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Understanding the Polygon with the Eyes of Blinds*

Tuğba Horzum [i]
Necmettin Erbakan University

Ahmet Arıkan [ii]
Gazi University

Abstract

This study investigated the concept images of blind students about the polygon concept. For this purpose, four open-ended questions were asked to five blind middle school students. During the interviews, geometric shapes were presented with raised-line materials and blind students were given opportunities to construct geometric shapes with magnetic sticks and micro-balls. Qualitative research techniques applied in grounded theory were used for analyzing documents pictures, which were taken from magnetic geometric shapes that blind students constructed, raised-line materials and researchers' observation notes and interviews. As a result, it is determined that blind students have more than one concept image for the polygon concept. They scrutinized the polygon concept analytically not with a holistic perspective. They were often conflicted about triangle, rectangle, square, circle and circular region whether or not being a polygon. They also encountered with the difficulties associated with the combination of polygon sides' endpoints consecutively.

Keywords: blind students, concept definition, concept image, polygon, geometry education

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[i] **Tuğba Horzum**, Assist. Prof. Dr., Necmettin Erbakan University, Department of Mathematics and Science Education, Konya, Turkey.

Correspondence: thorzum@gmail.com

[ii] **Ahmet Arıkan**, Prof. Dr., Gazi University, Department of Mathematics and Science Education, Ankara, Turkey.

INTRODUCTION

In our changing and developing world, individuals understanding geometry and admiring geometry in nature and its effects in daily life may have the opportunity to shape their own future, because geometry develops individuals' logical and intellectual abilities (National Council of Teacher of Mathematics [NCTM], 2006). So, individuals begin to see, know and understand their physical world (Ubuz, 1999). However, it is obvious that it is not easy to understand the physical world for the visually impaired students (VISs), i.e. partially sighted or blind students (BSs). The communication of mathematical knowledge such as writing algebraic and geometric notations, graphics, diagrams mostly in visual forms (Edwards, Stevens & Pitt, 1995), makes the situation even harder. Moreover, without the ability to read and write the symbols representing the mathematical concepts, mathematics does not exist for VISs (Kapperman & Sticken, 2003). But everyone has a different way of learning. VISs, especially BSs, learn by touching, hearing and most importantly via cognitive processing because of their inability of seeing or partially seeing ability. Therefore, teachers should configure BSs' comprehension of mathematical and especially geometric concepts much better. In fact, mathematics has a socially shared and logically configured conceptual system (Godino, 1996). For this reason, the importance of teaching and learning concepts in BSs cannot be ignored. However, there are some remarkable studies upon teaching mathematics and especially geometry to VISs. According to these studies, VISs, especially BSs faced some challenges. The first challenge is about the absence of textbook, note taking tools, access to symbols and technological assistance like electronic braillewriter, drawing tools, auditory or raised-line or tactile materials (Dulin, 2008; Kohanová, 2008; Pritchard & Lamb, 2012; Rule, Stefanich, Boody & Peiffer, 2011). The second challenge is about contextual situations. For instance, teachers have difficulties about guiding BSs to an understanding of concepts for which they have no context such as estimation, perspective, solids of rotation (Dulin, 2008; Pritchard & Lamb, 2012). In addition, they have difficulties mostly in generalizing, translating activities into mathematical knowledge, and translating and transferring three-dimensional objects into two-dimensional iconic forms (Kohanová, 2008). Besides, the major part of the VISs form their own particular mathematical language in accordance with their conditions and requirements (Kohanová, 2008). The third challenge is about VISs' mathematics teachers and their teaching. Because the mathematics and geometry use so many drawings, graphs, diagrams, symbols, charts, and other illustrations to present content and relationships, these have been particularly challenging for VISs, and difficult for many special education teachers who are unfamiliar with the content (Rule et al., 2011). Nevertheless, the achievement of VISs is directly affected by their teachers' teaching effectiveness. Indeed, teacher of these students are not specially educated in this field in Turkey like in some other countries, they often have to use the 'trial and error' method to find the best way of teaching as Kohanová (2008) referred. Research shows that if the instruction is tailored to individual needs, any students can reach his/her cognitive potential in this process (Pritchard & Lamb, 2012; Spindler, 2006) because, blind individuals' visual images and memories have astonishingly remarkable capacity (Haber, Haber, Levin, & Hollyfield, 1993; Landau, Gleitman, & Spelke, 1981; Landau, Spelke & Gleitman, 1984; Millar, 1985). According to Kohanová (2007), VISs can explain objects like cube, prism, pyramid, cylinder, triangle, circle, trapezoid, square, and rectangle better and more precise than the sighted students in the concepts of shape and location. Moreover based on senses, they can name and distinguish basic geometric shapes and solids (Srichantha, Inprasitha & Ariratana, 2008). Furthermore, Kennedy (1993) observed the ability to draw 3D objects of BSs. This capacity of BSs can be used for mathematical thinking and understanding. By this means, the opportunities for BSs to understand geometry and thus physical world better can be provided. For this reason, educators need to understand how VISs understand the concepts and need to know what is going on in VISs' minds to get a quality education. In fact, there is disharmony between concepts formulized by mathematicians and interpreted by students (Tall, 1992). At this point the concept image (CI), which is first brought out by Vinner and Hershkowitz (1980), emerges. Tall and Vinner (1981) defined the CI as one's total cognitive structure related to a mathematical concept such as processes, evokings, features and mental images. These structures are formed as a result of individual's experiences and continuous changes with a new stimuli, that is, they are formed as a result of students experiences occurred with concepts' definitions and examples (Vinner & Dreyfus, 1989). Namely knowing the concept definition does not guarantee comprehension of that concept (Vinner,

