International Journal of Progressive Education, Volume 21 Number 3, 2025

Mathematical Modeling Studies on Geometry Subjects: What, How, When. A Systematic Literature Review

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Abstract

The aim of this study is to conduct a systematic review of mathematical modeling studies on geometry between 2007 and 2023. For this purpose, trends in the scope and methods of the studies to be examined were determined. A systematic literature review was conducted through ERIC, Web of Science and Scopus databases that the researcher's institution had access to. In this context, it was realized that there were a limited number of articles. All studies from the databases that met the inclusion criteria determined within the scope of the research were included. As a result of the analysis, it was seen that most of the studies dealing with geometry topics were conducted in 2022, and qualitative analysis was mostly adopted in the studies. When the application levels of the studies were examined, it was seen that conducting research with high school students was more preferred. When the aims of the studies examined are considered within the framework of findings and results, it is possible to classify the aims in 4 subcategories. These are, respectively, the transfer of mathematical knowledge outside the classroom and the effect of the techniques used in this process on the skills in the modeling process, the transfer of mathematical situations to real life situations, the effect of modeling problems on the problem solving process and activity design, and the examination of the transition between the modeling cycle steps in the modeling problem solution phase. As emphasized in the literature, the aims of the studies were generally on the development of mathematical skills through mathematical modeling. When the implementation options of the articles in the classroom were examined, geometry topics in mathematical modeling were mostly given in a technology-supported environment. In addition, it is seen that Geogebra is the most prominent application among different technological tools during modeling. In this direction, it is thought that the study will provide a holistic perspective to the data of the researches conducted so far and will give an idea to future mathematical modeling studies in geometry.

Keywords: Geometry, Mathematical Modelling, Mathematical Modelling Activities, MEAs, Systematic Review,

DOI: 10.29329/ijpe.2025.1317.1

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INTRODUCTION

Mathematics is used to make sense of many situations or events in life. From health care to graphic design, architecture to dance, music to sports, and geography to economics, the level of mathematical thinking and problem-solving has increased significantly. Therefore, the foundations of everyday life are becoming increasingly mathematical, and the need to understand and use mathematics in everyday life and work is becoming increasingly important (National Council of Teachers of Mathematics [NCTM], 2020). This role of mathematics has led to parallel developments in the world of education (PISA, 2022) and has changed the understanding of problem-solving (Lee, Yeo & Hong, 2014). For this purpose, it has become essential in mathematics education to raise individuals who can acquire the mathematical knowledge and skills they need in daily life, express their knowledge, be aware of the relationship between mathematics and the real world, solve real-life problems, and actively use their knowledge in their daily lives (NCTM, 2020; Panadero, 2017).

The terms mathematical model and mathematical modeling should be understood for a good understanding of mathematical modeling. A model is defined mathematically as a conceptual tool that can be shaped, modified, managed, and reused to control, predict, describe, explain, and structure systems (Lesh & Doerr, 2003). Equations, diagrams, computer programs, or other embodied tools are examples of models in this context. A mathematical model, on the other hand, is a representation of mental constructs transformed into a mathematical form that allows interpretation and analysis of a context (Lesh & Doerr, 2003; Sriraman, 2005). Modeling, on the other hand, is the process of organizing, coordinating, systematizing, and organizing problem situations in mind in the process of interpreting events and problems, finding a pattern, and creating models using different schemas in mind (Lesh & Doerr, 2003; Lesh & Zawojewsky, 2007). As a result, while models represent a structure, modeling creates a prototype of a structure (Lesh & Doerr, 2003). One of the sub-branches of the concept of modeling is mathematical modeling. Mathematical modeling is defined as the process in which the inferences derived for the solution of a real-life problem are mathematized and analyzed, the solution is interpreted according to the real-life situation, and these steps are cyclically rearranged to create a physical, symbolic, or abstract model of the real-life situation expression (Borromeo Ferri, 2006; Maaß, 2006). Mathematical models are used in the mathematical modeling process to find solutions to real-life problems.

The geometry learning domain gives students a perspective on the schemas they have in their mental world and enables them to interpret them (Kemp & Vidakovic, 2021a). With geometry topics, students gain knowledge about geometric shapes and structures and make sense of the properties and relationships of these geometric shapes and structures. Geometry, which mostly engages comprehension, perception, identification, interpretation, and spatial thinking skills, is also necessary for mathematical modeling studies because it enables students to analyze and interpret their approaches, evaluations, and results (Niss et al. l. 2007). Students can learn about geometric shapes using objects that can be seen, held, and manipulated (Rusiman et al. l., 2017). For example, they can explore movements such as sliding, somersaulting, and spinning and learn to manipulate them physically and mentally systematically. To this end, with well-designed activities and appropriate tools, students can make and explore conjectures about geometry, thereby improving their geometry skills (NCTM, 2020).

It has been emphasized by many studies that problem situations taken from real life situations and different disciplines improve students' interpretations of the process (Fredriksen, 2020; Sevinc & Lesh, 2018). Geometry knowledge effectively represents and solves real-world problems (Van De Walle, Karp & Bay-Williams, 2012; Clements, 1997). Geometric ideas are used in maps, planning routes, designing floor plans, and creating works of art. Combining these similar ideas with modeling in creating and applying modeling problems is essential. Studies have shown that combining the geometry dimension of mathematical knowledge with modeling problems will positively affect the teaching process (Tezer & Cumhur, 2017; Ortiz & Machin, 2022; Passarella, 2022). Since mathematics is based on a holistic perspective (NCTM, 2020), combining these two areas can lead to new knowledge formations. In this context, working with mathematical modeling in geometry learning areas (Latifi et al., 2022) is essential and valuable. In addition, the visual plans presented allow the content of the discussions in the classroom to be realistic. In this way, they can be guided and debugged, and new perspectives can be opened up. This emerging meaning of visualization in real-world-based learning (Henn, 1998) shapes students' evaluation of the process and their conclusions about the process. Students' mastery of the holistic framework presented by these approaches can positively affect the teaching process (PISA, 2022).

From this perspective, it is important to examine the modeling problems dealing with geometry and present studies that will form a road map in shaping the teaching process. When the literature was examined, it was seen that there were studies on more specific topics such as a systematic literature review on dynamic geometry software to improve mathematical problem-solving (Susanto, Hartati & Standsyah, 2023) and a systematic literature review on the use of Geogebra in mathematics education (Yohannes & Chen, 2023). However, no systematic literature study on geometry in general was found. Considering the contribution of geometry to abstract thinking skills, blending it with the contexts in modeling problems will be essential and will guide future studies in this field. This study aims to systematically review mathematical modeling studies on geometry. For this purpose, trends regarding the scope, results and methods of the studies will be determined. In this direction, this study is thought to provide ideas for future mathematical modeling studies on geometry.

Research Problems

In line with the aim of this study, answers to the following problems were sought:

1. What are the bibrographic characteristics of the studies analyzed?

    a.What is the distribution of the analyzed studies according to databases and years?

    b. What is the distribution of the analyzed studies according to research methods?

    c. How did the data analysis types of the analyzed studies vary?

    d. How did the study group of the analyzed studies vary?

2. How were the studies be analyzed?

3. How were the data of the reviewed studies be collected?

4.) How were the reviewed studies implemented in the classroom?

    a)What is the learning environment of implementation of the studies in the classroom?

    b. What are the technologies used?

    c. How were the practices in the studies analyzed?

5.)What are the aims of the studies examined?

6.) What are the findings and results of the reviewed studies, how do the aims overlap?

METHOD

Research Model

In this study, a systematic literature review of research on geometry topics in mathematical modeling was conducted. A systematic literature review is a comprehensive, unbiased, and technical scientific research process based on predetermined criteria (Dixon-Woods, 2011). In this process, sources related to a research question are brought together and synthesized (Liberati et al., 2009; Moher et al., 2015). By bringing together the information related to the research question in this way, a summarized and analyzed version of the literature on the subject is created (Cooper, 2010). In this context, this study, as stated in the Centre Methods for Conducting Systematic Reviews EPPI (2010), general descriptive information about the subject, the results of the studies, and the data on research methods were revealed to determine the status of the studies dealing with geometry topics in mathematical modeling in the literature.

Data Collection Process

Articles that studied mathematical modeling of geometry subjects were examined. The Preferred Reporting Items for Systematic Reviews and Meta-analysis (PRISMA) approach was revised and used for the study. The presentation of the systematic literature review could be simplified with the guidelines determined by the PRISMA approach (Liberati et al., 2009). The first process specified in the approach is determining the number of articles in the databases selected for the study. The second process is to scan the number of records after removing duplicates. The third process is determining the number of articles due to eligibility and evaluation. The last process involves the inclusion of the articles in the analysis process (Liberati et al., 2009).

