considered as the basic paradigm of MKT, SCK is also a need for the efficiency of the in-class teaching activities. In this sense, teachers or pre-service teachers should be able to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems (Hill, Ball & Schilling, 2008). Petrou and Goulding (2011) define SCK, which is the focus of the conceptualization of mathematical knowledge, as the type of knowledge that teachers or pre-service teachers need for effective mathematics teaching and use in the classroom environment.

It is hard to say that the scope and limitations of SCK, which is defined as the mathematical knowledge used or to be used by teachers in the classroom, are clear. In her study on pre-service teachers' specialized content knowledge of the associative property of multiplication, Ding (2016) addressed representation and explanation as two components of this type of knowledge. Ball et al. (2008) also examined the examples of explaining and justifying mathematical ideas like "explanation of the reason for using the invert and multiply method in division of fractions" within the scope of specialized content knowledge. Lin, Chin and Chiu (2011) suggest that SCK consists of three components, i.e. representation, justification and explanation, and define these components as follows:

Representations: Choosing and using mathematical representations accurately and effectively

Justification: Descriptions and justifications of mathematical ideas

Explanation: Providing mathematical expressions of common rules and procedures

The term representation refers both to process and product. Representations are used to emphasize mathematical ideas and support learning and they can be divided into two categories as internal and external representations. Internal representations refer to internal organization of knowledge, while external representations refer to the external tools used in modelling mental processes such as real world contexts, manipulative models, diagrams and verbal or symbolic expressions. Since each of these representation types has certain limitations, the gaps can be minimized only by using multiple representations during the teaching process. Representations are also the most important elements of the specialized content knowledge which is one of the three components of subject matter knowledge together with common content and horizon content knowledge. Edward (1998) stated that external representations, especially in terms of communication skills, are conventionally used more than internal representations. This study only focuses on external representations. The representations used in this study were limited to the following most commonly used three representations: number line, counters and real world context. Number line and neutralization models are used extensively in understanding of addition and subtraction in integers. Two-color counters are used in general for the neutralization model. Real world context are also a useful representation of in order to make sense of the operations. These representations will help revealing pre-service teachers' specialized content knowledge of integer addition and subtraction in a more detailed way. The pre-service teachers' representations and explanations about these operations constitute the basis for how they make sense of these concepts.

Tasks such as explanation and justification are effective in ensuring meaningful and high-level learning. Such tasks are given importance in studies due to their contribution to encouraging individuals to justify their thoughts and hypotheses and to deep understanding and thinking (Schwarz, Hershkowitz & Prusak, 2010; Yackel & Hanna, 2003). Lewis (1998) indicates that personal explanations of students about their mathematical thoughts contribute to the improvement of their learning. Explanations and justifications of pre-service teachers offer strong clues to their ability to ensure high-level learning of students in the future courses they will deliver. At this point, pre-service teachers' explanations and justifications for integer addition and subtraction operations may refer to the structural relations rather than general superficial characteristics and to the relationships between quantities rather than just numbers. Venkat (2015) points out that focusing on representations and explanations might simultaneously support teachers' mathematical learning and their learning about the mathematics they will teach.

In fact, representations should not be considered separately from justifications and explanations. These three components are intensively used during the process of teaching and/or learning mathematics subjects. Pre-service teachers' specialized content knowledge not only includes using multiple representations, but also explanations and justifications. In a sense, the intersection of the characteristics of common content knowledge (CCK) and horizon content knowledge (HCK), which are the other two dimensions of specialized content knowledge and content knowledge, is not an empty set. In the CCK dimension, it is possible to solve a question without providing any justification or using any representatives. The SCK dimension, which indicates deeper and more comprehensive knowledge of any mathematics subject, involves understanding the important of concepts and expressing the meanings lying behind the operations. In this study, which examines pre-service mathematics teachers' specialized content knowledge of integer addition and subtraction, the theoretical framework proposed by Lin et al. (2011) which covers these three components was taken as basis.