1991) but helps understanding concepts truly. Thereby, mathematical knowledge is not a phenomenon transferred only formally. Furthermore, the CI of individuals may be inconsistent about its content and may evoke images appearing contradictory to each other in different times that a student is not aware, if those individuals do not see these relations and transitions (Rösken & Rolka, 2007; Tall & Vinner, 1981). The reason for this may be the idea of Vinner and Dreyfus (1989) that 'CI generally consists of typical examples, but not of a concept definition'. In fact, Fujita and Jones (2007) points out that learners' thinking is likely to be influenced by specific examples called prototype. According to Hershkowitz (1990), each concept has one or more prototype examples that are attained first, and therefore, exist in the CI of most subjects. Thus, learners use prototypical judgement either in a visual form or in the form of knowledge of the properties or attributes of the geometric figure (Hershkowitz, 1989, 1990). It should be conceived here that every geometrical concept has also a visual image. In this context, Fischbein (1993) explained geometrical figures as figural concepts because of their double nature. Thus, geometrical figure is not only a visual image but also a concept itself. However, visual images of that concept may be more dominant than the concept (Türnüklü & Berkün, 2013). Therefore, it can be said even in the existence of a formal definition that the relationship between the visual prototype and the definition of the shape is often disconnected. As a result of this, most individuals have difficulties and numerous misconceptions even in naming, classifying and determining the most basic geometry concepts (Akuysal, 2007; Carreño, Ribeiro & Climent, 2013; Erez & Yerushalmy, 2006; Erşen & Karakuş, 2013; Fujita, 2012; Fujita & Jones, 2007; Heinze & Ossietzky, 2002; Hershkowitz, 1989; Horzum, 2016, 2018; Kartal & Çınar, 2017; Monaghan 2000; Shaughnessy & Burger, 1985; Türnüklü & Berkün, 2013; Ward, 2004). One of the geometrical concept challenging for individuals is polygon. For example, 7th grade students did not accept the polygons with special names such as rhombus, square rectangles as polygons (Akuysal, 2007). According to this, the students thought that the geometric shapes with five sides or more are polygon. Carreño and her colleagues (2013) found that when a preservice teacher was asked to draw polygonal and non-polygonal shapes, she had a regular and convex polygon image. The results obtained from various age groups showed that individuals have problems with hierarchical classification of quadrilaterals and with naming, drawing/constructing, defining polygons (Erez & Yerushalmy, 2006; Erşen & Karakuş, 2013; Jujita, 2012; Fujita & Jones, 2007; Heinze & Ossietzky, 2002; Monaghan, 2000). Related to these results, the 'horizontal length' of typical images of rectangles disturbed individuals' perceptions of 'inclusion relationships' of quadrilaterals and caused them not likely to accept that a square is a special type of rectangle (Hershkowitz, 1989; Monaghan, 2000; Ward, 2004). Ward (2004) reported that K-8 preservice teachers' CIs for right triangles were comprised of gravity-based right triangles, and for hexagons were regular, convex, and gravity-based too. In addition, some researchers (Kartal & Çınar, 2017; Shaughnessy & Burger, 1985) reached the conclusion that defining polygons with straight sides could not prevent the individuals to decide a shape with curve being polygon. Individuals also have misconceptions related to the sum of interior and exterior angles of polygon. Students thought the sum of the exterior angles increases as the number of sides increases just like the interior ones (Hadas, Hershkowitz & Schwarz, 2000). Despite all these difficulties and misconceptions, the concept of polygon has a major place in teaching geometry especially at a primary and middle school level. It is important that the side, corner and polygon CIs are structured in a healthy way in the minds of the individuals up to middle school level. Because of containing most of the geometry concepts and being included in many concepts such as prism, pyramid, teaching and learning polygon is significant effect on geometry education. Considering that sighted individuals, even preservice teachers and teachers have conceptual difficulties, it is important to study polygon concept in VISs to provide the equitable teaching. Besides, the experiences that the first author of this investigation had in her volunteer lessons for three years to the VISs have directed to this study. For example, some of VISs mentioned that there are polygons called in Turkish as 'birgen', 'ikigen', 'üçgen' and so on. These naming were obtained by suffixing the word '-gon' at the end of the numbers. In this way, it was observed that these VISs ignored the attribute of having 3 or more sides which is the most critical critical attribute for the polygon. Thus, learning the images of the VISs about the polygon may give clues to mathematics teachers, special education teachers and teacher educators for teaching geometrical concepts to VISs.

Mathematics education and the visually impaired students

Six basic principles for a qualified mathematics education were introduced in reform movements, which were started by NCTM in 1989 and have reached today: Equity, teaching program, teaching, learning, evaluation and technology. One of these, enacting equity in education has increasingly become a common concern for practitioners and policy makers in the World (Herrera, Jones & Rantala, 2006, p.7). Thus, the importance of education for every person is emphasized with some documents such as *Individuals with Disabilities Education Act* (IDEA, 2007) which were signed as *The Education for All Handicapped Children Act* in 1975 and were organized a few time, *Salamanca Statement* (UNESCO, 1994), *Principles and Standards for School Mathematics* (NCTM, 2000) and *No Children Left Behind Act* (2001). That is to say, many countries such as Australia, Canada and USA support disabled individuals' education through regulations and state policies and there are also detailed regulations for disabled individuals' education in Turkey as well. *Basic National Education Act* (with number 1739) and Special Education Services Regulation (MoNE, 2006) are the most important ones. For example, *Basic National Education Act* (with number 1739) indicates that "all children have right to have education regardless of their disabilities". Considering the fact that many students are struggling with mathematics and fail over and over again (PISA, 2015; TIMMS, 2016), it can be guessed that disabled students may have to cope with more difficulties than even from students without any disability. But whatever their personal characteristics, previous experiences or physical difficulties, all students must find opportunities to receive mathematics education and learn mathematics (NCTM, 2000, p.12).

Visual impairment is one of the disabilities that individuals may face. VISs, especially BSs cannot benefit from their visual senses, and so they are disadvantaged because they have to learn concepts only related to hearing, touching etc. As a matter of fact, visual impairment affects a person's development in other areas somehow and generally all dimensions of development are negatively affected by disabilities (Brian & Haegele, 2014; Erol, Riedler & Eryaman, 2016; Lieberman, Houston-Wilson & Kozub, 2002). While mental functions of most of them are normal, their cognitive, social and language skills are negatively affected as they cannot receive and understand any visual information from their environment (Kızar, 2012; Sucuoğlu & Kargın, 2006). For that reason, they may face many serious problems especially about area and space concepts. VISs can use the reflection of a voice through their hearing ability and know the distance and direction of the object; however, it may not be possible for them to know what the object is without touching it or asking information from other persons with direct experience (MEGEP, 2013, p.13). However, all concepts cannot be learned through touching and hearing. Especially abstract ones are difficult to learn for VISs. According to Rule and her colleagues (2011), VISs have difficulties because of the abstractness of mathematical or geometrical concepts. Nevertheless, there are limited number of studies that handle VISs' understanding about concepts such as triangle and some other geometrical concepts (Argyropoulos & Argyropoulos, 2002; Horzum, 2013, 2016). It is determined that VISs have the right understanding as well as misconceptions about the related concept. For example, they knew the angle as a point, a side, an interval between the rays/line segments (Argyropoulos & Argyropoulos, 2002; Horzum, 2013, 2016). VISs also believed that the interval between the rays/line segments is a line segment and this situation led VISs to perceive sides as angles in triangles, and even triangle as a straight angle (Horzum, 2013, 2016). Consequently, correct expression of definition is important in academic sense, scrutiny of personal descriptions and properties of concept directed by the conceptual understandings are also significant. Thus, one can gather deep knowledge upon BSs' learnings by providing clear description of cognitive structure about concepts. With this, BSs' conceptual comprehensions and misconceptions can be revealed. In this regard, as one of the basic concept of geometry and being introduced since primary school and containing most of the geometry concepts, the polygon is handled in this study. In this investigation, what the images of BSs for the polygon are investigated through qualitative approaches. It is important to emphasize that comparing the sighted and visually impaired individuals is not the aim of this study.