The study applied a four-step procedure for the systematic literature review. The development of inclusion criteria was the first step of the process. In order to organize the data systematically, screening criteria and key search strings were also determined in this context. The keywords used in the study were "Mathematical Modelling" and "Geometry" since the search was conducted in English databases. The criteria are presented in Table 1.

Table 1. Inclusion criteria of the studies included in the study

	Inclusion criteria
Publication type	Academic refereed journal articles
Access	Studies that can be accessed in full-text
Broadcast language	English
Workspace	Related to mathematics education
The preferred method in the study	Mathematical modeling
Subject	Geometry

The second step of the process was to search for articles related to the research question in line with the inclusion criteria. The searches were conducted through the Education Resources Information Center (ERIC), Web of Science, and Scopus databases that the researchers had access to at their institutions. Since the study was conducted for refereed articles published in academic journals, conference proceedings, newspaper clippings, books, projects, master's and doctoral theses, and reports were not examined. Thus, the relevant studies in the literature were tried to be identified through 3 databases. Based on the inclusion and exclusion criteria developed in the review step, the studies in the selected databases were screened in this context. The research team reviewed the databases in detail by scanning abstracts and full texts according to the inclusion and exclusion criteria. After searching with keywords, specific keywords were selected to ensure that as many publications as possible were relevant to the research questions. The information about the databases and the keywords used during the search process is given in Table 2. When keywords were searched, keywords were schematized to fit the desired information. An example of this is the use of conjunctions, both in parentheses and in quotation marks (e.g. ("Mathematical Mode*ing"). In this way, it is indicated exactly which words should be found in the customized field in the search. In addition, since the software of some words may change, a detailed search was made by replacing the letters that may differ with an * sign. In addition, another commonly used form of the word was also included in the search in case of a change in the concept used in the articles (e.g. "Modeling", "Modelling").

Table 2. Databases and words searched

Database	Screened Words and screening stages
Web of Science	**1) "Mathematical Mode*ing" And "Geometry" (Topic)** 2)Open Access 3) Article, conference paper, book chapters 4) Language: English 5) Related to the field of "Mathematics" and "Mathematics Education" (In this process, many engineering-related articles were eliminated.) 6) Elimination of studies that are not related to the research topic in the abstract review
ERIC	1) **"Mathematical Modeling" and "Geometry"** **"Mathematical Modelling" and "Geometry"** **("modeling" and "modeling" were reviewed separately because the * command did not work)** 2) Open Access 3) Article, conference paper, book chapters 4) Language: English 5) Related to the field of "Mathematics" and "Mathematics Education" 6) Elimination of studies that are not related to the research topic in the abstract review
Scopus	1) **(ALL ("Mathematics Education" AND "Mathematical Mode*ing") AND ALL ("Geometry"))** 2) Open Access 3) Article, conference paper, book chapters 4) Language: English 5) Related to the field of "Mathematics" and "Mathematics Education" (In this process, many engineering-related articles were eliminated.) 6) Elimination of studies that are not related to the research topic in the abstract review

In the third stage, the scanned articles were evaluated within the scope of the study's relevance to the research question. For this purpose, the identified studies were examined and mapped according to categories. The studies were defined and mapped according to publication years, databases, application styles, research questions, methods, findings, and study results. As a result of the definitions and mapping, it was evaluated whether the articles supported the research question of the study. The process also identified and eliminated geometric model studies instead of mathematical models. Articles dealing with geometry topics in mathematical model studies were revealed. In this process, identical articles across the three databases were also identified and eliminated. In the fourth and final stage, articles related to the study's research question were selected. The review identified studies that included keywords at least once in their texts and references. It was observed through deeper analysis that these articles were not in the trend of the study's research question. At this stage, the internal consistency of the remaining studies was also examined. In this context, the reliability of the results, as well as the external reliability and consistency, were examined. In this context, the studies were included in the systematic review, and this process is presented in Figure 1.

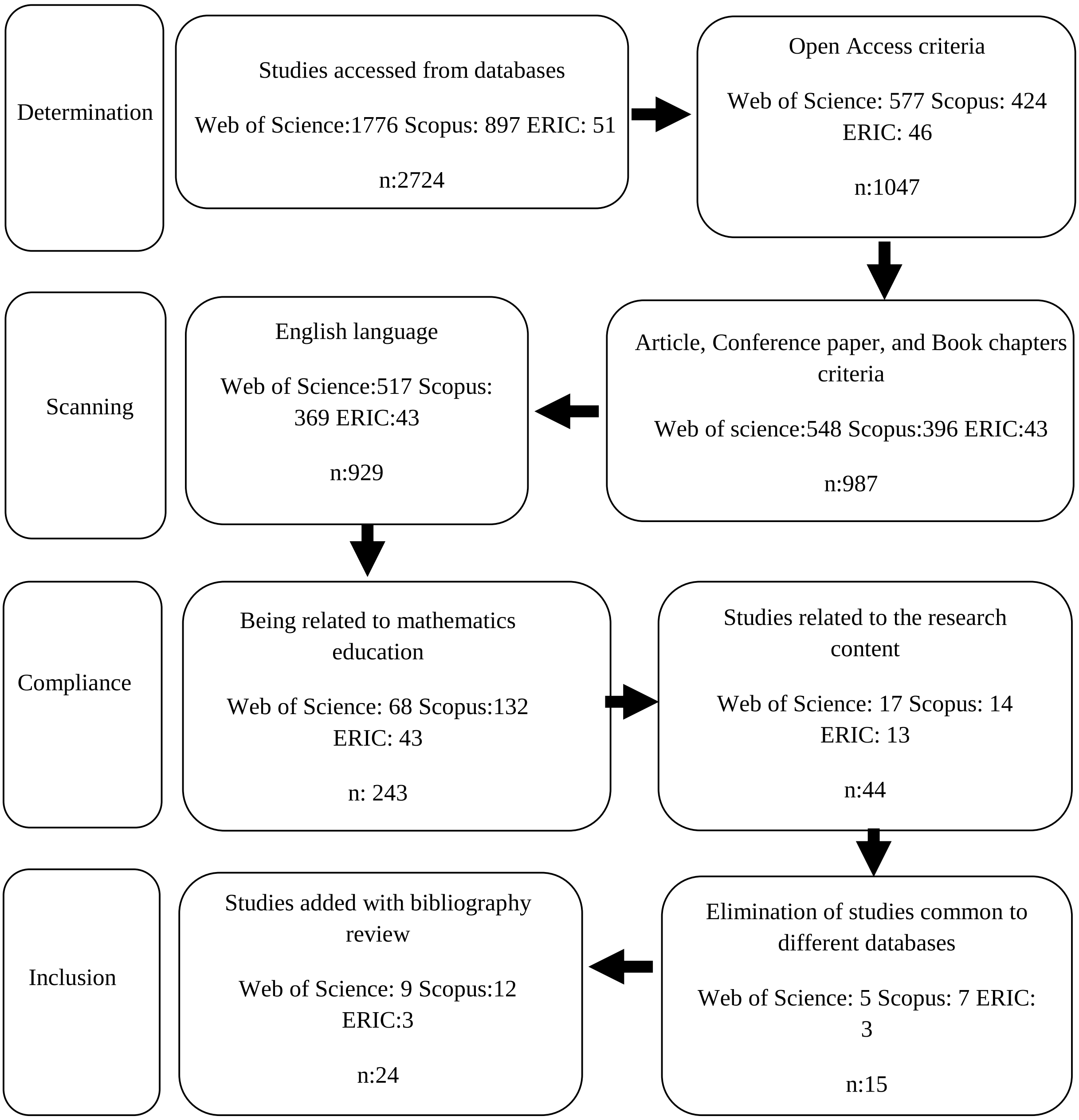

Figure 1. Stream Selection Process Flow Diagram (adapted from Liberati et al., 2009, Prisma)

(At the beginning of the screening process, 391 studies related to the research were obtained when logged into the Scopus database at the beginning of the screening process, and 897 studies related to the research were obtained when logged in without logging in. In this study, the screening process was continued without logging in).

A total of 2724 studies were accessed by searching the databases. The accessed studies were first evaluated according to the open access criteria and continued with the studies with open access. Among the studies that met the open access criteria, only studies with articles, conference papers and book chapters were included. Other studies such as reports, proceeding papers, meeting abstracts etc. were eliminated. Of the remaining 987 studies, only those in English were included and the remaining 58 studies were eliminated because they were not in English. The final 929 studies were analyzed according to their content. Studies outside mathematics education (e.g. physics applied, calculus, computer software, engineering chemical, mechanics, cell biology, materials science composites, etc.) were eliminated. With the elimination of out-of-field studies, 243 studies remained. Among these studies related to mathematics education, 44 studies that related mathematical modeling problems with geometry subjects were examined for the purpose of the study. The number of studies examined decreased to 15 by eliminating the studies that were in different databases at the same time. During the review, 9 studies included in the bibliographies of the studies and related to the subject were also included in the review, and a total of 24 studies were analyzed in detail. At the end of the process, A total of 24 international articles searched in Scopus, Eric, and WOS were included.