Integer Addition and Subtraction

Integers constitute one of the basic math topics in secondary education. Bolyard (2005) states that understanding integers is important for laying the foundation of various other topics to be covered in the future like algebra. However, basic concepts and operations involving integers are among the math concepts that are most difficult to understand in secondary education. Although integers is a math topic, they are frequently used to represent many real world situations, such as temperature, profits and losses of money and location. Studies reveal that students do not understand addition and subtraction at a conceptual level especially due to the intensive use of operational approaches. One of the reasons for having difficulty is that students proceed to addition and subtraction operations without learning integers and their characteristics at the conceptual levels. Qualifications of mathematics teachers and their content knowledge are important in helping students overcome the difficulties they have. Mathematical knowledge of pre-service or in-service teachers is addressed in general terms as understanding mathematical concepts and establishing relationship between concepts and procedures.

Studies reveal that students do not understand integer addition and subtraction at a conceptual level (Ferguson, 1993; Wilkins, 1996; Shore, 2005; Bofferding, 2014). Operational approaches are more frequently used in teaching this topic. Results from the Second Mathematics Assessment of the National Assessment of Educational Progress (Carpenter, Corbitt, Kepner, Lindquist & Reys, 1981) show that 75% of 13-year olds correctly added two negative integers but only about 43% correctly added a negative integer and a positive integer.

SCK for Integer Addition and Subtraction

Although it is generally accepted that pre-service or in-service mathematics teachers' specialized content knowledge is important, there is still a need for detailing on the basis of topics. In this sense, this study focuses on pre-service mathematics teachers' specialized content knowledge of integer addition and subtraction. In their study, Mitchell, Charalambous and Hill (2013) examined the behaviors of teachers in classroom environment and stated that the use of representatives is a must in teaching integers. They also developed a typology of the tasks performed by teachers involving the use of representations in teaching integers. It is necessary to use representation in deal with integers operations, since features of the operations are not easy for students. Stephan and Akyuz (2012) pointed out that integer operations offer some challenges for representation use, because unlike natural numbers, students cannot construct the meaning of integer operations by mere abstraction from real objects. Number lines reveal the ordinal meaning and counters reveal the cardinal meaning of numbers; therefore, Davidson (1987) points out the necessity of understanding both ordinal and cardinal meanings of integers during the conceptualization of these numbers. In this sense, both teachers and pre-service teachers should have a deep conceptual understanding regarding why and how addition and subtraction of integers is performed.

Method

Participants

The study was carried out using the case study model of descriptive research methods. The study sample included 42 fourth-grade PSTs studying in teaching program at a state university located northeast of Turkey. 27 participants were female and 15 participants were male. The participants aged 20 to 24 years. All participants had previously taken courses related to teaching concepts in secondary education mathematics and had basic knowledge and approaches related to teaching integers. Especially the course "special teaching methods" the pre-service teachers took in the third grade in both terms is extremely important for the development of their specialized content knowledge. This course aims to help pre-service mathematics teachers gain a deeper and conceptual understanding of integers and operations as well as of many other concepts in secondary mathematics. In this study, we tried to reveal the specialized content knowledge of integer addition and subtraction that pre-service teachers who took this course previously have by means of their explanation, justification and representation abilities. In this context, the data were collected by open-ended questions and semi-structured interviews.

Instrument

Considered as a type of knowledge specific to teaching, specialized content knowledge is addressed as the mathematical knowledge teachers use/will use in the classroom environment. In this study which examines this type of knowledge whose scope and limitations are not clear, the categorization of specialized content knowledge proposed by Lin et al. (2011) (Explanation, Justification and Representation) was used. An analytical data collection tool was developed to identify pre-service teachers' specialized content knowledge of integer addition and subtraction by taking the relevant literature into account. The data collection tool which included all three dimensions mentioned above consisted of a total of 16 items. Written responses of the pre-service mathematics teachers were received in two separate sessions. Following the first session held for the representation dimension, another session was held after a while for the explanation and justification dimensions. Both sessions took about 60 min. Table 1 shows some examples taken from this data collection tool.