METHOD

This qualitative investigation is a case study to investigate deeply the CIs related to the polygon of BSs. A case study requires investigating and analyzing a group or an event deeply (Merriam, 2009). The case in the current study is the BSs chosen to determine CIs related to the polygon.

Participants

This investigation was realized with five congenitally BSs aged between 12-15 studying at a school for the visually impaired in Central Anatolian region of Turkey. All participants were selected through in-course/out-course observations, the opinions of mathematics teachers of visually impaired students and voluntariness of students and named as S_1,S_2,S_3,S_4,S_5 for anonymity and confidentiality reasons. During the 2-month observations, the first author observed the social activities, friendship relationships, and habitats of the students in their boarding school without interfering with the lives of the participants. Besides, The performances in the mathematics courses during 2-month observations were a factor in the selection of these stıdents. The participants volunteered to participate and they were students who had taken the mathematics courses which the polygon handled in elementary education (1-4 grades) and the secondary school 5 grade. According to school records, all of these students were successful in the overall assessment of the mathematics courses in the earlier years.

Data collection tool and process

Different data collection methods were used in order to get various information about BSs' understanding of the polygon. The semi-structured interviews were applied as the main data collection tool. Besides, observation notes and pictures of geometric shapes created with magnetic materials (micro-balls and magnetic-sticks), raised-line materials and geometry board were used as supporting data collection tool. In the interviews, BSs were asked four different questions to solve (see Appendix). While preparing these questions, three justification were taken into consideration. The first one of them is that the students' awareness about the existence of concave and complex polygons is beneficial in terms of learning the concept of polygon (Argün, Arıkan, Bulut & Halıcıoğlu, 2014, p.86). Second, the identification of examples and non-examples of a given concept, problem solving and mathematical proofs can encourage students to use the formal concept definitions (Vinner, 1991). Third, non-examples were useful if they were sequenced by matching them with examples in such a way as to focus on the critical attributes (Tennyson, Steve & Boutwell, 1975). Then for the validity of interview questions, it was consulted with the three experts having a PhD degree in mathematics education, a geometry scholar, a PhD student in mathematics education, and two teachers teaching mathematics to VISs. In order to determine the appropriateness of interview questions, a pilot study was realized with two low–vision students who took mathematics courses in elementary, middle and high schools and required corrections were made. As a result, in the first question BSs were asked to draw (meaning here to construct) three polygon and then they were asked to indicate why these shapes are polygon. In the second question, BSs were asked to determine whether the given 15 different shapes (examples, non-examples) are polygon or not and to state the reasons for being or not being a polygon. Thus, it was questioned which definitions and characteristics were used by BSs for the concept of polygon. It is important to specify that 5^{th},6^{th},7^{th},10^{th},12^{th},13^{th},14^{th},15^{th} figures do not provide the definition of polygon given in the data analysis section. For example, the 5^{th} figure, in other words circle, does not have any sides which should be line segment for being a polygon. The 6^{th} figure specifies a rectangular area. In the 7^{th},12^{th} and 14^{th} figures, combining of the sides at the endpoints is not provided. The consecutive junctions of the sides is not provided in the 10^{th} and 15^{th} figures. The 12^{th},13^{th} and 14^{th} figures have curve segments. Besides, 4^{th} and 9^{th} figures are complex polygons and 1^{st},2^{nd},8^{th} and 11^{th} figures are non-convex polygons. The third and fourth questions were used to deepen the results of the first two questions. According to this, it was aimed to obtain deeper information about the participants' CIs about the angles of polygon through a concave quadrilateral given in the thirdquestion. Finally, it was aimed to obtain deeper information about the participants' CIs related to the sides properties of polygon with the fourth question.

During the interviews, the participants was requested to use geometry board or the magnetic materials to constitute the geometrical shapes. So, it is assumed that constructing geometric shapes with these materials is equivalent to drawing of that shape. Because, Klingenberg (2007) specified that the drawing is difficult for blind individuals. In addition, some of the questions were presented to the students through the raised–line materials. Before the interviews, magnetic and raised-line materials were introduced and BSs were asked to think of the micro-balls as points and magnetic-sticks as thin threads. This is second assumption of this investigation. Moreover, participants were allowed to interact with these materials for a while before the interviews. Participants were asked to answer questions with as much detail as possible. Interviews were audiotaped and videotaped with permission of participants and their families. Additional interviews were performed in the situations in which the students were not clear. BSs' non-verbal behaviors during interviews were also observed. Lastly, document analysis included analyzing every kind of representation used by participants when answering the questions such as pictures which were taken from magnetic geometric shapes that BSs constructed and raised-line materials. For this, the audio and video-recorded data were transcribed and the data were checked by the researchers. Later on, the pictures of geometrical shapes constructed by participants, observations of researchers and the implications of students were inserted to the data into transcribed data.