Data Analysis

Document analysis was used in this research. The articles deemed appropriate for the study were categorized. In this context, the articles were categorized into six main dimensions. The main categories are publication years, databases, implementation options, type of study, method, findings, and results framework. By the purpose of the study, some of the six main categories were examined in detail by determining sub-categories. The theoretical frameworks of the articles obtained from the keyword search and those remaining due to the evaluation were coded this way. The identified categories were presented in tables and graphs in the findings section, explained and interpreted. Expert opinions were consulted intermittently during all these stages.

Validity and Reliability of the Study

After the researchers determined the criteria to be screened in the research process, necessary arrangements were made regarding the research criteria by evaluating them with the opinions of field experts. After the arrangements, the research questions were screened and the findings were coded. The researchers created the codings separately and compared at the last stage. During the comparison, inter-rater reliability was calculated. While making this calculation, the formula used by Miles and Huberman (2015) was considered. The formula used is Reliability = Agreement / (Agreement + Disagreement). If the numerical value resulting from the formula is above 70%, the study is considered reliable (Miles & Huberman, 2015). For this study, the calculation resulted in 93%. Thus, it was seen that inter-coder reliability was achieved. Finally, a standard code was obtained by scanning again for codes that differed due to the comparisons.

FINDINGS

In this part of the study, the data obtained from the analyzed studies will be presented together with the problem situations. In the findings section, codes for research articles are coded with the letters "J", codes for book chapters with the letters "B", and codes for conference presentations with the letters "C". First, the bibrographic characteristics of the studies will be examined. Then other questions will be explained.

Bibrographic Characteristics of The Studies

Table and graphic information regarding the distribution of the studies analyzed according to databases and years are presented below.

Table 3. Distribution of Studies According to Databases

Database	Related Study(s)	Frequency
Eric	J18, J19, C2	3
Web of Science	J1, J2, J3, J4, J5, J6, J7, J8, B1	9
Scopus	J9, J10, J11, J12, J13, J14, J14, J15, J16, J17, B2, B3, C1	12
Total		**24**

When Table 1 is examined, it is seen that 12 of the 24 studies on geometry were published in the Scopus database, 9 were published in the Web of Science database and 3 were published in the

ERIC database. It is seen that 50% of the articles analyzed are included in the Scopus database. This indicates that Scopus database is preferred in the studies conducted in this field.

Information on the distribution of the studies analyzed by years between 2007 and 2023 is presented in the graph below.

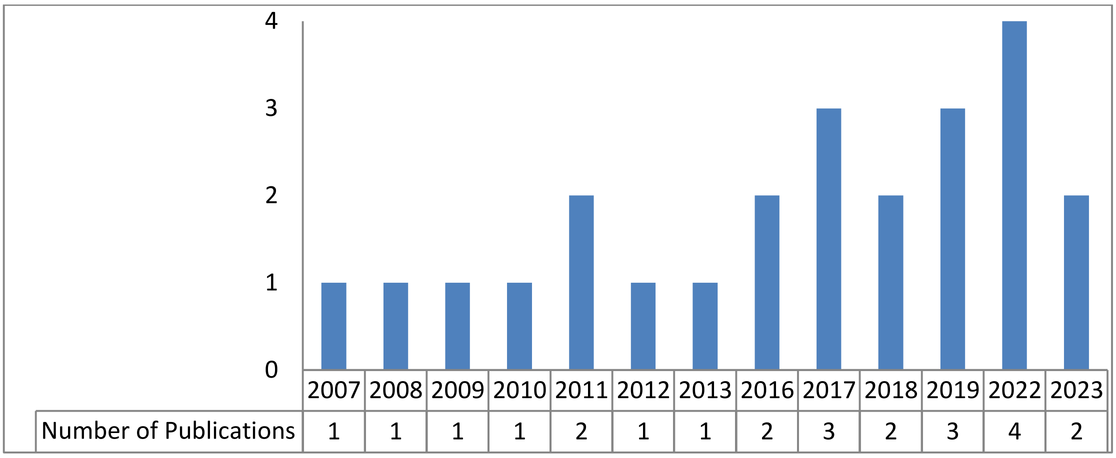

Figure 2. Distribution Chart by Years

When Figure 2 is analyzed, it is observed that between 2007 and 2023, which constitutes the limitation of this study, publications were regularly published every year except 2014 and 2015. In addition, compared to other years, it is observed that the highest number of studies (n=4) was conducted in 2022.

a. The analyzed studies according to methodology

The table below shows studies according to research methods.

Table 4. Methodologies of the Studies

Qualitative	J1, J2, J4, J5, J10, J11, J12, J16, J17, J18, J19, B2, B3, C1, C2
Quantitative	J3, J6, J13, J14, J15, B1
Mixed	J7, J8, J9

When the research methods of the studies examined were examined, it was seen that 15 studies were designed as qualitative analysis, six studies as quantitative analysis, and three studies as mixed.

b. The data analysis of the studies

The table below presents the types of analysis used in the studies and the distribution of analysis types according to the type of analysis.

Table 5. Data Analysis Types of the Studies by Type of Analysis

	Types of Analysis		Related Study(s)	Frequency
Qualitative Analysis	Parallel analysis		J2	1
	Tailored Analysis		J4, J10, B2	3
	Hypothetical Learning Trajectory		J17	1
	Comparative Analysis		J16, J18	2
	Descriptive Analysis		J12	1
	Content Analysis		J11, J19, B1, B3, C1, C2	6
	Unspecified		J1	1
	Total			15
Quantitative Analysis	Statistical Analysis	Confirmatory factor analysis	J6	1
		Independent t Test	J14	1
		Chi-Square	J15	1
		Variance Analysis	J13	1
	Multilevel Modeling		J3	1
	Total			5
Mixed	Content Analysis		J5, J7, J8	1
	Independent Group t-Test		J8	1
	Descriptive statistics		J9	1
	Descriptive Analysis		J9	1
	Total			4

When the distribution of the studies according to data analysis is examined, it is seen that qualitative analysis types are preferred more than other analysis types. While content analysis was used the most among qualitative analysis types, it was also observed that specially developed analyses were preferred (Packel & Wagon, 1994; Explanation Building (Burns, 2000); Kaiser-Meßmer (1986)). After these, comparative analysis is used. Descriptive analysis, parallel analysis, and hypothetical learning trajectory were used in one study. Regarding quantitative analysis, two categories emerged: statistical analysis and multilevel modeling. Statistical analyses are the most preferred type of analysis and are divided into four categories: confirmatory factor analysis, independent sample t-test, Chi-Square test, and variance analysis. In mixed design research, there are content analysis, independent group t-tests, descriptive analysis, and descriptive statistics types, and all of these types were used in one study.

c. The study groups according to the type of analysis

The table below shows the distribution of the sample/participant numbers of the studies according to the type of analysis.

Table 6. Classification of the Study Group by Type of Analysis

	Number of Sample/Participants	Related Study(s)	Frequency
Qualitative Analysis	1-10	J2, J5, J10, J12, J19, B2, B3, C1, C2	9
	15-50	J11, J16, J17, J18	4
	Unspecified	J1, J4	2
	Total		15
Quantitative Analysis	1-10	B1	1
	50-61	J13, J14, J15	3
	450-710	J3,J6	2
	Total		6
Mixed	1-10	J7	1
	50-100	J9	1
	300-350	J8	1
	Total		3

When the study groups is examined, it is seen that the number of participants in qualitative analyses is less than other types of analysis. In contrast, there are studies with numbers up to 709 people in quantitative analyses. In quantitative studies, it was observed that J3 worked with 473 samples, while J6 worked with 709 sample groups. Another finding was that J8 worked with a sample group 301 in mixed-design studies. In addition, it is seen that two studies (J1, J4) in qualitative research should have included information about the number of participants used in the research.

d. The implementation level of the studies

The table below shows the application levels of the studies examined and the distribution of these studies according to the types of analysis.

Table 7. Implementation Level of Studies by Analysis Type

	Implementation Level	Related Study(s)	Frequency
Qualitative Analysis	1st-4th grade	J16	1
	High School	J1, J2, J10, J17, J19, C1, C2	7
	University	J4	1
	Teacher Candidate	J11	1
	Teacher	J12, J16, J18, B2, B3	5
	Parent	J16	1
	Unspecified	J5	1
	Total		17
Quantitative Analysis	5th-8th grade	J13, J14	2
	High School	J3, J6, B1	3
	University	J15	1
	Total		6
Mixed	Preschool	J8	1
	5th-8th grade	J9	1
	High School	J7	1
	Total		3

When the application level is examined, it is observed that there are studies conducted with high school students in all three categories. Most studies conducted at this level are the studies evaluated with qualitative analysis. Another finding is that the number of studies conducted with teachers is also high (n=5). However, the number of studies conducted with pre-service teachers is not as high as those conducted with teachers. In addition, it is seen that the number of studies in the categories of preschool and 1st-4th grade is low (n=1).

e. The data collection tools used in the studies

The table below presents the data collection tools used in the studies and the type of analysis in which the data collection tools were used.