Table 1. Examples from data collection tool

	Explanation-Justification	Representations		
		Real world context	Number line	The counters
Integer addition	Tuna found the sum (+5) + (-3) = -2. Do you think his answer is correct? (If you think the answer is not correct) How do you explain it to Tuna?	Write a real world situation that can be represented by the expression (-5)+(-3) =?	Use a number line model to show (-5) + (-3) =?	Use counters to model (-5) + (-3) = ?
Integer subtraction	Ceyda answered the subtraction operation (-5)– (+3) = -2. Do you think her answer is correct? (If you think the answer is not correct) How do you explain it to Ceyda?	Write a real world situation that can be represented by the expression (-5) - (+3)=?	Use a number line model to show (-5)- (+3) =?	Use counters to model (-5) - (+3) = ?

It is possible to construct four different types of questions using numbers with the same signs and numbers with the different signs. However, the data collection tool used in this study included 2 types of questions for addition and subtraction operations in order to limit the number of questions asked in line with the above-mentioned purpose. The pre-service teachers were asked two types of questions: numbers with the same signs (i.e. (-5)- (-3)=?) and numbers with the different signs (i.e. (+5)+(-3)=?). While developing the data collection tool, the three dimensions mentioned above were taken into account and the representations, i.e. counters, number line and real world situations, were used. Besides, the pre-service teachers were asked to provide justification and explanation by taking account of the answers of the wrongly answered questions. Finally, interviews were held with three pre-service mathematics teachers (coded as $PEMT_1$, $PEMT_2$, $PEMT_3$) who performed differently in terms of the representation ability in order to examine their specialized content knowledge of integer addition and subtraction in more detail.

Data collection and data analysis

The analysis of the data collected was performed at three stages: (i) identifying whether the answers of PEMTs were correct/wrong; (ii) identifying and coding the representations, justifications and explanations used by the pre-service teachers; (iii) qualitative and quantitative interpretation of the data (Creswell, 1994). First, a framework including the representation, justification and explanation dimensions was formed for the data analysis in line with the theoretical framework of the study. For the analysis of the data in the representation dimension, we tried to identify correct/incorrect uses in all three representation types. In the justification dimension, first the answers of pre-service teachers to the relevant questions were analyzed. Incorrect justifications were analyzed in the "incorrect" category, while correct justifications were analyzed under two subcategories, i.e. incomplete and complete. For example, any "yes" answer to the question "*Tuna found the sum (+5)+(-3)= -2. Do you think his answer is correct?*" was analyzed as incorrect. The statements like "*I would want him to move 5 steps forward and then 3 steps back on the number line. And I would make him to express that point as the result of the operation.*" were considered as incomplete, while statements like "*I would explain this on a number line model. First I would tell Tuna to face towards the positive direction while standing at point zero. And then I would tell him to move 5 steps forward and then 3 step back without changing his direction. I would tell him that the point he is at is the result of the operation*" were analyzed in the complete category. Besides, the statements in which only incomplete or complete justifications were considered were analyzed under three categories, i.e. only operational, partial/incomplete and complete statements. However, the data obtained for the reliability of the study was coded twice by the researcher at different times and the coding reliability percentage was found as 86%. For the remaining 14% difference, the researcher and a field expert discussed and reached consensus. The data obtained were organized and defined based on the framework formed accordingly. The data were described in detail by using direct quotations, where necessary. At the final stage, the findings were explained and correlated.

Results

Pre-service elementary mathematics teachers' ability to explain integer addition and subtraction using representations

Table 2 shows the data on PEMTs' ability to explain integer addition and subtraction using representations.