Data analysis

The data were first one-by-one and then comparatively scrutinized and the analysis was supported with data analysis techniques of grounded theory- that is open coding, axial coding and selective coding. The process of data collection and analyzing was carried out simultaneously. In this process, the answers for the first and second question were separately categorized and sub-categorized. These categories and sub-categories were compared with each other. After this coding process, common categories were formed for the first and second questions. However, after the axial coding it was found that some of the categories were still unclear, undiscovered or superficial. Therefore, additional interviews were applied to clarify these uncertainties and to deepen the superficial statements. For example, some of the BSs who defined polygon as a geometrical shape, having sides referred in the first question that the sides of the polygon should be line segments. But some of them did not say anything about the nature of the polygon' sides and referred in the second question that the 5th and 12th figure were polygon. In this situation, it was needed to ask these BSs about the nature of the polygon's sides. Then 3rd and 4th questions were applied to deepen the obtained categories which were about the understanding the polygon as a geometrical shape having sides and angles. Similar methods such as analyzing firstly the data from third question and then data from 4th question, after that additional interviews were applied. After all these processes were completed, the categories and sub-categories were combined to form common categories and they were compared among the students constantly. Finally, additional interviews were held to determine the dominant images of BSs who used contradictory expressions at the same or different times. In these interviews, to determine the dominant CIs of BSs, the questions that created contradictions in different time periods were asked at the same time. When this method does not work, interviews were conducted with questions similar to the questions that constituted the contradiction. However, if BSs still specified conflicting expressions, it was assumed that the BSs had two different CIs in their minds.

During the data analysis process, understandings of BSs on polygon were merely focused on. According to this, data analysis was done by taking into account the definitions of polygon, concave polygon, convex polygon, complex polygon, and the exterior angle of a polygon. According to Argün and his colleagues (2014, p.84), the definition of the polygon is as follows: Let's consider three non-linear $A_1, A_2, \dots A_n$ points (corners) in the same plane, where n is a natural number, n≥3. $[A_1A_2] \cup [A_2A_3] \cup \dots [A_{n-1}A_n] \cup [A_nA_1]$ is called polygon in which $[A_1A_2], [A_2A_3], \dots, [A_{n-1}A_n],\ [A_nA_1]$ are line segments. Yet, there is widespread opinion among the mathematics educators that the circle is a polygon, but the definition of the polygon concept handled in this study reveals that the circle can not be a polygon. Because, we can not construct the circle through the points with finite number and line segments combining them (ibid, p.86). Besides, polygons can be classified as convex and non-convex polygons. According to Downing (2009, p.254), 'most useful polygons are convex polygons; in other words, the line segment connecting any two points inside the polygon will always stay completely inside the polygon'. And he describes the non-convex polygon as a polygon that is not convex (concave), that is, it is caved in. On the other hand, complex polygon is a polygon whose sides cross over eachother one or more times ("TurtleDiary.com", n.d.; "Tutorvista.com", n.d.). Lastly, an exterior angle of a polygon is an positive angle formed by one side of the polygon and the line that is the extension of an adjacent side (Downing, 2009, p.125).

The coherence was provided by checking the relationships between the themes obtained from data and sub-themes forming the themes. To increase the internal validity of the research, the researchers separately analyzed the data. In addition, a third researcher independently rechecked obtained codes. Using the formula – Reliable = Agreement/(Agreement + Disagreement) suggested by Miles and Huberman (1994), consistency percentage was calculated as 89%. To increase external reliability of the research, what was performed throughout the research was comprehensively presented without making any direct comments.

RESULTS

The results are presented in three sections: the first polygon as a geometrical shape having sides, the second polygon as a geometrical shape having angles and the third polygon as a geometrical shape having at least three corners. These themes have been presented as study findings. It is important to note that BSs described the polygon as a geometrical shape– together or separately – having sides, angles and at least three corners. But these images are presented separately here.

Polygon as a geometrical shape having sides

All BSs defined the polygon as a geometrical shape having sides. This situation was called polygon as a geometrical shape having sides. BSs who had this image, took into consideration of some situations such as the nature of sides, the number of sides, junction of sides at their endpoints and finally naming polygons by the number of sides.

With this image, BSs stated that the sides of a polygon might be a line segment and a curve segment. BSs often used the statements such as 'side' and 'line' for the 'line segment' concept. For example, S_1 said 'This circle…is not a polygon because it does not have any sides' for 5^{th} figure at the second question. Similarly, S_1 used following expressions for 13^{th} figure as well: 'This is not a polygon. Is this a side? Does it have 2 sides? No, this is not polygon. [Paying attention to curve segment]". Moreover, S_2, who identified the side with beeline, said that 6^{th} figure was not a polygon and, she defended this with the expressions of 'The side is not evident. Whatever its corner, side, brink are, should be evident'. On the other hand, she stated that 8^{th} figure was a hexagon – comparing with the geometrical shapes she had previously stated that it was not a polygon - with the following sentence: 'But, what this is may come up with something when you have made something with the beelines'.

Majority of the participants who pointed that the sides of a polygon should be line segment agreed upon that circle is not a polygon. On the other hand, as for some of the participants, they stressed on the fact that a circular region cannot be polygon. Unlikely, as for S_3 who said that a circle was not a polygon, constituting a relationship between a circle and a circular region, she claimed that a circle is a circular region. For example, S_3 used following statement for 5^{th} figure at the second question 'Circular region does not have side. Therefore it is not a polygon'.

S_2 and S_5 mentioned that the sides could be both a line and a curve segment uttered the concept of a 'curve segment' in distinctive ways. These expressions emerged in form of 'being round', 'being a circle or circular region'. For example, when the first problem was asked, S_2, saying 'I have tried to make a circle', made following geometric shapes (Figure 1). Upon being asked by the researcher to S_2 why the shape that she made was a polygon, she gave the following explanation 'It begins from the circle and goes on with quadrilaterals, pentagons and hexagons. Circle has no side and corner'. On the other hand, S_2 argued for not being a polygon of the 3^{th} figure at the second question with the following defensive sentences; 'It does not have any sides, nor a corner, if it were a circle. Then, I would say yes'. Thus, although she was also aware that a circle does not have any side, S_2 defended that circle is a polygon.