Table 8. Collection Tools of the Studies According to the Type of Analysis

	Data Collection Tool	Related Study(s)	Frequency
Qualitative Analysis	Audio and video recorders	J10, J12, J18, B3, C1, C2	6
	Interview Form	J2, J10, J16, J18, J19, B2, B3, C1	8
	Photo.	J12	1
	Written reports, Worksheets	J2, J4, J11, J17, J18, J19	6
	Observation, Field Notes	J4, J10, J16, J19, C2	5
	Unspecified	J1	1
	Total		27
Quantitative Analysis	Modeling Test	J6	1
	Achievement Test	J14	1
	Audio and video recorders	B1	1
	Scale	J3, J15	2
	Written reports, Worksheets	B1	1
	Unspecified	J5, J13	2
	Total		8
Mixed	Observation, Field Notes	J9	1
	Audio and video recorders	J7	1
	Special tool (prepared indexes)	J8	1
	Total		3

When the data collection tools are analyzed, it is seen that the interview form was used the most in qualitative studies (n=8), followed by audio and video recorders and written reports (n=6). In addition, observations and field notes were also used, and one study used photographs to collect data. One study, which did not specify the data collection tool, evaluated with qualitative analysis. In the studies that collected data through quantitative analysis, it was observed that scales were used the most (n=2). In addition, two studies did not specify the data collection tool, and the other data collection tools had equal frequency (n=1). These are modeling test, achievement test, audio and video recording device and written reports. It was observed that audio and video recorders and written reports were common data collection tools in the qualitative study. Regarding mixed design studies, observation, field notes, audio and video recorders and a specially prepared index were used with equal frequency (n=1). The specially prepared index is the "learning styles index for children (Güneş, 2014)". In addition, it can be interpreted that the data collected with audio and video recording devices were present in all three types of analysis.

1. Learning Environments of Implementations

The learning enviroments of implementations of the studies in the classroom and the distribution of these according to the types of analysis are presented below.

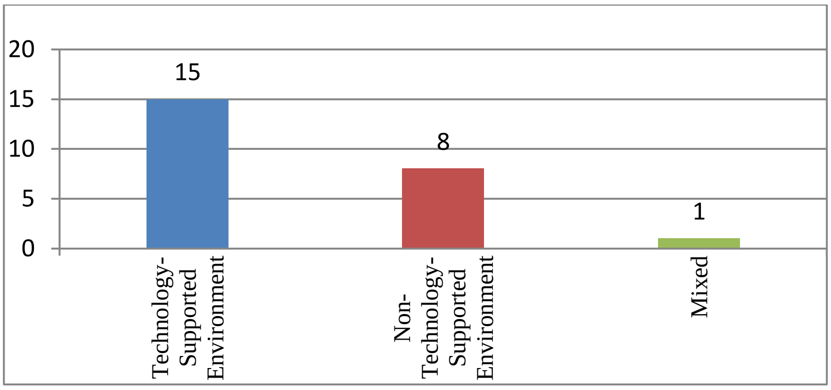

Figure 3. Implementation options in the classroom

When Figure 3 is examined, we see information about the implementation options of the studies in the classroom. It was observed that 15 studies were conducted in a technology-supported environment, eight studies were conducted in a non-technology-supported environment, and 1 study conducted data collection processes without the use of technology along with the use of technology. This reveals that mathematical modeling practices are largely carried out in technologically supported environments. Although Griffrath and Siller (2018) stated that the technology-supported environment may have a positive feedback for mathematical modeling problems and solution processes, it was stated that the effect of digital tools should be measured with different mathematical modeling studies.

a. Applications and types of analysis used in technology-supported environment

The tables below present the analysis types of the methods used in the technology-supported and non-technology-supported environments of the studies examined. The table below presents the classification of the applications used in the technology-supported environment and the types of analysis using these applications in the 24 studies examined.

Table 9. Applications Used in Technology-Supported Environment

	Application Name	Related Study(s)	Frequency
Qualitative Analysis	GeoGebra	J1, J2, J4, J11, J16, J18, J19	7
	Excel	J2	1
	Nspire	J10, C2	2
	A.R. (Augmented Reality)	J16	1
	Photoshop	J2	1
	Paint Tool	J2	1
	Maxima	J4	1
	Geometer's Sketchpad	J5, B3	2
	Total		16
Quantitative Analysis	Geometer's Sketchpad	J13	1
	Geogebra	J6, J15	2
	Total		3
Mixed	Gamification	J9	1
	Total		1

When Table 9 is examined, it is observed that Geogebra (n=9) is the most commonly used application in the studies conducted in a technology-supported environment. Seven studies used qualitative analysis, and two used quantitative analysis. One of the reasons why Geogebra is widely used is its contribution to learning beyond the classroom and its possibilities (Budinski, 2017). Geometer's Sketchpad (n=3) and Nspire (n=2) followed Geogebra. It is observed that 2 of the studies using Geometer's Sketchpad program were evaluated with qualitative analysis, and the other one with quantitative analysis. All of the studies using Nspire were evaluated using qualitative analysis. It is seen that other programs are equally distributed and have a frequency of n=1. Another finding was that gamification was used in the mixed design study in which the technology-supported program was used. In addition, it was also observed that some studies (e.g., J2 and J4) used more than one program simultaneously.

b. Methods and types of analysis used in non-technology-assisted environments

The table below presents the methods used in non-technology-supported environments and the types of analysis related to these methods in the studies examined.

Table 10. Techniques Used in Non-Technology-Supported Environments

	Techniques Used	Related Study(s)	Frequency
Qualitative Analysis	Paper and Pen	J17, C1	2
	Product Creation	J17	1
	Drawing Methods	J12	1
	Attitude/Belief Review	B2	1
	Total		5
Quantitative Analysis	Drawing Methods	J3	3
	Paper and Pen	J13, J14	2
	Product Creation	J13	1
	Total		6
Mixed	Learning Styles Index	J8	1
	Paper and Pen	J7	1
	Drawing Methods	J7	1
	Total		3

When Table 10 is examined, it is seen that the most preferred technique in traditional methods is the analysis process using paper and pencil (n=4). It was observed that 2 of the studies were analyzed with qualitative analysis, 2 with quantitative analysis, and 1 with mixed design. After this, it is seen that drawing methods (n=3) and then product creation (n=2) are preferred. It can be interpreted that drawing methods are equally distributed among all three types of analysis. It is observed that the other techniques used have the same ratio.

2. *Purposes of the studies*

The table below shows the distribution of the studies according to their objectives.

Table 11. Objectives of the Studies

Objective	Related Study(s)	Frequency
Transfer of mathematics outside the classroom	J1	1
Associations between mathematical terms and real-life situations	J2, J4	2
The effect of mathematical knowledge on thinking skills	J3, J7, J13, J16, J17, B3	6
The impact of technological tool use on modeling	J4, J5, J6, J9, J10, J11, J18, J19, B1, C2	10
Impact of cultural background on modeling	J12	1
Effect on mathematics achievement/problem-solving situations	J14	1
Determining modeling skills according to grade levels	J8	1
Identify transitions between modeling cycle steps	C1	1
Determining beliefs and attitudes about modeling skills	B2	1
Activity design/Model elicitation	J15, J17	2
Total		26

When the data on the purpose obtained from the studies are examined, it is seen that the most critical purpose is to examine the effects of technological tool use on mathematical modeling. Since the use of technological tools is one of the most effective ways to connect the world of mathematics with the world of mathematics learners (Jackiw & Sinclair, 2009), its integration into the teaching process is important. In addition, determining the effect of mathematical knowledge on modeling is another important objective. In their study, Lesh and Caylor (2007) revealed the change in students' thinking style within the framework of their own knowledge and ideas instead of the studies developed by students with external guidance. At the same time, it was also observed that students included knowledge, beliefs, values, metacognitive processes as well as mathematical knowledge while interpreting the mathematical modeling problem.Objective statements related to product creation, activity design, and modeling associated with mathematical terms or real-life situations are also included in the scope of the objectives of the studies. However, when the objectives of the studies were examined, a study was not found that aimed to examine a relationship or effect between geometry knowledge and mathematical modeling.

3. The findings and results of the studies

In the table below, the findings obtained from the studies examined and the results obtained within this framework are presented with relevant publications.