Table 2. Frequency table of representations component

Types of representations		Number line		The counters		Real world context	
		Correct	Incorrect	Correct	Incorrect	Correct	Incorrect
The addition operation	(+5)+ (-3)	18 (%42.9)	24 (%57.1)	26 (%61.9)	16(%38.1)	20 (%47.6)	22(%52.4)
	(-5) + (-3)	20 (%47.6)	22 (%52.4)	32(%76.2)	10 (%23.8)	30(%71.4)	12(%28.6)
The subtraction operation	(+5)- (-3)	20 (%47.6)	22 (%52.4)	26(%61.9)	16(%38.1)	14 (%33.3)	28(%66.7)
	(-5)- (-3)	16(%38.1)	26(%61.9)	30(%71.4)	12(%28.6)	28(%66.7)	14(%33.3)

A general comparison of the use of three representations shows that the pre-service teachers use counters more accurately in all four operations (Table 1). 32 pre-service teachers (76.2%) used counters correctly while explaining (-5)+(-3), while 30 pre-service teachers (71.4%) used counters correctly while explaining (-5)-(-3). Another remarkable finding is that the rate of correct modelling is lower (61.9%) in explaining the operations with numbers with the different signs ((+5)+ (-3) and (+5)-(-3)) . In other words, the pre-service teachers used counters more successfully while explaining addition and subtraction of numbers with the same signs compared to addition and subtraction of numbers with the different signs.

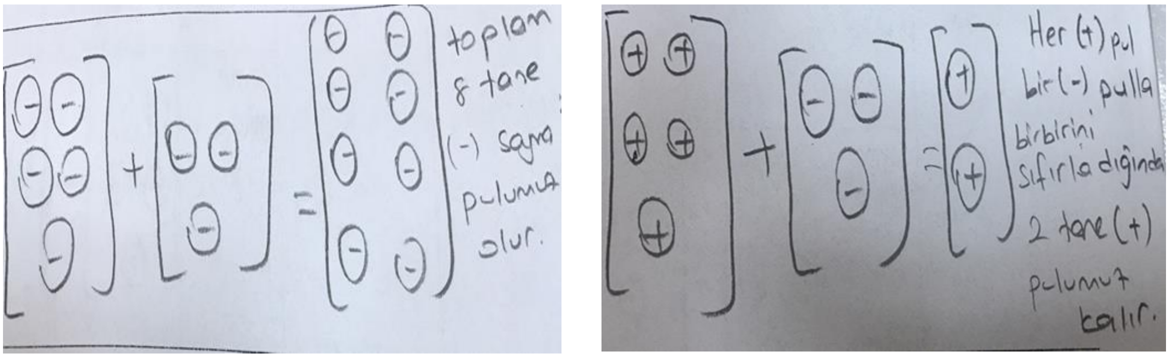

Figure 1. Modeling with the counters of pre-service teachers’

The pre-service teachers had difficulty especially in using counters to explain the process of forming zero-pair while adding and subtracting integers. For example, Figure 1 shows the interview held with a pre-service teacher who correctly modelled (-5)+(-3) by using counters. The statements of the pre-service teacher about the modelling of (+5) + (-3) confirm this finding.

Researcher: Could you explain how did you solve (-5)+(-3)?

PEMT$_1$: I used counters to solve (-5)+(-3).

Researcher: How did you get the answer?

PEMT$_1$: Since the numbers have the same sign, I combined the counters of the same color. The total number of counters gives us the answer.

Researcher: What do you think of this model?

PEMT$_1$: To be honest, I am a little confused about the model.

Researcher: What do you mean?

PEMT$_1$: I am not confused about (-5)+ (-3) because the counters are of the same color. However, for (+5)+ (-3), the process of getting number pairs from counters of different colors is a little complex.