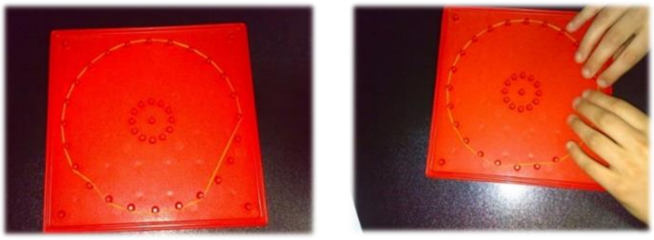

Figure 1. Polygon created by S_2

Secondly, BSs stressed on some criteria regarding the number of sides of a polygon. These are number of sides being at least 2 or 3 or 4 or 5. Here, particularly what the remarkable case is that the BSs use statements that may conflict with each other in different questions/times. For example, while S_2 stated the number of sides of a polygon to be at least 3 and at least 4, S_3 and S_4 defended it to be at least 3 and at least 5 sides. When asked to make a polygon at the first question, first making a square, S_2 made the following statement; 'Triangle is not a polygon. The numbers of sides is not much. But, as far as I could remember quadrilateral, pentagon, hexagon, and octagon, these are polygon. In my opinion, number above four are in it'. However later on, forming a circle and triangle, she used the expressions of 'This time, I am pricely decisive. There is nothing with 2 sides; one that has higher than 2 sides certainly is a polygon I am sure'. On the other side, for the same question S_3 formed respectively hexagon and pentagon and then said 'Rhombus is four-sided...yet I am not sure. Can it…be a polygon…I guess not though'. In this way, S_3 reported that a polygon should have at least five sides. However in the additional interview, S_3 referred the presence of at least 3 sides saying 'We used to say whether or not it was a polygon according to the number of sides. It is supposed to be greater than two. Because, two does not constitute a full shape, there is a gap in-between'. Another understanding related to the number of sides, the situation expressed by S_5 is that the polygon should have at least two sides. At the first question, S_5, first making a square, said that the square consists of four sides. Then saying, 'If it has 2 sides, it likes a little more polygon. But if it was a one-side shape, people would not call it polygon', she pointed that a polygon should have more than one side.

Thirdly, BSs stated that the polygons are named according to numbers of sides that can be differentiated. Here the polygons BSs emphasized were the triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, decagon and hendecagon. According to this, BSs who stated that if a polygon has three sides, then it will become a triangle, expressed that triangles are named according to their side sizes. Participants pointed the types of triangles such as 'equilateral', 'isosceles' and 'scalene' triangles referring to the equality of side lengths. Participants mentioned that another name for the equilateral triangle is smooth triangle and angle measurements hereof are equal. On the other hand, BSs stated that in case where a polygon has four sides, it would be called quadrilateral. BSs stated that the rectangle is different from square and it has two short and two long sides and it is not a regular polygon. For example, S_3 made the following explanation for 10^{th} figure 'That inner square and outer one as well. It can be either a square or rectangle. Yes, both of these are square. They have four sides. Unlikely the rectangle, all of its sides are equal in size. Two of it in the rectangle are equal namely, two of them are short, and two of them are long'.

For the pentagon, hexagon and octagon, BSs explained mentioning whether or not they are regular. For instance, S_2 said for pentagon 'Pentagon. Even if its five sides are not equal, you know it is required to have five sides. Pentagon has two types: Ones with equal sides and ones with unequal sides'. On the other hand, S_3 made the distinction for the hexagon: regular and damaged hexagon. It can be understood here that S_3 has an understanding of a hexagon with equal sides and equal angles and S_3's understanding as for the hexagons which do not match with this situation, it can be comprehended with her statements that they are considered to be damaged:

'Hexagon has six sides. Even if we change it in this way, it becomes a hexagon again. We can increase their lengths. Surely, you are required to increase all equally. There seems to be damaged if the sides are different, it would be okay even if it were wider [showing gap between two sides] though, it has six sides too. It gets a little damaged'.

Lastly, all participants addressed to the junction of the sides at their endpoints. Except from the S_3, all students gave conflicting statements for combining of the sides at the endpoints. For instance, for the square he made, S_1 emphasized combining of the sides at the endpoint with the statement of 'Let's assume that this side does not join with the other one. It does not become a square'. On the other hand, for 7th figure at the second question S_1 said 'Triangle…yes. One second…here is not existing [getting surprised]. When here is missing...did you intentionally eliminated here? Then, this is not a polygon, it has 2 sides. It does not have the third one...then polygon...even so polygon...1, 2, 3'. Hence, even though S_1 firstly paid attention junction of the line segments at the endpoints, later on he stated that this combination does not have a characteristic of being a polygon. As a result, in process of forming the polygons, while paying attention for the junction line segments at their endpoints and even orally defending this, it can been seen that S_1 did not pay attention to such combination in determining whether or not a given shape was a polygon.

Some participants who took into consideration of junction of sides at their endpoints paid attention the sides not to be linear as well. For example, S_3 tried to place the sides of hexagon and pentagon she formed in a negative direction (Figure 2).

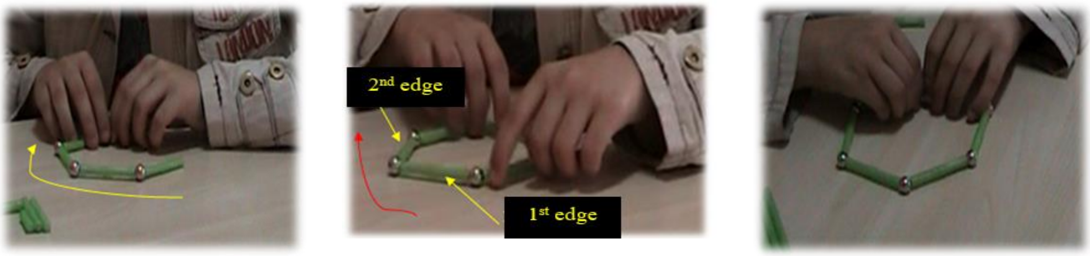

Figure 2. S_3' understanding regarding the placement of sides

Polygon as a geometrical shape having angles

BSs who had a polygon image as a geometrical shape having angles, addressed to the cases where the sum of angle measurements differed and kinds of different angles existed. However, BSs except from S_5 stated that number of angles in a polygon can alter. Here, BSs questioned the existence of equality in terms of the number of interior and exterior angles by addressing the relationship between interior and exterior angles. In this place, S_2, who referred four interior angles, made contradictory explanations at the third question remarking the existence of first only one and then four exterior angles (Figure 3).