Table 12. Findings and Results of the Studies

Findings	Results	Related Study(s)
Positive and negative approaches to the solution stages of the modeling problem with technology-supported applications were presented.	**Positive** It was found that it contributed to the modeling and visualization processes.	J1, J9, J10, J11, J14, J16, J19, C1, C2
	Negative No results were found regarding the effect of technology on intuitive thinking processes and creative thinking processes. Studies that negatively affected the process of transfer to mathematics were observed.	J6, J10, J19, B1
Information on transferring the real-life situation to the model is presented.	**Positive** It was concluded that understanding and analyzing real-life problem situations was supported during the transfer to the model.	J2, J4, J18
	Negative Difficulties were reported in establishing the relationship between data and real-life situations.	B2
Approaches to interpreting changes in the solution process are presented.	**Positive** It contributed to transferring the real-life situation to the geometric model, creating a geometric solution process, and associating the model with algebraic notation.	J2
Findings related to determining students' knowledge/skills related to modeling are presented.	**Positive** It was concluded that students' knowledge levels increased and creative thinking skills improved.	J3, J7, J8, J14, J15, C2
Evaluations on the use of representations in the mathematical modeling process are presented.	**Positive** The modeling process contributed to exploring different phenomena and understanding and interpreting representations.	J5, J11
Findings related to interpreting the mathematical modeling process with cultural phenomena are presented.	**Positive** There is a deep connection between culture and mathematics.	J12
Obstacles that may arise during the modeling process are presented.	**Positive** Barriers due to lack of reflection or inaccurate or incomplete information were identified, and solutions were offered to overcome these barriers.	C1
The modeling process was examined in the context of model-building activities.	**Positive** Model elicitation activities based on rich contextual problems could be carried out and interpreted.	J13, J17
Findings related to the process obtained by combining the modeling process and the game are presented.	**Positive** The development of a sense of discovery and curiosity was supported. Students' creative thinking skills were positively affected.	B3

In general, the aims of the research have been on mathematical modeling and the development of mathematical skills (Nish, Blum, & Galbraith, 2007). Findings and results were also shaped in line with these aims. When the aims of the studies examined are considered within the framework of findings and results, it is possible to classify the aims into 6 subcategories. These are listed as "modeling and problem solving stages in technologically supported applications" (1), "information accessed in transferring the real-life situation to the model" (2), "studies conducted in designing a model suitable for the real-life situation" (3), "in-class interaction and examination of student thinking processes" (4), "necessary information while designing the model process" (5) and "development of a sense of discovery and curiosity" (6). These categories constitute sub-themes in which the items listed as "title" or "finding/result" in the table are naturally grouped according to their content and focus.

Each category, based on a specific conceptual framework, brings together similar findings from different studies and thus offers the opportunity to evaluate emerging trends in research from a more holistic perspective. Although these themes may seem to be independent in themselves, they have direct or indirect relationships with each other as they address different dimensions of the modeling process. For example, the study by Ekol (2011) deals with the effect of game-based instruction on exploration, developing a sense of curiosity and gaining experience through experimentation, while the study by Lesh and Caylor (2007) deals with the process steps developed by students within the framework of their own knowledge. In both studies, process planning focuses on the knowledge that students will reveal and make sense of through their existing knowledge. However, when these two studies are examined as sub-themes, it is observed that one study (B3) is in the game-based sub-theme and the other study (J13) is in the modeling activity sub-theme.

The first category addresses how technological tools (e.g. GeoGebra, Geometer's Sketchpad, and Nspire) are integrated into the modeling process and how they improve students' problem-solving skills through visualizing abstract concepts, experimenting, and receiving immediate feedback. For example, the study by Latifi et al. (2022) shows that in a class using GeoGebra, students experimented with different scenarios through their modeling-based approach to solving geometry problems. The results show that modeling problems positively affect the modeling process in a technology-supported environment, but there is no finding regarding the effect on thinking skills such as intuitive thinking and creative thinking. Another category is the transfer of mathematical situations to real-life situations and their relationship with the modeling process. This category examines the methods used to relate abstract mathematical concepts to real world problems. Transferring real-life situations to the model helps students find concrete answers to the “why” question and makes learning more meaningful. Tezer and Cumhur (2017) reported that students associate the geometric problems they encounter in daily life with the concepts taught in the classroom and thus understand the subject better. The third category is the studies conducted in designing models suitable for real-life situations. This category evaluates whether the model design is appropriate for the age and experience level of the student and whether it reflects real life situations. Appropriate model design helps deepen learning and support students' creativity. Güneş and Şahin (2019) emphasized that when examples appropriate for the age of the student are not selected in model design, the modeling process remains superficial and conceptual depth may decrease. The fourth category is the examination of classroom interaction and student thinking processes. This category reveals how technologically supported modeling activities affect classroom interaction and student thinking processes. Effective teacher guidance, group work and discussions enable students to take a more active role in the modeling process. Stillman et al. (2013) and Haas et al. (2013) observed that the instant feedback given by the teacher to the students during the modeling process contributed to students to be more successful in problem solving steps by increasing classroom interaction. Another category is the necessary information while designing the modeling process. This category examines the effect of planning the modeling activity, step-by-step instructions and teacher guidance on the modeling process. Well-structured information enables students to grasp each step in the modeling cycle more effectively. Passarella (2022) found that detailed debriefing sessions with students prior to the modeling process reduced students' error rates during the model building, testing and interpretation stages. The sixth and final category is the development of a sense of exploration and curiosity. This category examines how technologically supported modeling environments trigger creative thinking and sense of discovery by arousing curiosity in students. This affective factor allows students to try different scenarios and develop innovative solutions in the problem solving process. Lesh and Caylor (2007) reported that students experimented with various parameters in the modeling process and developed original methods beyond classical solutions. These six categories provide a comprehensive picture of how modeling activities work both in and out of the classroom and how they affect students' mathematical conceptualization, problem solving and critical thinking skills. Each category represents specific sub-themes of the findings presented in the table and thus provides a more systematic and in-depth analysis of the studies in the literature.

When the aims, findings and results of the studies were analyzed, it was seen that there was a progression in a way that complemented each other and that purpose-oriented findings and results were obtained. When the aims, findings and results of the studies in these six categories were examined, no study was found that directly related geometry knowledge to mathematical modeling studies. Considering the features of geometry knowledge such as being associated with daily life, contributing to creative thinking skills, and having a direct relationship with cultural elements, the limitations of the studies stand out.

DISCUSSION, CONCLUSION, AND RECOMMENDATIONS

In this study, a systematic review of research on geometry topics in mathematical modelling is presented. Web of Science, Scopus and Eric databases were analysed. The studies included in the research were published between 2007 and 2023 and consisted of a total of 24 studies, including 3 conference presentations, 2 book chapters and 19 research articles. When the databases where the studies were published were examined, 12 studies were taken from Scopus, 9 studies from Web of Science and 3 studies from Eric databases. When the distribution of the articles according to years is examined, the highest number of studies was published in 2022. Before 2007, there were no mathematical modelling studies dealing with geometry topics. The reason for this situation is that mathematical modelling is a current formation (Wess et all., 2021). No study related to the research topic was encountered in 2014, 2015 and 2021. Considering that the current study was implemented in 2023, the handling of geometry topics in mathematical modelling is gradually increasing. In 2021, it was observed that fewer studies were conducted compared to the previous years.

When the studies according to their methods is examined, it is seen that qualitative method was applied in 15 of the studies, quantitative method in 5 of them and mixed method in 4 of them. As Sokolowski (2015) claimed, it was seen that various learning environments were created from modelling activities designed for the deductive approach to modelling activities in which the inductive approach was adopted. It was seen that the sample group was distributed between 1-10 in the studies in which qualitative method was applied, while the distribution in the sample group was between one and 50-61 in the studies in which quantitative studies were applied. This result is supported by the findings of studies that emphasise that the participant group should be selected in small numbers in order to obtain in-depth information (Creswell & Poth, 2016). In two of the studies, no information about the number of participants was found. In line with the high number of qualitative studies, the intensity of qualitative analysis was observed in data collection tools. The data in the analysed studies were organised with different data collection tools. The most preferred data collection methods were written reports, worksheets, interview forms, audio and video recorders. It was also observed that the collected and organised data were analysed with different methods. With the determination of the use of rich methods used in the measurement of data, a result parallel to the study of Cevikbas, Kaiser and Schukajlow (2022) was found.

In parallel with Sokolowski's (2015) study, the sample of the analysed studies was mostly high school students. The reason for this is the expectation that geometry topics can be done by high school students (Alsina, 2007). The least applied samples in the studies were preschool, pre-service teachers and parents. At the primary school level, there were no studies conducted directly with primary school students, except for the children supported in J16 with parents (the children of the participants). This supports Stillman's (1998) assertion that students are not exposed to mathematical modelling until secondary school level. However, young children already have the basic competencies of modelling, and therefore the foundations of mathematical modelling can begin even before primary school (English, 2010; Lehrer, R., & Schauble, L. 2003; NCTM, 2020). In addition, studies have indicated that modelling activities are aimed at developing a product and allow children to express their ideas using multiple representations (English & Watters, 2004). In the product development process, it has been observed that many questions, assumptions, conflicts, revisions, and solutions are created (Zawojewski, Lesh, & English, 2002).