Number line, which is an important model for reflecting the structural characteristics of integer operations, was found to be the representation type that pre-service teachers had the most difficulty in using. Widjaja, Staceyn and Steinle (2011) state that number line is both a didactical model and an important means of representation due to its wide use in various math topics. This representation, which is most likely to be focused on in the studies on teaching integer operations, was correctly used by a maximum of 20 pre-service teachers (47.6%) (Table 1). More than half of the pre-service teachers used this representation wrong/inadequately while explaining all four operations. For example, only 38.1% of the pre-service teachers were able to model (-5)-(-3) correctly. Figure 2a shows how one of the pre-service teachers who had difficulty in using a number line modelled (+5)-(-3). Figure 2b shows how another pre-service teacher modelled (-5) –(-3) using a number line.

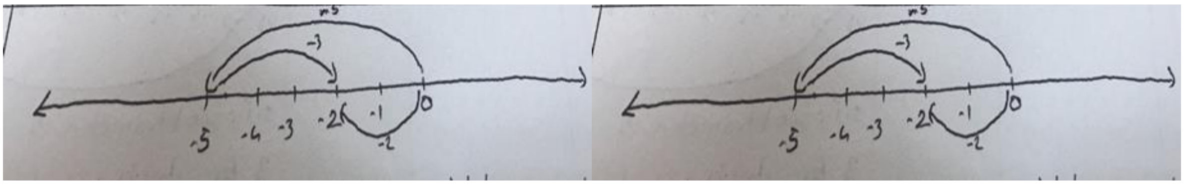

Figure 2 a-b: Pre-service teachers'modeling the operations (+5)-(-3) and (-5)-(-3) using a number line

Figure 2a shows that the pre-service teacher was not able to emphasize the meaning of subtraction when modelling (+5)-(-3) based on the equation (+5)+(+3). Figure 2b shows that the pre-service teacher used arrows pointing towards different directions to express (-5) and (-3). We see that the pre-service teachers mistook the signs of integers for the symbols of addition and subtraction. This forms the basis of the most significant challenges the pre-service teachers had in modelling with number lines. The following interview section shows the confusion that one of the pre-service teachers had during this process.

Researcher: Could you please explain how you modelled (+5)-(-3) using a number line?

PEMT$_1$: Since the first number is (+5), I moved 5 steps forward. Then I turned around and faced towards the negative direction since this is a subtraction operation. As the second number is (-3), I turned towards the direction opposite to the point I was at.

Researcher: What if the operation was addition, not subtraction?

PEMT$_1$: Then we should not change our direction.

Researcher: How did you find the answer?

PEMT$_1$: As a result, I was facing the positive direction. I arrived at 8 units. I mean, the result is +8.

Another point that is remarkable is that PEMTs usually used the metaphor of "walking" on the number line. The following interview section shows that one of the pre-service teachers who found the correct answer had difficulty in explaining how to distinguish between the symbol of subtraction and the minus sign of a negative integer on the number line.

Researcher: Are there two minus signs here, right?

PEMT$_1$: Yes

Researcher: The symbol of subtraction and the minus sign of a negative integer

PEMT$_1$:Yes

Researcher: Do you think there are any difference between these two?

PEMT$_1$: Actually, they are both minus signs. But one of is the subtraction operation's symbol and the other one is the sign of the integer

Researcher: Good… Well, were you able to show this difference on the number line?

PEMT$_1$: Since this is a subtraction operation, I faced towards the negative direction. While expressing (-3), I only turned towards the direction opposite to the point I was at.

Finally, Table 2 reveals that the pre-service teachers do not have sufficient content knowledge of how to construct real world situations. They mostly preferred using assets-debt and check-bill contexts. Mukhopadhyay, Resnick & Schauble (1990) indicated that students most easily use assets and debts as a foundation for negative numbers. The following interview sections shows why $PEMT_2$ preferred using these contexts.

Researcher: Why do you think it is important to relate integer operations with everyday life?

$PEMT_2$: It is important to relate these operations with everyday life. Relating them with real life contexts such as debit-credit helps students develop reasoning about these operations. It helps them concretize.

Researcher: Which contexts do you think will be most effective?