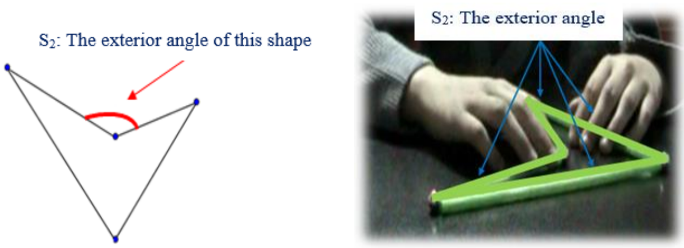

Figure 3. Conflicting explanation of S_2

S_3, who addressed that the number of angles could change, pointed that polygons were named according to number of angles. For example, she made the following explanation 'These are two triangles…look like having been divided from the center. This might be a polygon' for 15th figure at second question. Even though she explained the reason for being a polygon to have numerous sides, counting the angles, she said 'There are two right triangle here. 1,2,3,4,5…6 angle, hexagon'. Another topic the participants mentioned that the polygon have angles as many as its number of sides. S_3 expressed this situation for triangle as follows: 'Since there are 3 angles in triangle, pentagon has 5 angles, hexagon has 6 angles'.

Secondly, BSs emphasized presence of two angle types for the polygon such as interior and exterior angle. Participants pointed that the exterior angle was at the outer region of the polygon and the interior angle was at the inner region of the polygon. In this entire process, they focused on the angle which had smaller size. While talking about interior angles of the polygon, BSs thought that these angles had two different nature. According to this, the angles are sides and interval between two sides. For the exterior angle, participants made different interpretations depending on whether or not the polygon is convex. For instance, S_2 and S_3 addressed that the every sides are angles in a non-convex polygon. S_1, S_2 and S_3 referred that interval between two sides is angle. When hand motions of participants were examined, it was realized that BSs showed these intervals as an arc segment (Figure 4).

BSs also discussed presence of an exterior angle in a non-convex polygon. In a regular polygon, while S_4 was defending the presence of an exterior angle, S_1 and S_2 claimed that there could not be. This situation might be related to BSs' description of the exterior angle as an angle facing outward.

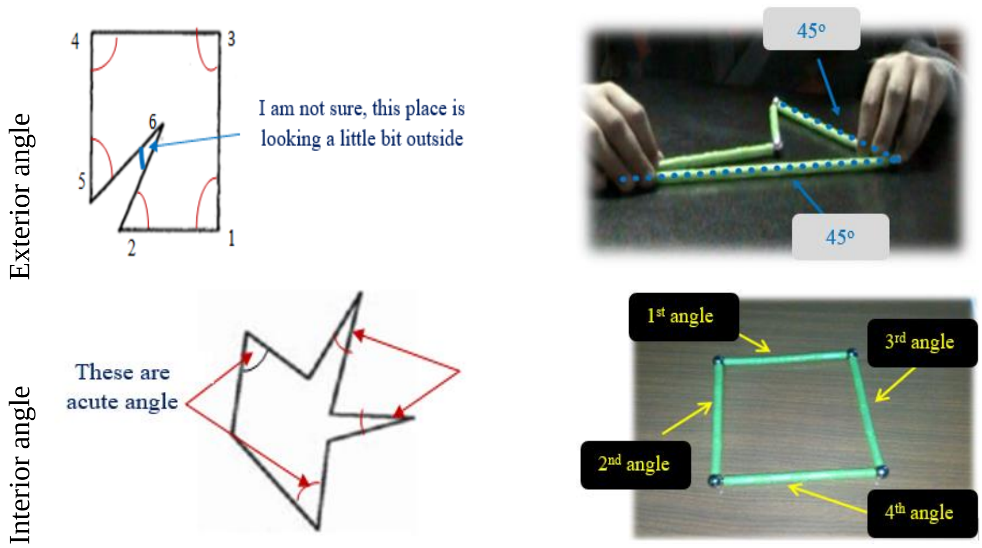

Figure 4. Interior and exterior angles mentioned by BSs in a polygon

Thirdly, BSs mentioned that sum of interior angles changes according to the number of sides. In this case, if the number of sides increases, the sum of interior angles increase too. In this place, S_2 and S_3 tried to find out sum of interior angles with more than one method. On the other side, while all of the participants excluding S_5 used memorized expressions for the sum of interior angles, S_2 tried to determine that sum according to the polygonal areas inside the polygon, S_5 with only *(n-2).180^o* formula and finally S_3 by dividing polygon into the known polygons. For instance S_5 said 'We have learned it from our teacher. Our formula is as follows. If it has five sides, I calculate it as five minus two equals to 3. Three multiplied by 180 produces the result of 540. Our formula (n-2).180'. On the other side, BSs who used memorized expressions, addressed to triangle and quadrilateral. According to this, the sum of interior angles of a triangle is 360^o or 180^o, and for quadrilateral it is examined with respect to the convexity. If the quadrilateral is convex, the sum of interior angles of a quadrilateral is 360^o, if it is non-convex, it is random. For example, S_3 said, 'When it has four sides, the interior angles' measurement becomes 360. If we had done one more side and create a pentagon, the sum of its interior angles increases…the more the number of sides, the larger the interior angle'. Figure 5 demonstrates the images of S_1's memorized or random expression of the sum of interior angles of a non-convex quadrilateral.

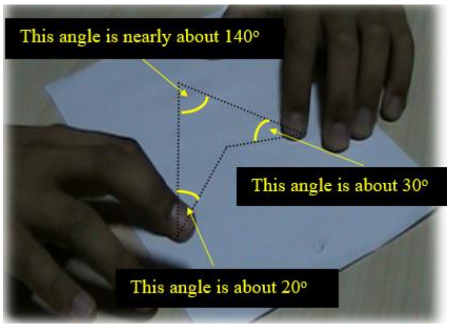

Figure 5. Randomly expressed sum of interior angles by S_1

BSs also stated that the sum of exterior angles of polygon might change. In this condition, the convexity of the polygon played a role. For a convex polygon, S_3 and S_4 stated that as the number of sides increases, so does the sum of exterior angles. On the other side, for a non-convex polygon, while S_4 pointed that each one of exterior angles is 180^o, S_2 and S_3 expressed that the sum of exterior angles should be equal to the sum of sides lengths. For instance, S_4 said for triangle, 'The exterior angle of every shape would be 180…there were three exterior angles…I think it becomes 540'. On the other hand, the explanation of S_3, who described the side as an exterior angle, is given with Figure 6.