When the aims of the analysed studies are considered within the framework of findings and results, it is possible to classify the aims into 4 subcategories. These are, respectively, the transfer of mathematical knowledge outside the classroom and the effect of the techniques used in this process on the skills in the modelling process, the transfer of mathematical situations to real life situations and their effect on the modelling process, the effect of modelling problems on the problem solving process and activity design, and the effect of modelling problems on the transition between the steps of the modelling cycle in the solution phase. As it is emphasised in the literature, the aims of the studies have generally been on the development of mathematical skills through mathematical modelling (Nish, Blum, & Galbraith, 2007). Findings and results were also shaped in line with these aims.

When the implementation options of the articles in the classroom were examined, geometry topics in mathematical modelling were mostly given in a technology-supported environment (Geiger, Faragher & Goos, 2010; Morante & Vallejo, 2012). In the technological environment, technology was generally presented to the samples with Geogebra software in the classroom (Hıdıroğlu & Güzel, 2016; Budinski, 2017; Ortiz & Machin, 2022; Haas, Lavicza & Kreis, 2023). During the implementation process of the studies examined, modelling competencies were improved by using technology (e.g. Cevikbas., Kaiser & Schukajlow (2022)). Another finding obtained is that technology was used more as a tool rather than a purpose in the educational environment because it facilitates access to mathematics as stated by Ang (2010). As a result, technology was used to achieve mathematics learning objectives (Ang, 2010). In addition, it was observed that Geogebra was used more in the studies analysed with qualitative analysis methods compared to the studies using quantitative analysis methods. In addition, the high variety of technological tools used in qualitative analysis methods supports the progress of the process in a student-centred way (Guerrero-Ortiz & Camacho-Machín, 2022). The diversification of the methods used in studies using technology also enables deep analyses of the situation. It can also accelerate the stage of accessing the information that needs to be related in the process. In environments where technology was not used, paper, pencil and drawing methods were generally applied. In the studies analysed, it was observed that new classroom environments were designed by interacting with real life problems instead of stereotyped problem situations. This finding overlaps with Bonotto's (2007) study. In addition, it is a striking finding that quantitative analysis methods are preferred more in environments where technology is not used. Because in environments where students are evaluated with quantitative analysis, students' individual views cannot be examined in detail as in qualitative analysis (Akyıldız & Ahmed, 2021). In this case, the effects of variables such as psychological conditions in the application process and the effect of the environment on the students cannot be fully understood. The low number of techniques used in the process can also be given as an example. As the diversity in the techniques used decreases, the information that students will construct in their mental worlds may be limited to a certain framework and narrow the solution process and the mathematical terminology used in the solution process.

In line with the reviews, mathematics has been emphasised through modelling applications (Makar & Confrey, 2007). With the applications, the sample groups were made aware of mathematics subjects (Lesh & Yonn, 2007). As a result, as stated by Chinnappan (2010), by determining and using appropriate strategies, it was tried to obtain quality learning outcomes for mathematics lessons. In the study conducted by Zapata-Grajales, Cano-Velásquez, and Villa-Ochoa in 2017, the real-life situation was examined by first defining the function and performing algebraic analysis, and then mathematisation was performed, and as a result, the relationship between geometric quantities was modelled and the relationship between the daily life situation was explained. In modelling studies, different methods were tried in technological or non-technological environments. With the methods applied, mathematical concepts were applied to real life situations. For this purpose, as in other studies, it was concluded that modelling activities created positive learning effects (Sokolowski, 2015).

In this study, conducting studies with sample groups at different grade levels and with different cultural backgrounds will enable students to examine the dynamics in geometry subjects with mathematical modelling in more detail and compare the results obtained. As classroom environments can be enriched with the use of dynamic software, students' mathematical actions can be explored with the guidance of these software. While modelling with digital tools, different student strategies can be examined more closely. As suggested by Morante and Vallejo (2012), technology-related information can be integrated into the curriculum in this context. In addition, research is needed to develop modelling competencies without the use of technology (Greefrath, Hertleif, & Siller, 2018). In order to make it easier for students to switch between the representations used in the solution process (for example, from graphical representation to algebraic representation), it is thought that learning environments where students can share their ideas in detail and receive instant feedback can also improve the modelling process. It is thought that more studies are needed to improve the content used for geometry subjects in the teaching and comprehension of mathematical modelling and the effect of mathematical modelling problems directly in the teaching of geometry subjects should be examined.

Conflict of Interest: The authors have not disclosed any potential conflicts of interest.

Funding: This study did not obtain any specific funding from public, commercial, or non-profit organizations.

CRediT Author Statement: The authors contributed equally to the article.

Ethical Statement: Ethical approval is not applicable, because this article does not contain any studies with human or animal subjects.

REFERENCES

Akyildiz, S. & Ahmed, K.H. (2021). An overview of qualitative research and focus group discussion. *Journal of Academic Research in Education*, 7(1), 1-15. DOI: 10.17985/ijare.866762.

Alisina, C. (2007). Less chalk, fewer words, less symbols ... more objects, more context, more actions. In W. Blum, P. Galbraith, H.W. Henn, & M. Niss (Eds.), *Modeling and applications in mathematics education: The 14th ICMI Study* (pp. 35–44). New York, NY: Springer.

Ang, K. C. (2010). *Teaching and learning mathematical modeling with technology*. In W. Yang, M. Majewski, T. Alwis, & W. P. Hew (Eds.), Proceedings of the Fifteenth Asian Technology Conference in Mathematics (pp. 1–11). USA: Mathematics and Technology.06.12.2023.http://atcm.mathandtech.org/ep2010/invited/3052010_18134.pdf

Blum, W. & Leiß, D. (2007). How do students and teachers deal with modeling problems? In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical modeling (ICTMA 12): Education, engineering, and economics* (222-231). Chichester: Horwood Publishing.

Bonotto, C. (2007). How to replace word problems with activities of realistic mathematical modeling. In W. Blum, P. Galbraith, H.W. Henn, & M. Niss (Eds.), *Modeling and applications in mathematics education: The 14th ICMI Study* (pp. 45-56). New York, NY: Springer.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modeling process. *Zentralblatt für Didaktik der Mathematik-ZDM*, 38 (2), 86-95.

Budinski, N. (2017). An Example of how Geogebra can be used as a tool for the stem. *The International Journal for Technology in Mathematics Education, 24*(3), 1–8. http://dx.doi.org/10.1564/tme_v24.3.07

Burns, R. (2000). *Introduction to research methods* (4th ed.). Melbourne: Longman.

Clements, D. H., (1997). "(Mis) Constructing constructivism". *Teaching Children Mathematics*, 14(4), 198–200.

Chinnappan, M. (2010). Cognitive load and modeling of an algebra problem. Mathematics Education Research Journal, 22(2), 8-23.

Cooper, H. M. (2010). *Research synthesis and meta-analysis: A step-by-step approach* (4th ed.). Sage.

Creswell, J. W., & Poth, C. N. (2016). Qualitative inquiry and research design: Choosing among five approaches. Sage publications.

Çevikbaş, M., Kaiser, G., & Schukajlow, S. (2022). A systematic literature review of the current discussion on mathematical modeling competencies: State-of-the-art developments in conceptualizing, measuring, and fostering. Educational Studies in Mathematics, 109, 1-32. https://doi.org/10.1007/s10649-021-10104

Dixon-Woods, M. (2011). Using framework-based synthesis for conducting reviews of qualitative studies. BMC Medicine, 9(39), 1–2.

Ekol, G. (2011). Understanding and promoting mathematical modeling competencies: an applied perspective. In G. Kaiser et al. (eds.), *Trends in Teaching and Learning of Mathematical Modelling*, International Perspectives on the Teaching and Learning of Mathematical Modelling, 57–64. DOI 10.1007/978-94-007-0910-2_7,

English, L.D. (2010). Young children's early modeling with data. *Mathematics Education Research Journal 22*(2), 24–47.

English, L.D., & Watters, J. (2004). Mathematical modeling with young children. *28th Conference of The International Group for the Psychology of Mathematics Education*, 335–342. Bergen, Norway: Bergen University College.

EPPI (2010). EPPI-Centre methods for conducting systematic reviews. London: EPPI.

E.W. Packel & S. Wagon, (1994). Animating calculus: Mathematica notebooks for the laboratory, W.H. Freeman, New York, 1994.

Fredriksen, H. (2020). Exploring realistic mathematics education in a flipped classroom context at the tertiary level. International Journal of Science and Mathematics Education, 19(2), 377-396.

Geiger, V., Faragher, R., Redmond, R., & Lowe, J. (2008). CAS enabled devices to be provocative agents in the process of mathematical modeling. In M. Goos, R. Brown, & K. Makar (Eds.), *Mathematics Education Research Group of Australasia Inc.* 219-226.