$PEMT_2$: Debit and credit bills

Researcher: Why do you think so?

$PEMT_2$: Because they are the easiest and most practical contexts.

We observed that the pre-service teachers were more successful in using number line to explain addition and subtraction of numbers with the same signs, just like they were in using counters. The rate of constructing real world contexts for both addition and subtraction with number with opposite signs is reduced by almost half compared to adding and subtracting numbers with the same signs. For example, only 14 pre-service teachers (33.3%) constructed a real world context correctly for (+5)-(-3). One of the pre-service teachers who correctly constructed a real life context for (5)+(-3) by stating "*A submarine dived up 5 m below the sea level. To explore more, it dived up 3m more. What is current the position of the submarine?"* did not prefer this context for (+5)+(-3), but constructed a real life context by stating "*I have 5 TL. I received a 3 TL electric bill. How much will I have when I pay the bill*?". The following interview held with this pre-service teacher confirms this finding.

Researcher: You preferred two different contexts for two different addition operations. Is there any particular reason why you did so?

$PEMT_2$: I actually thought to use the same contexts for both operations.

Researcher: Why didn't you use then?

$PEMT_2$: I was not able to express (+5)+(-3) with the sea level context.

Researcher: What is the difference between the real life contexts of (-5)+(-3) of (+5)+(-3)?

$PEMT_2$: We can do the same operation again when the signs are the same. But things get a little complex when the signs are different.

It is possible to establish strong connections with real life contexts and integer addition and subtraction operations. In other words, the real life contexts constructed by the pre-service teachers might offer strong clues about the meanings of these operations. Fuson (1992) divides the meanings of integer addition and subtraction operations into three: combine, change or compare. Representation of a safe in which there are documents showing assets and debts with integers is related to the combine meaning. The difference between profit and loss of a person or a company is related to the change meaning, while comparison of two situations using integers is related to the compare meaning. At this point, we observed that the pre-service teachers highlighted the combine and change meanings in the real world contexts they constructed and put almost no focus on the compare meaning. The context constructed by one of the pre-service teachers for (+5) +(-3) by stating "*If 5 TL credit and 3 TL debit bill is put in a safe, what is there in the safe?"* can be given as an example to the combine meaning.

The context constructed by one of the pre-service teachers for (-5) – (-3) by stating "*My current debt is 5 TL. If I pay 3 TL, how much do I owe?"* is related to the change meaning.

Pre-service elementary mathematics teachers' justifications and explanations for integer addition and subtraction

This section presents the findings obtained from the justifications and explanations of PEMTs for integer addition and subtraction. Table 3 shows the findings from the justifications of pre-service teachers for integer addition and subtraction. 2 pre-service teachers (4.8%) answered "yes" to the question "*Tuna found the sum (+5)+(-3)= -2. Do you think his answer is correct?*". 95.2% of the pre-service teachers answered two addition questions correctly. The rate of answering correctly decreases down to 85.7% for (+5)- (-3) and 81% for (-5)- (-3).

Table 3. Frequency table of justification component

		Incorrect		Incomplete		Complete	
		f	%	f	%	f	%
The addition operation	(+5)+ (-3)	2	4.8	31	73.8	9	21.4
	(-5) + (-3)	2	4.8	27	64.3	13	30.9
The subtraction operation	(+5)- (-3)	6	14.3	28	66.7	8	19
	(-5)- (-3)	8	19	24	57.2	10	23.8

Table 3 shows that the pre-service teachers are more successful in providing justification for addition operations compared to subtraction. It is possible to say that the above-mentioned difficulties the pre-service teachers had in using number line, counters or real-life contexts form the basis for their justifications. For example, the pre-service teacher who used the statement "*I would want him to move 5 steps forward and then 3 steps back on the number line. And I would make him to express that point as the result of the operation.*" while representing (+5)+ (-3) with a number line did not express the position at the beginning and did not highlight the different between the plus sign of the integer and the symbol of the addition operation. Such inadequate representations stem from the lack of representation abilities. When we examined their justifications, we saw that asset-debt and check-bill contexts were more frequently used. We also observed that they had lack of knowledge of the concept of integer while using integers in their justifications. For example, some pre-service teachers neglected the direction meaning of integers by using such statements "*(+5) represents the number of oranges that grow on an orange tree and -3 represents the number of apples he ate or the number of rotten fruits*". These findings offer a clue that the pre-service teachers have limited content knowledge of the meaning of integers rather than of operations.