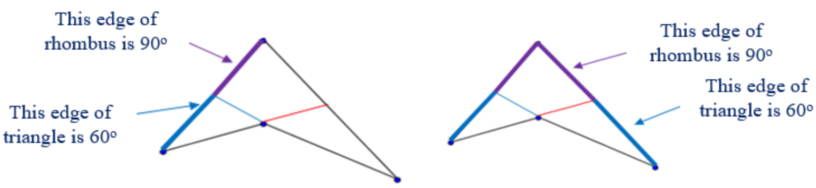

Figure 6. Calculating process of the sum of exterior angles by S_3

Polygon as a geometrical shape having at least three corners

BSs made statements thirdly about the three or more corners. According to this, some of participants stated that they named the polygon according to the number of corners and they referred the nature of corners. For instance, S_4 pointed to the existence of at least three corners in a polygon. S_2 pointed that naming the polygons depends on finding how many corners the polygon has and suffixing the word '-gon' at the end of that number. For example, counting the corners S_4 said, 'Those combined places are corners. Starting from there…1st,2nd,3rd…and 4th corner…quadrilateral…I mean, polygon' to determine whether 4th figure at the second question is a polygon or not. In additional interviews, saying 'Quadrilateral is a polygon. It is a closed shape...It has more than 3 corners' S_4 emphasized that the number of corners should be at least three. On the other hand, S_2 described the concept of corner as an angle and a junction of two sides to the outward (Figure 7).

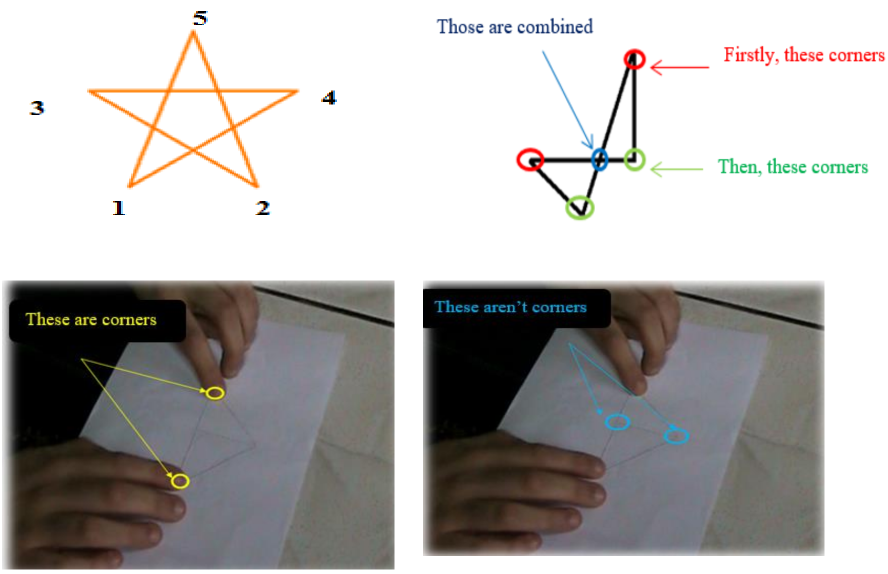

Figure 7. S_2' corner understanding in a polygon

DISCUSSION AND SUGGESTIONS

This study investigated the CIs of BSs about the polygon. These CIs make it possible to draw some profiles about BSs. It cannot be concluded that the participants had completely wrong understandings or they had completely deficient knowledge about the polygon. Even if comparing the sighted and BSs is not the aim of this study, the most substantial outcome required to be paid attention to is that BSs are not cognitively different from the students who see completely. Because the majority of the findings obtained in this study are similar to the results obtained from studies conducted with sighted individuals in different age groups. Indeed, some researchers (Haber et al., 1995; Landau et al., 1981; Landau et al., 1984; Millar, 1985) state that BSs have an amazing capacity for visual and mental images in conceptualization as in this study. Accordingly, it is found that BSs have more than one CI for the polygon. BSs' CIs about polygon provided clues of what the understandings that BSs had. In this context, it is determined that BSs scrutinized the polygon analytically not with a holistic perspective. Consequently, as with most research studies accomplished in the literature (Erez & Yerushalmy, 2006; Fujita, 2012; Heinze & Ossietzky, 2002; Horzum, 2013, 2018; Monaghan, 2000), it is identified that the participants used partial classification. For example, side properties came into prominence for square and rectangle, and so the participants defended that the square is not a rectangle. Furthermore, the rectangle definition of some participants 'quadrilateral possessing two long and two short sides' shows parallelism with the study by Fujita and Jones (2007). Nevertheless a clear result cannot be given since the understandings related to quadrilaterals were not studied in this study. Thus, a comprehensive research is needed regarding how BSs categorize the quadrilaterals and what they know about the quadrilaterals.

BSs addressed the descriptions for the polygon related to presence of only sides, only angles or only at least 3 corners and the descriptions in which two or three of these are together as well. In these images, BSs have correct understandings as well as some misconceptions and difficulties because of not knowing the formal definition of the polygon or failing to apply it. It is thought that this situation caused BSs to describe the polygon in different forms in the different time zones. Indeed, researchers (Rösken & Rolka, 2007; Tall & Vinner, 1981) stated that inconsistent images appearing contradictory to each other in different times might evoke. In the cope of this investigation, it can be suggested that teachers of the VISs should often emphasize and remind VISs the definition of the polygon in geometry lessons especially when teaching related concepts such as triangle, quadrilateral, prism, pyramid.

The most obvious of BSs' difficulties is that BSs fell into contradiction regarding whether a circle, circular region, triangle, rectangle and square are a polygon. This contradiction canalized BSs to think at least how many sides are required in the polygons. Following this reasoning, some participants state that a polygon should have at least two sides; some of them state at least 3, 4 or 5 sides. The use of language may cause these different understandings of the number of sides. These reasons emerged in two ways. First, some participants focused on the word of 'poly' in the polygon and interpreted the word of 'poly' by themselves: such as not being a polygon having single-side. Secondly, some participants focused on the absence of '-gon' suffix at the equilateral triangle, rhombus, rectangle, and square. This situation coincides with the result of Akuysal (2007) obtained from the seventh grade students. This result is 'thinking that the geometric shapes having special title cannot be a polygon and naming the geometric shapes whose number of sides are five or more are polygon'. The other situation related to the use of language, BSs mentioned the polygons with different names such as smooth triangle and damaged hexagon. According to Kohanová (2008), this is because students adapted their own language to their conditions and requirements. Here, the limitations on the use of Turkish language come into prominence. To get accustomed to the mathematical language, BSs' participation of the classroom discussions related to the polygon together with their teachers may contribute to the development of mathematical language skill.