Geiger, V., Faragher, R., & Goos, M. (2010). CAS-enabled technologies as 'agents provocateurs' in teaching and learning mathematical modeling in secondary school classrooms. *Mathematics Education Research Journal, 22*(2), 48-68.

Girnat, B. & Eichler, A. (2011). Secondary teachers' beliefs on modeling in geometry and stochastics. In G. Kaiser et al. (eds.), *Trends in Teaching and Learning of Mathematical Modelling*, International Perspectives on the Teaching and Learning of Mathematical Modelling, 75-84. DOI 10.1007/978-94-007-0910-2_9

Greefrath, G., Hertleif, C., & Siller, H-S. (2018). Mathematical modeling with digital tools-a quantitative study on mathematizing with dynamic geometry software. ZDM *Mathematics Education, 50*(1-2), 233-244. https://doi.org/10.1007/s11858-018-0924-6

Greefrath, G. & Siller, H-S. (2018). GeoGebra as a tool in modeling processes. *Uses of Technology in Primary and Secondary Mathematics Education, ICME-13 Monographs,* 363-374. https://doi.org/10.1007/978-3-319-76575-4_21

Guerrero-Ortiz, C. & Camacho-Machín, M. (2022). We are characterizing tasks for teaching mathematics in dynamic geometry systems and modeling environments. *Mathematics 10* (1239), 1-20. https:// doi.org/10.3390/math10081239

Güneş, G. & Şahin, V. (2019). The algorithm of mathematical modeling for learning styles of preschool children. *Education, International Journal of Primary, Elementary and Early Years Education 3-13, 47*(3), 277-292, DOI:10.1080/03004279.2018.1430844

Haas, B., Lavicza, Z & Kreis, Y. (2023). Parent's experience in remote learning during COVID-19 with digital and physical-mathematical modeling. *Research and Practice in Technology Enhanced Learning 18*(13), 1-21.

Henn, H.W. (1998). Crash tests and the head injury criterion. *Teachıng Mathematıcs and Its Applıcatıons, 17*(4), 162-170.

Hıdıroğlu, Ç.N. & Bukova Güzel, E. (2016). The Conceptualization of the Mathematical Modelling Process in a Technology-Aided Environment. *International Journal of Technology in Mathematics Education, 24*(1), 17-37.

Jackiw, N. & Sinclair, N. (2009). Sounds and pictures: dynamism and dualism in Dynamic Geometry. ZDM Mathematics Education 41, 413–426. DOI 10.1007/s11858-009-0196-2

Jones, G., Langrall, C., Thornton, C., & Nisbet, S. (2002). Elementary school children's access to powerful mathematical ideas. In L. D. English (Ed), Handbook of International Research in Mathematics Education (pp. 113–142). Mahwah, NJ: Erlbaum.

Kaiser-Meßmer, G. (1986). *Anwendungen im mathematikunterricht (Applications in mathematics classrooms)*. Bad Salzdetfurth: Franzbecker.

Kalinec-Craig, C., Prasad, P.V., & Luna, C. (2019). Geometric transformations and Talavera tiles: a culturally responsive approach to teacher professional development and mathematics teaching. *Journal of Mathematics and the Arts, 13*(1-2), pp. 72-90. https://doi.org/10.1080/17513472.2018.1504491

Kemp, A., & Vidakovic, D. (2021). Ways secondary mathematics teachers apply definitions in Taxicab geometry for a real-life situation: Midset. *The Journal of Mathematical Behavior, 62*, 100848. https://doi.org/10.1016/j.jmathb.2021.100848

Latifi, M., Eseghir, A., Elmaroufi, A., Hattaf, K., & Achtaich, N. (2022). Modeling with differential equations and Geogebra in high school mathematics education. *Journal of Educational and Social Research, 12*(3), 47-61. DOI: https://doi.org/10.36941/jesr-2022-0065

Lee, N.H., Yeo, J.S.D., & Hong, S.E. (2014). A metacognitive-based instruction for primary four students to approach non-routine mathematical word problems. ZDM - the International Journal on Mathematics Education, 46(3), 465–480.

Lehrer, R., & Schauble, L. (2003). Origins and evolutions of model-based reasoning in mathematics and science. In R. A. Lesh & H. Doerr (Eds.), Beyond constructivism: A models and modeling perspective on teaching, learning, and problem-solving in mathematics education (pp. 59-70). Mahwah, NJ: Lawrence Erlbaum

Lesh, R. & Caylor, B. (2007). Introduction to the special issue: Modeling as application versus modeling as a way to create mathematics. *Int J Comput Math Learning, 12* (2007), 173–194. DOI 10.1007/s10758-007-9121-3

Lesh, R., & Doerr, H. M. (2003). Foundations of models and modeling perspective on three mathematics teaching, learning, and problem-solving. In R. Lesh & H. M. Doerr (Eds), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning and teaching*, (pp. 3–34). Mahwah, NJ: Lawrence Erlbaum.

Lesh, R., & Yoon, C. (2007). What is distinctive in (our views about) models and modeling perspectives on mathematics problem solving, learning, and teaching? In W. Blum, W. P. Galbraith, P., Henn, H.W. & Niss M. (Eds.), Modeling and applications in mathematics education: The 14th ICMI Study (pp. 161-170). New York, NY: Springer.

Lesh, R.A., & Zawojewski, J. (2007). Problem-solving and modeling. In F. Lester (Ed.), Second handbook of research on mathematics teaching and learning: A Project of the National Council of Teachers of Mathematics. Charlotte, NC: Information Age Publishing.

Liberati, A., Altman, D. G., Tetzlaff, J., Mulrow, C., Gøtzsche, P. C., Ioannidis, J. P. A., ... Moher, D. (2009). The PRISMA statement for reporting systematic reviews and meta-analyses of studies that evaluate health care interventions: *Explanation and elaboration. PLoS Medicine, 6*(7), e1000100. https://doi.org/10.1371/journal.pmed.1000100

Maaß, K. (2006). What are modeling competencies? *Zentralblatt für Didaktik der Mathematik [ZDM], 38*(2),113-142.

Makar, K., & Confrey, J. (2007). Moving the context of modeling to the forefront: Preservice.

Teachers' investigations of equity in testing. In W. Blum, P. Galbraith, H.W. Henn, M. Niss (Eds.), *Modeling and applications in mathematics education: The 14th ICMI Study* (pp. 485-490). New York, NY: Springer.

Miles, M. B. & Huberman, A. M. (2015). Qualitative data analysis (Translation from Second Edition). (Translated by S. Akbaba-Altun and A. Ersoy). Ankara: Pegem A Akademi Publications.

Moher, D., Shamseer, L., Clarke, M., Ghersi, D., Liberati, A., Petticrew, M., Shekelle, P., Stewart, L. A. and PRISMAP Group (2015). Preferred reporting items for systematic review and meta-analysis protocols (PRISMA-P) 2015 statement. Systematic Reviews, 4(1), 1-9.

Morante, A. & Vallejo, J.A. (2012). Modeling the landing of a plane in a calculus lab. *International Journal of Mathematical Education in Science and Technology, 43* (7), 936 949, DOI: 10.1080/0020739X.2011.633626

National Council of Teachers of Mathematics NCTM (2020). Standards for School Mathematics Teacher Preparation.

Niss, M., Blum, W., & Galbraith, P. (2007). Introduction. In W. Blum, P. Galbraith, H.W. Henn,& M. Niss (Eds.), *Modeling and applications in mathematics education: The 14th ICMI Study* (pp. 3-33). New York, NY: Springer.

OECD (2023). *PISA 2022 Results (Volume I): The State of Learning and Equity in Education*, PISA, OECD Publishing, Paris, https://doi.org/10.1787/53f23881-en.

OECD (2023), *PISA 2022 Results (Volume II): Learning During - and From - Disruption*, PISA, OECD Publishing, Paris, https://doi.org/10.1787/a97db61c-en.

Ortiz, G. & Machin, M. (2022). Characterizing tasks for teaching mathematics in dynamic geometry systems and modeling environments. *Mathematics*, 10(8), 1239. https://doi.org/10.3390/math10081239

Panadero, E. (2017). A review of self-regulated learning: Six models and four directions for research. Frontiers in Psychology, 8(422). http://dx.doi.org/10. 3389/fpsyg.2017.00422.