Only incomplete or complete justifications were taken into account in the explanation dimension of specialized content knowledge. The statements of two pre-service teachers regarding the addition operation were not included in the analysis shown in Table 4. Similarly, statements of eight pre-service teachers regarding (+5)- (-3) and six pre-service teachers regarding (-5)- (-3) were not included in the analysis.

Table 4. Frequency table of explanation component*

Types of explanations		Procedural		Partial explanations		Complete explanations	
		f	%	f	%	f	%
The addition operation	(+5)+ (-3)	17	42.5	18	45	5	12.5
	(-5) + (-3)	14	33.3	21	50	7	16.7
The subtraction operation	(+5)- (-3)	11	30.6	19	55.9	6	17.6
	(-5)- (-3)	14	41.2	15	44.1	5	14.7

* Explanations of pre-service teachers' incomplete or complete justifications

Table 4 reveals that the pre-service teachers mostly highlighted their operational knowledge while explaining operations of integer addition and subtraction. For example, one of the pre-service teachers used the following statement for (+5)+(-3) which is focused on operational knowledge: "*I would subtract the larger number from the smaller one while adding two integers with different signs. I mean, I would subtract 3 from 5 and find 2. I would add the plus sign in from of the larger number before 2.*" When examined from the perspective of operational knowledge, another remarkable point is that the subtraction operation is transformed into addition. The interview held with a pre-service teacher who adopted this approach for subtraction operations confirms this finding.

Researcher: "How do you find the answer to (+5)- (-3)?"

PEMT$_3$: "The signs in (+5)- (-3) are distributed and the answer will be found (+5)+(3)=8"

Researcher: So you transformed subtraction into addition.

PEMT$_3$: "It is quite logical… the numbers with the same signs are added. The result is found by adding the common sign."

Researcher: Does not multiplication, which is a further operation than addition and subtraction, come up here?

PEMT$_3$: "Hmm… yes, but the operation is solved more easily in this way"

Explanations and quotations from the interviews show that the pre-service teachers usually adopt this approach. Among the main reasons why they adopt this approach may be the failure of addressing addition and subtraction operations at the conceptual level. Besides, we observed that explanations of almost half of the pre-service teachers about each of the addition and subtraction operations were incomplete (Table 4). Based upon these findings, we can say that the pre-service teachers do not have sufficient content knowledge of integer addition and subtraction.

Discussion and Recommendations

Studies on in-service or pre-service teachers' content or pedagogical knowledge show that their teaching knowledge is one of the factors that affect their in-class decisions and their preferences in achieving their objectives. This study examines preservice elementary mathematics teachers' content knowledge of integer addition and subtraction in the context of developing multiple representations, explaining mathematical reasons lying behind the concepts and justifying them. The findings of the study present some indicators about how PEMTs understand integer addition and subtraction in multiple representations as well as about the changes that might be made in the teaching programs. When their representations, justifications and explanations were examined together, it was found that the pre-service teachers do not have sufficient content knowledge. We believe that studies on this issue will contribute to pre-service teachers' professional development, and thus the assessment of the important elements of SCK. In this sense, the findings of this study point out some critical points.