The participants were able to define, name and usually distinguish the geometrical objects as stated by some researchers (Kohanová, 2007; Srichantha et al., 2008). However, this does not mean that BSs had entirely accurate conceptual understanding. Indeed, it was determined that the participants had some misconceptions/conflicts due to the effect of blindness and using frequently their intuitions as well. For example, BSs utilized different strategies to figure out the sum of polygon's interior angle. While some participants used $(n\text{-}2)180^o$ formula, n number of sides, some of them tried to find out that sum by separating the given shape into polygon parts they knew (equilateral triangle, square, rhombus). Some participants used memorized expressions, some stated it with the space inside the polygon. However, for any polygon the sum of interior angles is a function of n, number of sides: $S(n)=(n\text{-}2)180^o$. Besides, students set up an expectation that the exterior angles of polygons are like the interior ones, while BSs are expected to say that the sum of exterior angle of a polygon should be constant and 360^o. They stated that exterior angle should also increase as the number of sides, similar to study of Hadas and colleagues (2000), and each one of the exterior angles is 180^o and the sum of exterior angle is equal to the sum of side lengths. This result may be due to the difficulty of determining the exterior angle in non-convex polygons. Because, an exterior angle of a polygon is a positive angle formed by one side of the polygon and the line that is the extension of an adjacent side (Downing, 2009, p.125). Whereas in the non-convex polygon, negative angles are in question and the sum of exterior angles of a polygon is still constant and 360^o. Here the situation, which particularly attracts attention, is that the sides are identified with the angle. Another misconception is related to side nature of polygon and reflections of this situation. While all participants (mostly while forming the polygon) expressed that the sides were a line segment, at the same time, some of them defended (particularly in the second question) that it might be a curve as in harmony with the studies in literature (Kartal & Çınar, 2017; Shaughnessy & Burger, 1985). As a result of this, the participants express that circle, even a circular region is a polygon. However, majority of participants who stated that the nature of side is a line segment also expressed that a circle and a circular region cannot be polygon either. On the other hand, one participant claimed that the circle and circular region are the same concepts. This situation reveals that there is a serious problem in the concept of dimension and there is a necessity of preparing appropriate learning environments for this. Therefore, teachers of VISs should design activities and teaching environments to overcome the lack of knowledge about properties of polygons and the relation between the different types of polygons. The fourth misconception is related with the nature of polygon corners. Only two of the participants touched upon to the polygon corners and one of them expressed that the corner (namely point) is an angle. The angle concept, which is characterized in different ways throughout the study being interval, point, and side, shows that BSs had difficulties with the angle concept as in the BSs' literature (Argyropoulos & Argyropoulos, 2002; Horzum, 2013, 2016). The fifth misconception of participants is related to whether the sides combine at their endpoints. All participants paid attention to ensure the sides to be combined at their endpoint and stressed on this when constructing the polygon, except only one of them did not pay attention to this combination. And they also defended that such combination was not available at all in the second question. This situation reveals the positive aspect of the magnetic materials used in the study because when forming polygons, the participants used these materials and realized that this combination was provided with the gravitational force of the magnets. Additionally, these materials provided clues for the participants that the sides of a polygon should be a line segment and will not be able to form a circle/circular region with these materials. With this perspective, the magnetic materials used in the study forced the participants to recognize their erroneous thoughts and positively affected them in taking decision. Therefore, the magnetic materials might be useful in teaching particularly polygon and the other geometrical concepts to BSs (also to the students with sense of sight). Finally, BSs stated that regular polygons such as square and regular hexagon could not have exterior angles. This situation shows that the participants focused on the figural representations. According to the statement of Hershkowitz (1989, 1990); this situation is the result of the visual-perceptual limitations which affect the determination capabilities of the individuals. Besides, these dominant images related square and regular hexagon overlap with the results of some studies (Carreño et al., 2013; Ward, 2004). For this reason, by adapting to VISs, irrelevant attributes such as position and dimension can be presented and non-examples or unusual examples can be given to VISs to understand the critical attributes of polygon.

This study provided an opinion related to how polygon understandings of BSs was shaped and with what misconceptions they addressed. The findings acquired which are similar with other studies show that there is a cognitive process usually encountered in terms of certain misconceptions and typical cases that might be come across in the understandings of polygons. To enable this cognitive process to be determined, a study which can be realized with larger and distinctive working groups may be suggested. And it seems important to have a suitable teaching design-particularly geometry lessons- in order to form correct images of polygon.

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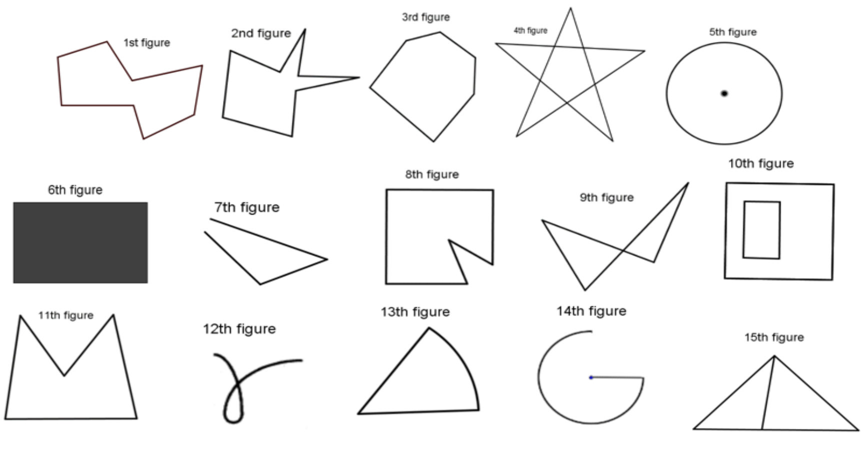
Appendix

1. Draw three polygons. Which features of these shapes cause them to become polygon?

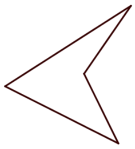
2. Determine whether or not the following planar shapes are polygons according to the given example. State the names of those which are polygons and explain why those not being polygons are not Lpolygons.



3. In the following geometrical shape;

   a) Find the sum of interior angles of this geometrical shape.

b) Find the sum of exterior angles of this geometrical shape.



4. As the number of sides increases in a polygon;

a) How does the sum of its interior angles change?

b) How does the sum of its exterior angles change?