Passarella, S. (2022). Supporting emergent modeling by implementing model eliciting activities: the case of 3D-Euclidean geometry. *Research in Mathematics Education, 24*(3), 347-366, DOI:10.1080/14794802.2021.1994869

Rellensmann, J., Schukajlow, S., & Leopold, C. (2020). Measuring and investigating strategic knowledge about drawing to solve geometry modeling problems. *ZDM Mathematics Education, 52*(1), 97-100. https://doi.org/10.1007/s11858-019-01085-1

Rellensmann, J., Schukajlow, S., Blomberg, J., & Leopold, C. (2023). Does strategic knowledge matter? Effects of strategic knowledge about drawing on students' modeling competencies in geometry. *Mathematical Thinking and Learning, 25*(3), 296-316, DOI: 10.1080/10986065.2021.2012741

Rincón Flores, E.G., Montoya, M.S.R., & Mena, J. (2016). Challenge-based gamification and its impact on teaching mathematical modeling. In Proceedings of the Fourth International Conference on Technological Ecosystems for Enhancing Multiculturality, Salamanca, Spain, November 2, 2016; pp. 771-776. DOI: http://dx.doi.org/10.1145/3012430.3012605

Rusiman, M. S., Hau, O. C., Abdullah, A. W., Sufahanı, S. F., & Azmı, N. A. (2017). An analysis of time series for the prediction of Barramundi (Ikan Siakap). *Price in Malaysia. Far East Journal of Mathematical Sciences, 102* (9), 2081-2093.

Segaran, J.S. & Kwan Eu, L. (2022). I am exploring initial mathematical modeling process using Geogebra of form four students. *Malaysıan Onlıne Journal Of Educatıonal Sciences, 10*(3), 47-59.

Sevinc, S. & Lesh, R. (2018). Training mathematics teachers for realistic math problems: A case of modeling based teacher education courses. *ZDM Mathematics Education, 50*, 301-314. https://doi.org/10.1007/s11858-017-0898-9

Siregar, R.N., Suryadi, D., Prabawanto, S., & Mujib, A. (2023). We are increasing student's self-regulated learning through a realistic mathematical education. *Jurnal Program Studi Pendidikan Matematika, 12*(1), 1197-1212

Sokolowski, A. (2015). The Effects of Mathematical Modeling on Students' Achievement-Meta-Analysis of Research. *The IAFOR Journal of Education, 3*(1), 93-114.

Sriraman, B. (2005). Conceptualizing the Notion of Model Eliciting. Proceedings of the Fourth Congress of the European Society for Research in Mathematics Education. Sant Feliu de Guíxols, Spain.

Stillman, G. (1998). Engagement with task context of applications task: student performance and teacher beliefs. *Nordisk Matematikk Didaktikk Nordic Studies in Mathematics Education, 6*(3/4), 51-70.

Stillman, G., Brown, J., & Galbraith, P. (2013). Identifying challenges within transition phases of mathematical modeling activities at year 9. In R. Lesh et al. (eds.), *Modeling students' mathematical modeling competencies*, International perspectives on the teaching and learning mathematical modeling, pp. 385–397. DOI 10.1007/978-94-007-6271-8_33

Susanto, N. C. P., Hartati, S. J., & Standsyah, R. E. (2023). Systematic literature review: application of dynamic geometry software to improve mathematical problem-solving skills. *Mathline: Jurnal Matematika dan Pendidikan Matematika, 8*(3), 857-872.

Tezer, M. & Cumhur, M. (2017). Mathematics through the 5E instructional model and mathematical modeling: The geometrical objects. EURASIA Journal of Mathematics Science and Technology Education, 13(8):4789-4804. DOI: 10.12973/eurasia.2017.00965a

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. (2012). Elementary and middle school mathematics: Teaching developmentally (8th ed.). Boston, MA: Allyn and Bacon.

Wess, R., Klock, H., Siller, H.S. & Greefrath, G. (2021). Mathematical modeling. In R. Wess, H. Klock, H.S. Siller and H. Greefrath (Eds.), International perspectives on the teaching and learning of mathematical modeling: measuring professional competence for the teaching of mathematical modeling (pp: 3-17). Switzerland AG, Springer. https://doi.org/10.1007/978-3-030-78071-5_1

Yohannes, A. & Chen, H. L. (2023). Geogebra in mathematics education: a systematic review of journal articles published from 2010 to 2020. *Interactive Learning Environments*, 31(9), 5682–5697. https://doi.org/10.1080/10494820.2021.2016861

Zapata-Grajales, F.N., Cano-Velásquez, N.A & Villa-Ochoa, J.A. (2017). Art and geometry of plants: experience in mathematical modeling through projects. *EURASIA Journal of Mathematics, Science and Technology Education, 14*(2), 585-603. DOI:10.12973/ejmste/76958

Zawojewski, J.S., Lesh, R., & English, L. (2003). A model and modeling perspective on the role of small group learning activities. In R. Lesh and H.M. Doerr (Eds.), Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching (pp. 337-358). Mahwah, NJ: Lawrence Erlbaum Associates.

ATTACHMENTS

ANNEX 1: Publication Information

The studies were coded in the order of Web of Science, Scopus, and ERIC database. While coding, a random coding was made (articles were coded as J, book chapters as B, and conference presentations as C).

Database	Publication Name	Year of Publication	Authors	Publication Code
Web of Science	An Example of how Geogebra can be used as a tool for STEM	**2017**	Natalija Budinski	**J1**
Web of Science	Art and Geometry of plants: Experience in mathematical modeling through projects	**2017**	Fabio Nelson Zapata-Grajales, Natalia Andrea Cano-Velásquez, Jhony Alexander Villa-Ochoa	**J2**
Web of Science	Does strategic knowledge matter? Effects of strategic knowledge about drawing on students' modeling competencies in the domain of geometry	**2023**	Johanna Rellensmann, Stanislaw Schukajlow, Judith Blomberg, Claudia Leopold	**J3**
Web of Science	Modeling the landing of a plane in a calculus lab	**2012**	Antonio Morante, José A. Vallejo	**J4**
Web of Science	Sounds and pictures: dynamism and dualism in Dynamic Geometry	**2009**	Nicholas Jackiw, Nathalie Sinclair	**J5**
Web of Science	Mathematical modeling with digital tools-a quantitative study on mathematising with dynamic geometry software	**2018**	Gilbert Greefrath, Corinna Hertleif, Hans-Stefan Siller	**J6**
Web of Science	Measuring and investigating strategic knowledge about drawing to solve geometry modeling problems	**2019**	Johanna Rellensmann, Stanislaw Schukajlow, Claudia Leopold	**J7**
Web of Science	The algorithm of mathematical modeling for learning styles of preschool children	**2019**	Gökhan Güneş, Volkan Şahin	**J8**
Web of Science	GeoGebra as a tool in modeling processes	**2018**	Gilbert Greefrath and Hans, Stefan Siller	**B1**
Scopus	Challenge-based gamification and its impact on teaching mathematical modeling	**2016**	Elvira G. Rincón Flores, María Soledad Ramírez Montoya, Juanjo Mena	**J9**
Scopus	CAS-enabled technologies as 'agents provocateurs' in teaching and learning mathematical modeling in secondary school classrooms	**2010**	Vince Geiger, Rhonda Faragher, Merrilyn Goos	**J10**
Scopus	Characterizing tasks for teaching mathematics in dynamic geometry systems and modeling environments	**2022**	Carolina Guerrero-Ortiz, Matías Camacho-Machín	**J11**
Scopus	Geometric transformations and Talavera tiles: A culturally responsive approach to teacher professional development and mathematics teaching	**2019**	Crystal Kalinec-Craig, Priya V. Prasad, Carolyn Luna	**J12**
Scopus	Introduction to the special issue: Modeling as an application versus modeling as a way to create mathematics	**2007**	Richard Lesh, Beth Caylor	**J13**
Scopus	Mathematics through the 5E instructional model and mathematical modeling: The geometrical objects	**2017**	Murat Tezer Meryem Cumhur	**J14**

Scopus	Modeling with differential equations and Geogebra in high school mathematics education	**2022**	Mohamed Latifi, Abdelhak Eseghir, Abdelouahed Elmaroufi, Khalid Hattaf, Naceur Achtaich	**J15**
Scopus	Parent's experience in remote learning during COVID-19 with digital and physical-mathematical modeling	**2023**	Ben Haas, Zsolt Lavicza, Yves Kreis	**J16**
Scopus	Supporting emergent modeling by implementing model eliciting activities: the case of 3D-Euclidean geometry	**2022**	Simone Passarella	**J17**
Scopus	Identifying challenges within transition phases of mathematical modeling activities at year	**2013**	Gloria Stillman, Jill Brown, Peter Galbraith	**C1**
Scopus	Secondary teachers' beliefs on modeling in geometry and stochastics	**2011**	Boris Girnat, Andreas Eichler	**B2**
Scopus	Understanding and promoting mathematical modeling competencies: an applied perspective	**2011**	George Ekol	**B3**
ERIC	The Conceptualization of the Mathematical modeling process in a Technology-aided Environment	**2016**	Çağlar Naci Hıdıroğlu, Esra Bukova Güzel	**J18**
ERIC	Exploring initial mathematical modeling process using Geogebra of form four students	**2022**	Jeevalata Sathera Segaran , Leong Kwan Eu	**J19**
ERIC	CAS-enabled devices as provocative agents in the process of mathematical modeling	**2008**	Vince Geiger, Rhonda Faragher, Trevor Redmond, Jim Lowe	**C2**