Comparison of the pre-service teachers in terms of their representation ability shows that number line is the representation model that the pre-service teachers had the most difficulty in using, while the counters are the means of representation that they were most successful in using. This finding is in parallel with the findings of previous studies (Birenbaum & Tatsuoka, 1981; Liebeck,1990) which revealed that counters were more effective than number lines. Janvier (1983) indicates that the use of counters seems logical to students, claiming that they are hard to use in integer addition and subtraction. However, this study revealed that the pre-service teachers used counters more effectively especially while adding numbers with the same sings. The need for forming zero-pairs while using counters for the operations with numbers with opposite signs and the failure to make sense of this process are the main challenges they were confronted. Moreover, the pre-service teachers usually mistook the signs of integers for the symbols of addition and subtraction when modelling operations with number line. Number line is also important due to its role in providing the opportunity to observe the thinking process of pre-service teachers regarding addition and subtraction and in revealing the potential challenges they may encounter. Another point that is remarkable is that PEMTs usually used the metaphor of “walking” on the number line. Billstein, Libeskind, and Lott (2010) indicate that the metaphor of "walking” on the number line is frequently used to visually enact integer operations. Diezmann, Lowrie, and Sugars (2010) emphasizes the importance of students’ development of appropriate representational apprehension.

Another remarkable finding is that, while using both counters and number lines, the pre-service teachers were not able to make sense of the concepts of magnitude and sign which are the characteristics of integers.

Researcher: What do you think positive and negative integers have in common?

PEMT$_3$: Actually, they are both integers.

Researcher: Well, what could be the difference between positive and negative integers?

PEMT$_3$: They are on the opposite sides of zero.

Researcher: I want you to think how you would model (+5) and (-3) using a number line and counters. (Showing the previous modellings) What is different between the number line and counters when modelling the sign and size of the number?

PEMT$_3$: Hmm. Now I realize. I used counters of opposite colors to show them. But I used arrows when showing them on the number line.

This interview reveals that the pre-service teachers have a lack of knowledge of the meanings lying behind modelling with the counters and number lines. The interviewed pre-service teacher stated that the sign of an integer is shown using arrows on the number line or using counters of opposite colors. However, the preservice teacher was not able to state that the magnitude of the number should be shown with the number of counters or the length of the arrows.

Steiner (2009) points out that pre-service teachers find it more difficult to correctly write word problems for integer addition and subtraction than to perform the operations symbolically. This study found that the pre-service teachers mostly preferred using debit/credit and gains/losses contexts and were more successful in constructing real world contexts for operations involving numbers with the same signs. Furthermore, it is possible to establish strong connections with real life contexts and integer addition and subtraction operations. We also observed that, among the categories proposed by Fuson (1992) for the meanings of integer addition and subtraction, the pre-service teachers highlighted the combine and change meanings more, neglecting the compare meaning.

Most of the pre-service teachers were found to have had difficulty in explaining the reason to someone who did not solve an integer addition or subtraction equation correctly. Their justifications were usually found to be incomplete. They were observed to be more successful in justifying addition operations than subtraction operations. The studies (Nunes, 1993; Shawyer, 1985) show that subtraction is harder to understand than addition due to the multiple use of the minus sign “-”. Actually, this not only holds true for integers, but also for all number groups.

Finally, the findings reveal that the pre-service teachers mostly highlighted their operational knowledge while explaining operations of integer addition and subtraction. Symbolic presentation of integer addition and subtraction operations without associating them with the real world is what Skemp (1987) call ‘instrumental understanding’ and it can be said to be one of the main reasons for rote learning. In this sense, proper use of number line and counters, together with proper contexts, is of critical importance in the development of relational or conception understanding. The components of SCK defined in this study which considers SCK to be of critical importance in revealing the in-depth knowledge of mathematics teachers or pre-service teachers for mathematics teaching may be reinforced with other subjects and further studies may be carried out with teachers in the context. To help students develop a conceptual understanding of integers and operations, especially pre-service teachers should be provided with the opportunity to improve their mathematical understanding of this topic. Therefore, particular importance should be attached to developing contents for the undergraduate level theoretical and practical courses which aim to improve skills and knowledge regarding this topic.

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